COSMOLOGY I & II Help sheet 2025

1 eV $=11600$ K $= 1.60\times10^{-19}$ J $= 5.07\times10^{6}$ m^{-1} $= 1.52\times10^{15}$ s^{-1} $= 1.78\times10^{-36}$ kg

$c = 1 = 2.9979\times10^{8}$ m/s

$\hbar = 1 = 197.327$ MeVfm

1 pc $= 3.09\times10^{16}$ m

1 a $= 3.156\times10^{7}$ s

$h \equiv H_0/(100\text{km/s/Mpc})$

$(100\ \text{km/s/Mpc})^{-1} = 9.78\times10^{9}$ a

$= 2998$ Mpc$=4.69\times10^{32}$ eV^{-1}

$T_0 = 2.725$ K $= 2.348\times10^{-4}$ eV

$T_{\nu0} = (4/11)^{1/3}T_0$

$z_{\rm dec} = 1090$

$m_e = 0.511$ MeV

$m_N = 938$ MeV

$B = m_p + m_e - m_H = 13.6$ eV

$\zeta(3) = 1.20206$

$G_{\rm F} = 1.17\times10^{-5}$ GeV^{-2}

$M_{\rm Pl} \equiv (8\pi G_{\rm N})^{-1/2} = 2.436\times10^{18}$ GeV

$g_n = g_p = g_e = 2, \qquad g_H = 4$

$Q = m_n - m_p = 1.293$ MeV

$g_*(T \ll m_e) = 3.363$

$g_{*S}(T \ll m_e) = 3.909$

$g_*(1\ \text{MeV}) = g_{*S}(1\ \text{MeV}) = 10.75$

$\tau_n = t_{1/2}/\ln 2 = 878$ s

$n + \nu_e \leftrightarrow p + e^-$

$\mu_e \ll T \qquad$ (when $T > 30$ keV)

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left(\frac{\mathrm{d}r^2}{1-Kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2\right)$$

$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

$$n_i = \left\{\begin{array}{c} 1 \\ 3/4 \end{array}\right\}\frac{\zeta(3)}{\pi^2}g_iT^3 \quad (T \gg m_i)$$

$$n_i = g_i\left(\frac{m_iT}{2\pi}\right)^{3/2}e^{-\frac{m_i-\mu_i}{T}} \quad (T \ll m_i)$$

$$3\frac{\dot{a}^2}{a^2} + 3\frac{K}{a^2} = 8\pi G_{\rm N}\rho + \Lambda$$

$$3\frac{\ddot{a}}{a} = -4\pi G_{\rm N}(\rho + 3p) + \Lambda$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

$$\rho = \frac{\pi^2}{30}g_*T^4$$

$$s = \frac{2\pi^2}{45}g_{*S}T^3$$

$$g_*^{\ 1/2}tT^2 = 1.51M_{\rm Pl}$$

$$n - \bar{n} = \frac{gT^3}{6\pi^2}\left[\pi^2\frac{\mu}{T} + \left(\frac{\mu}{T}\right)^3\right] \quad (T \gg m)$$

$$n - \bar{n} = 2g\left(\frac{mT}{2\pi}\right)^{3/2}e^{-m/T}\sinh\frac{\mu}{T} \quad (T \ll m)$$

$$\Gamma = n\langle\sigma v\rangle$$

$$\int\frac{dx}{\sqrt{1+x^2}} = \text{arsinh(x)}$$

$$\int_{-\infty}^{\infty}e^{-ax^2}dx = \sqrt{\frac{\pi}{a}}$$

$$\varepsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta = M_{\rm Pl}^2\frac{V''}{V}$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$$

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \quad p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

$$H^2 = \frac{V(\varphi)}{3M_{\rm Pl}^2}$$

$$3H\dot{\varphi} = -V'(\varphi)$$

$$\mathcal{R} = -H\frac{\delta\varphi}{\dot{\varphi}}$$

$$\mathcal{P}_g(k) \equiv \left(\frac{L}{2\pi}\right)^3 4\pi k^3\langle|g_{\mathbf{k}}|^2\rangle$$

$$\left(\frac{2\pi}{L}\right)^3\sum_{\mathbf{k}} \to \int d^3k$$

$$\mathcal{P}_{\delta\varphi}(k) = \left(\frac{H}{2\pi}\right)^2_{aH=k}$$

$$\mathcal{P}_{\mathcal{R}}(k) = \left(\frac{H}{\dot{\varphi}}\right)^2\left(\frac{H}{2\pi}\right)^2_{aH=k}$$

