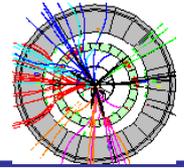


Beyond Standard Model

- ◆ **Dark matter: WIMPs & axions**
- ◆ **Energy scales & couplings**
- ◆ **Electroweak symmetry breaking**
- ◆ **Grand unified theories**
- ◆ **Supersymmetry**
- ◆ **Extra dimensional models**



The Standard Model is a very successful theory of particle interactions

- Electroweak interaction tested at 0.1-1 % level
- Basic strong interaction ingredients are confirmed, however perturbative calculations get very difficult at low Q^2 due to large α_s .
- SM describes well all observed phenomena upto now with the exception of ν oscillations
- Observed interactions are a dynamical consequence of symmetries ("gauge principle")

Is the Standard Model "the final theory"? **No, most probably!**

At least since SM doesn't include gravity!!

Gravity is extremely weak: $m_Z^2 / m_{Pl}^2 \approx 10^{-38}$. WHY?

Why the observed hierarchy of fermion masses ?

Why 3 families of fermions ?

Why this bizarre gauge group combination ?

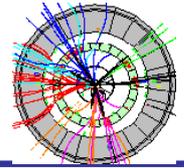
gauge group $\Rightarrow G = SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$

matter fermions \Rightarrow

$q_L =$	(2	,	1/3	,	3)
$u_R =$	(1	,	4/3	,	3)
$d_R =$	(1	,	-2/3	,	3)
$l_L =$	(2	,	-1	,	1)
$e_R =$	(1	,	-2	,	1)



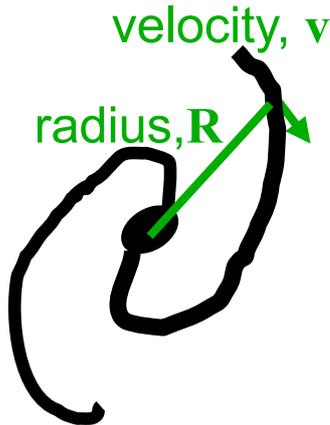
Dark matter



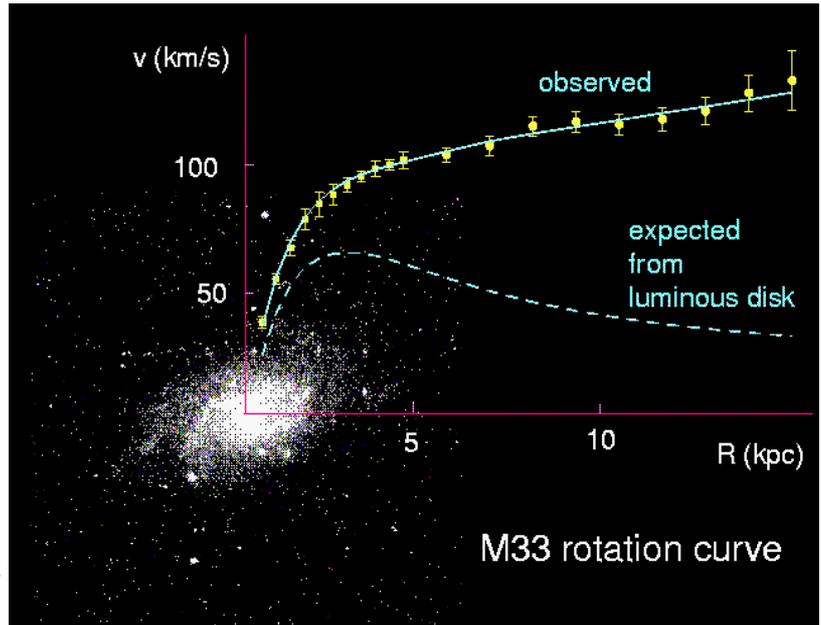
Possible indications of physics beyond SM ?

