Departmental exam in "Statistical Methods" 19.1.2018

NB! Students are allowed to use a calculator as help when completing the exam!

1. Define following concepts (briefly):
   a) the central limit theorem (i.e. why are errors usually Gaussian) [2p]
   b) systematic errors (define also difference w.r.t. random errors) [2p]
   c) the maximum likelihood method (suurimman uskottavuuden menetelmä) [2p]
2. (a) Describe the Kolmogorov-Smirnov test. What can one conclude with such a test? When can it be used and what should one be aware of? [3p]
   (b) Describe neural networks. Argue why neural networks (at least in theory) is the most optimal way to distinguish 2 event classes from each other. [3p]
3. a) The weights of eggs produced by a farmer's hens have a standard deviation of 10 g. He feeds some hens a vitamin supplement, which is cost-effective if weights increase by 2 g. He measures 25 eggs from the vitamin-fed hens and average has increased by 3 g. Is increase significant? [2p]
   b) In a time-of-flight (TOF) detector, the time used by a charged particle to traverse the detector is measured. These measurements are then used for separating different types of particles having the same momentum using their rest mass differencies (special relativity applied). Suppose a test statistic w built from the TOF measurements follows a Gaussian distribution centered at 0.0(3.0) for pions(kaons), with a standard deviation equal to 1.5 for both hypotheses. Assume a kaon selection with a $w \geq 1.5$ requirement.
   (i) What is the kaon selection efficiency when requiring $w \geq 1.5$? [1p]
   (ii) The probability for a pion to be tagged as a kaon (i.e. $w \geq 1.5$)? [1p]
   (iii) What is the per centage of kaons in the subsample selected by $w \geq 1.5$ from a random particle sample consisting 95 % of pions and 5 % of kaons?[1p]
   (iv) What w requirement on the same sample would select ~50 % kaons?[1p]
4. a) Pekka's boss checks his workers twice per hour to make sure they work. Pekka is tired and needs a short nap. If he takes a 10 minute nap, what is the probability of getting away with it? How long could the nap maximally be for the probability of getting caught to be less than 1 in 10? [2p]
   b) A Breit-Wigner distribution describes the energy distribution of an unstable quantum state in e.g. atomic, molecular, nuclear or particle physics

$$dN/dE \quad \propto \quad \frac{(\Gamma_s)^2}{(E - E_s)^2 + (\Gamma_s/2)^2},$$

   where E_s and Γ_s are the mean energy of the state and the uncertainty on the mean energy of the state. Latter is related to the Heisenberg uncertainty principle and inversely proportional to the lifetime of the state. Describe in detail how to generate the E_s distribution of any quantum state in your Monte Carlo code using the inverse transform method [4p].
5. A group of physicists is trying to solve a longstanding debate between two famous physicists. Noble T. Menows model predicts a distribution in a measured variable x, labelled "Menow pred.", and Noble S. Mines model another distribution in x, labelled "Mine pred.". Table contains probability

that x is contained in a certain bin for the two models. The group obtains an x distribution, labelled "data (events)", after background removal.

bin	1	2	3	4	5	6	7	8	9	10
bin starts at	-5.0	-4.0	-3.0	-2.0	-1.0	0.0	1.0	2.0	3.0	4.0
bin ends at	-4.0	-3.0	-2.0	-1.0	0.0	1.0	2.0	3.0	4.0	5.0
data (events)	1	2	9	35	49	57	28	15	4	0
Menow pred. (%)	0.15	0.75	4.5	15.8	28.8	28.8	15.8	4.5	0.75	0.15
Mine pred. (%)	0.5	2.5	8.0	15.0	24.0	24.0	15.0	8.0	2.5	0.5

(i) Plot the two models normalized to data together with the data. Can any of the two models be excluded based on the data? Which model is more consistent with the data? Assume measurement uncertainty in a bin to be square root of the number of observed events in that bin. Hint: P-value = 0.05 corresponds to $\chi^2 > 16.9\,(18.3)$ for nine (ten) degrees-of-freedom. [3p]
(ii) What statistics (= number of events) is needed (on average) to exclude both models if the true distribution lays in between, i.e. is:

bin	1	2	3	4	5	6	7	8	9	10
true distrib. (%)	0.3	1.2	6.3	15.3	26.9	26.9	15.3	6.3	1.2	0.3

Plot the corresponding "average measurement" with statistical uncertainties together with the two models. How many years of data taking does that correspond to (assume a yearly rate of 200 events)? [3p]

6. A pendulum is used to determine the gravitational acceleration g. If the pendulum swings in a small angle, the time of one swing (the period) T depends only on the pendulum length L and g, $T = 2\pi\sqrt{\frac{L}{g}}$. Some students did measurements with 5 different pendulums. The results are shown below:

Length (cm)	85.0±1.0	75.0±0.8	70.0±0.7	60.0±0.6	50.0±0.5
Period (s)	1.850±0.011	1.737±0.011	1.678±0.011	1.554±0.011	1.419±0.011

(i) Determine the gravitational acceleration g and its uncertainty at the location where the experiment was made, neglecting the uncertainties of the pendelum length. What is the χ^2_{min} value? [2p]

(ii) Calculate the variance of g using the RCF-bound assuming no bias. [2p]

(iii) Based on (i) and (ii), do you suspect that the students altered measurements (or overestimated uncertainties) to improve the result? Motivate.[2p]
NB! The RCF bound: $(1 + (\partial b/\partial g))^2 \,/\, \mathrm{E}\left[(\partial^2\chi^2/\partial g^2)/2\right]$, where b = bias.

Table of values of the confidence level for different values for the inverse of the cumulative gaussian distribution:

central		one-sided	
$\Phi^{-1}(1-\gamma/2)$	$1-\gamma$	$\Phi^{-1}(1-\alpha)$	$1-\alpha$
1	0.6827	1	0.8413
2	0.9544	2	0.9772
3	0.9973	3	0.9987