COSMOLOGY II Homework 1 2025

Due on November 3. The exercises are returned on the course Moodle page. You will get help for the exercises in the exercise sessions on Wed at 16-18. The help sheet at the bottom of the course webpage may come in handy. It will also be available in the course exam.

1. **The oldness problem.** Assume that the universe contains only radiation (and has spatial curvature but no cosmological constant) and $g_* = 100$. Suppose that close to the Planck density, $\rho = 10^{-4} M_{\rm Pl}^4$, the density parameter was $\Omega = \Omega_r = 0.9$.

   a) What is the age of the universe when Ω becomes smaller than 10^{-2}?
   b) What is the age of the universe when the temperature falls below $T = 2.7$ K?

2. **Free massless scalar field.** Consider a spatially flat universe dominated by a homogeneous scalar field with $V(\phi) = 0$, but $\dot{\phi} \neq 0$. (Again, no cosmological constant.) Find the scale factor and the scalar field as a function of time.

3. **Slow-roll parameters.**

   a) Demonstrate that $\varepsilon(\varphi) \ll 1$ and $|\eta(\varphi)| \ll 1$ are necessary conditions for the slow-roll approximation to hold.

   b) Explain why these conditions are not sufficient.

4. **E-folds of inflation.**

   a) Derive eq. (7.46) of the lecture notes.

   b) Consider the inflation model with with the potential $V(\varphi) = \frac{1}{2} m^2 \varphi^2$. Assume that $m = 2 \times 10^{13}$ GeV. Take inflation to end when $\varepsilon = 1$ and find the value of φ at the end of inflation. Then find the value of ε and η when $N = 50$. What scale k (in units of H_0^{-1}) does this value of N correspond to, assuming instant reheating?