$$n(k) - 1 \equiv \frac{d\ln\mathcal{P}_{\mathcal{R}}}{d\ln k}$$

$$\left(\frac{\delta T}{T}\right)_{\rm obs} = \frac{1}{4}\delta_\gamma - \mathbf{v}\cdot\hat{\mathbf{n}} + \Phi(t_{\rm dec}, \mathbf{x}_{\rm ls}) + 2\int\dot{\Phi}dt$$

$$e^{i\mathbf{k}\cdot\mathbf{x}} = 4\pi\sum_{lm}i^lj_l(kx)Y_{lm}(\hat{\mathbf{x}})Y^*_{lm}(\hat{\mathbf{k}})$$

$$\ddot{\Phi}_{\boldsymbol{k}} + H(4+3c_s^2)\dot{\Phi}_{\boldsymbol{k}} + c_s^2\frac{k^2}{a^2}\Phi_{\boldsymbol{k}} + [2\dot{H} + (3+3c_s^2)H^2]\Phi_{\boldsymbol{k}} = 0$$

$$\delta_{\boldsymbol{k}} = -\frac{2}{3}\frac{k^2}{(aH)^2}\Phi_{\boldsymbol{k}} - 2\frac{1}{H}\dot{\Phi}_{\boldsymbol{k}} - 2\Phi_{\boldsymbol{k}}$$

$$\Phi_{\mathbf{k}} = -\frac{3+3w}{5+3w}\mathcal{R}_{\mathbf{k}}, \quad (w \equiv p/\rho)$$

$$\int d\Omega\ Y^*_{lm}(\theta,\phi)Y_{l'm'}(\theta,\phi) = \delta_{ll'}\delta_{mm'}$$

$$\sum_m\left|Y_{lm}(\theta,\phi)\right|^2 = \frac{2l+1}{4\pi}$$

$$\int_0^{\infty}\frac{dz}{z}j_l(z)^2 = \frac{1}{2l(l+1)}$$

Table 1: The particles in the Standard Model of particle physics

Quarks	t	172.57 ± 0.29 GeV	$\bar{t}$	spin $\frac{1}{2}$	$g = 2 \cdot 2 \cdot 3 = 12$	
	b	4.183 ± 0.007 GeV	$\bar{b}$	3 colours		
	c	1.273 ± 0.005 GeV	$\bar{c}$			
	s	93.5 ± 0.8 MeV	$\bar{s}$			
	d	4.70 ± 0.07 MeV	$\bar{d}$			
	u	2.16 ± 0.07 MeV	$\bar{u}$			
						72
Gluons	8 massless bosons			spin 1	$g = 2$	16
Leptons	τ^-	1776.93 ± 0.09 MeV	τ^+	spin $\frac{1}{2}$	$g = 2 \cdot 2 = 4$	
	μ^-	105.658 MeV	μ^+			
	e^-	510.999 keV	e^+			
						12
	ν_τ	< 0.12 eV	$\bar{\nu}_\tau$	spin $\frac{1}{2}$	$g = 2$	
	ν_μ	< 0.12 eV	$\bar{\nu}_\mu$			
	ν_e	< 0.12 eV	$\bar{\nu}_e$			
						6
Electroweak gauge bosons	$W^\pm$	80.3692 ± 0.0133 GeV		spin 1	$g = 3$	
	Z^0	91.1880 ± 0.0020 GeV				
	γ	0 $(< 1 \times 10^{-18}$ eV)			$g = 2$	
						11
Higgs boson	H^0	125.20 ± 0.11 GeV		spin 0	$g = 1$	1

$$g_f = 72 + 12 + 6 = 90$$
$$g_b = 16 + 11 + 1 = 28$$
$$g_* = \tfrac{7}{8} g_f + g_b = 106.75$$

History of $g_*(T)$

$T \sim 200$ GeV	all present	106.75	
$T < 170$ GeV	top annihilation	96.25	
$T \sim 160$ GeV	EW crossover	(no effect)	
$T < 125$ GeV	H^0	95.25	
$T < 80$ GeV	$W^\pm$, Z^0	86.25	
$T < 4$ GeV	bottom	75.75	
$T < 1$ GeV	charm, τ^-	61.75	
$T \sim 150$ MeV	QCD crossover	17.25	(u,d,g$\rightarrow \pi^{\pm,0}$, $37 \rightarrow 3$)
$T < 100$ MeV	$\pi^\pm$, π^0, μ^-	10.75	$e^\pm$, ν, $\bar{\nu}$, γ left
$T < 500$ keV	e^-	(7.25)	$2 + 5.25(4/11)^{4/3} = 3.36$