Due on October 13 by 24.00. These are the last exercises.

1. **Neutrino chemical potential.** In the standard analysis, neutrino chemical potential is assumed to be negligible. Neutrino chemical potential affects BBN in two ways: 1) The energy density of neutrinos is larger, as shown in eq. (4.17), leading to faster expansion of the universe. 2) The chemical potential of the electron neutrino is reflected in the neutron and proton chemical potentials and thus in the n/p ratio. The chemical potential redshifts as the neutrino temperature. Define $\xi_i \equiv \mu_{\nu_i}/T_\nu$. Suppose that we have $\xi_e = 0.1$ (around the current upper limit), $\xi_\mu = \xi_\tau = 0$. Use a simplified nucleosynthesis model where the reactions $n+\nu_e \leftrightarrow p+e^-$, etc. are in equilibrium down to $T = 0.8$ MeV, when the neutrinos decouple instantaneously, after which neutrons decay (into protons) with a lifetime of 878 s, and nucleosynthesis, where all neutrons go into ^{4}He nuclei, happens instantaneously at $T = 70$ keV. Use the $T \propto t^{-1/2}$ expansion law, which applies after electron annihilation, for the whole period. How much ^{4}He is produced in this case? Is it more or less than in the standard ($\xi_e = 0$) case?

2. **WIMP miracle.** Consider a WIMP with a constant thermally averaged self-annihilation cross section times velocity, $\langle \sigma v \rangle = \sigma_0$, mass m and $g = 2$. Assume that the WIMP decouples instantaneously when $\Gamma = H$, that $m \gg T_\mathrm{d}$ and $g_*(T_\mathrm{d}) = g_{*S}(T_\mathrm{d}) = 100$, and that $\eta = 6 \times 10^{-10}$. Find the dark matter energy density today ρ_dm0 relative to the baryon energy density today ρ_b0, as a function of σ_0 and m. Give the numerical value for $\sigma_0 = 10^{-38}$ cm^2 and $m = 100$ GeV.

3. **Baryon-symmetric universe.** Consider a universe with zero baryon number. Use $g = 4$ (this includes protons and neutrons), $m_N = 0.94$ GeV, $\langle \sigma v \rangle = m_{\pi^0}^{-2}$ with $m_{\pi^0} = 0.135$ GeV and $g_*(T_d) = 10.75$.

   a) Find the decoupling temperature (i.e. the temperature where the reaction rate Γ = Hubble parameter).

   b) In analogy with the WIMP calculation above, find the present number density of nucleons plus antinucleons left over from annihilation, relative to photons. Compare it to the value of η in the real universre.