

Due on Monday September 29 by 24.00.

1. Assuming that $H_0 = 70$ km/s/Mpc, find the age of the universe when the expansion of the universe is completely dominated by a) radiation, b) matter, c) curvature and d) vacuum energy.
2. **Age of the positively curved universe.** Find the age-redshift relation for a positively curved universe ($K > 0$) with only matter (no radiation or vacuum energy). Calculate t_0 if $H_0 = 70$ km/s/Mpc and a) $\Omega_0 = 1.1$, b) $\Omega_0 = 2$.
3. **The Λ CDM model.** Suppose that we have $H_0 = 70$ km/s/Mpc, $\Omega_{m0} = 0.3$ and $\Omega_{\Lambda 0} = 0.7$, so that $\Omega = \Omega_m + \Omega_\Lambda = 1$ and the universe is spatially flat.
 - a) Find the age of the universe today and at redshift $z = 1090$.
 - b) When is the matter density equal to the vacuum energy density? (Give both t and z .)
 - c) The scale factor has an inflection point, where $\ddot{a} = 0$, at which the expansion starts to accelerate. When does this happen, in t and in z ?(Hint: Use the substitution $x^{3/2} = b \sinh \phi$ for the integral $\int \frac{x^{1/2} dx}{\sqrt{b^2 + x^3}}$.)