Dark (i.e. non-luminous & non EM radiation absorbing) **matter** in the universe see e.g. PDG review on dark matter

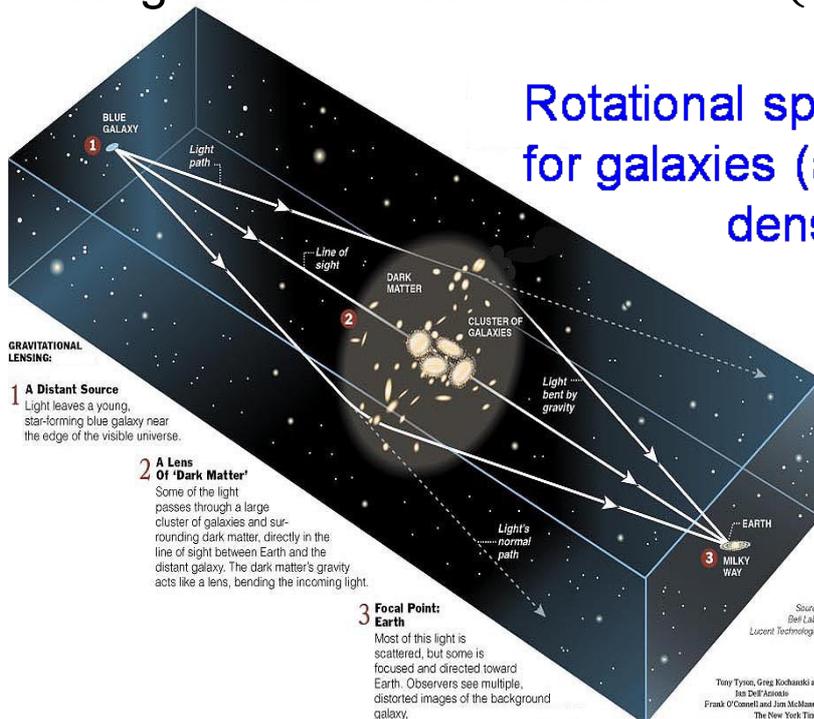
Seen e.g. in galaxy mass distribution



$v(R) \propto \sqrt{M(R)/R}$,
 $M(R)$ enclosed mass



Stars & gas predicts $v(R) \propto 1/\sqrt{R}$ but $v(R) \approx \text{const.}$ for most galaxies \Rightarrow dark halo with $M(R) \propto R$ or $\rho(R) \propto R^{-2}$

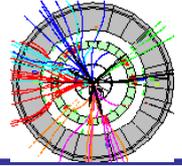


Rotational speed measurements for galaxies (above) and mass density measurements from gravitational lensing (left) indicate \Rightarrow

Luminous stars contain only small fraction of total galaxy mass



Dark matter & structure formation



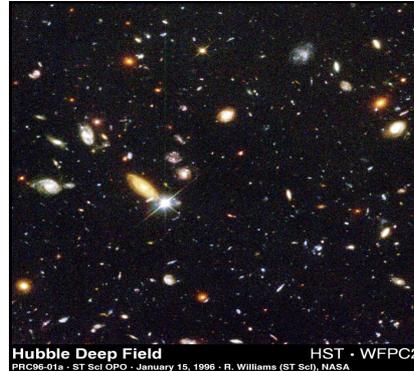
Big Bang



gravity



Present Structures



Dark matter dominates matter in our universe \Rightarrow governs structure formation
2 extreme forms of dark matter possible:
hot (relativistic) and cold (non-relativistic)

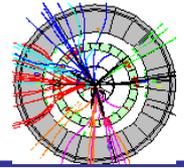
Relativistic particles escape from structure formation \Rightarrow galaxy formation indicate most dark matter cold: CDM

Dark matter must be stable on cosmological time scales, interact weakly with radiation (“electrically neutral”) & matter (no strong interaction) plus have right relic density

Baryonic candidates: primordial black holes e.g. Massive Compact Halo Objects (MACHOs) – not sufficient density, stranglets e.g. a uuddss-quark particle with mass $< 2m_{\Lambda_0}$.

Non-baryonic candidates: sterile singlet neutrino (ν mixing angle $\theta \ll 1$), dark photons (vector boson with mass $< 2m_e$ & only decay to 3γ possible), weakly interacting massive particles (WIMPs), axions – particle physics discoverables

An obvious WIMP would be a heavy neutrino but a SU(2) doublet neutrino ($m_\nu > m_Z/2$) gives too small relic density.
Historical candidate: lightest supersymmetric particle (LSP)

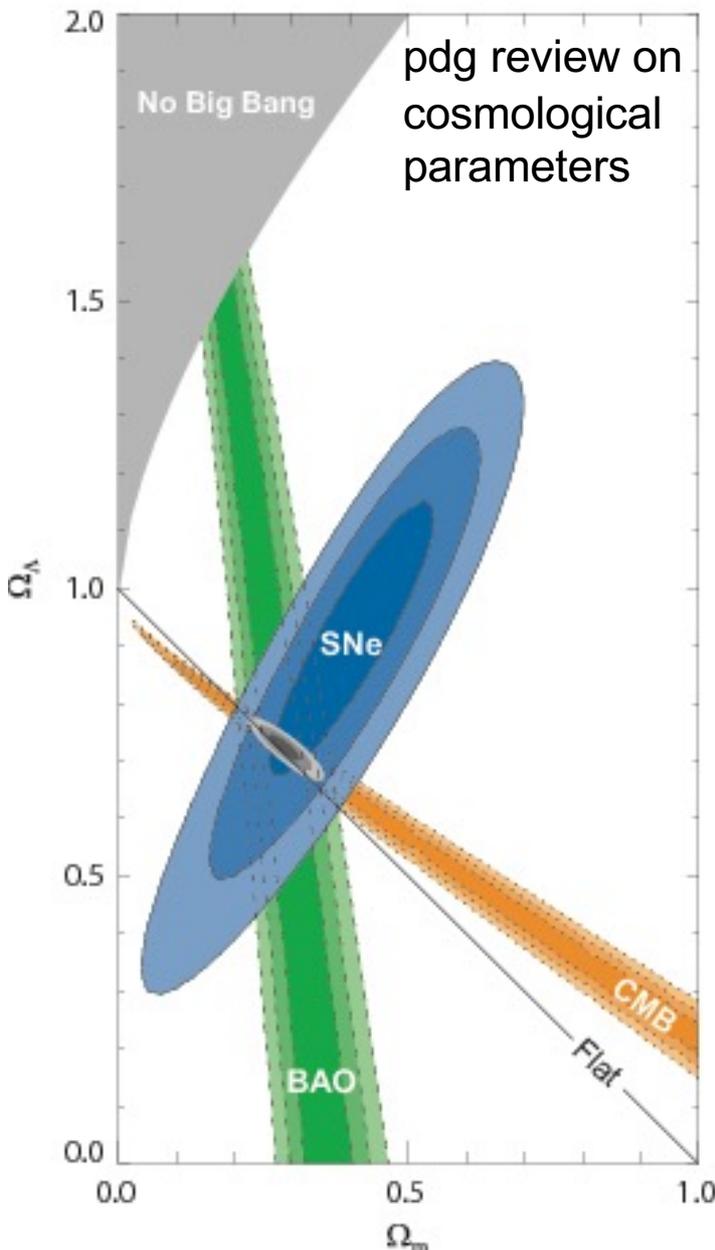


Supernova measurements (SNe):

measure brightness → distance: $B = L/4\pi d^2$
measure host galaxy redshift → recession velocity
test nonlinearity of Hubbles law at large distances

Cosmic microwave background (CMB):

measure size of CMB anisotropy (last baryon- γ scattering surface) → estimate of energy/matter density of universe



Galaxy clustering, baryonic acoustic oscillations (BAO):

measure galaxy clustering as "tracer" of dark matter distribution vs redshift → estimate of matter density

$$\Omega_x = \rho_x / \rho_{\text{critical}}$$

critical density

for flat universe

$$\rho_{\text{critical}} = 3H^2/8\pi G_N$$

$$H = h \cdot 100 \text{ km/s/Mpc}$$

CMB + lensing (Planck):

$$h = 0.674 \pm 0.005$$

$\Omega_{\text{tot}} = 1.011 \pm 0.006$ so agrees with flat universe

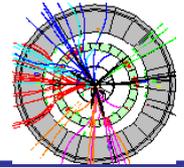
Cosmological constant:

$$\Omega_\Lambda = 0.685 \pm 0.007$$

$$\Omega_m = 0.315 \pm 0.007$$



Matter & energy in the universe



Dark energy (\equiv cosmological constant) constitutes the largest fraction of energy/matter in our universe !!

$$\Omega_{\text{total}} = \underbrace{\Omega_{\text{M}}}_{\text{matter}} + \underbrace{\Omega_{\Lambda}}_{\text{dark energy}} \sim 1$$

MATTER / ENERGY in the UNIVERSE

Matter:

$$\Omega_{\text{M}} = \underbrace{\Omega_{\text{b}}}_{\text{baryons}} + \underbrace{\Omega_{\text{v}}}_{\text{neutrinos}} + \underbrace{\Omega_{\text{CDM}}}_{\text{cold dark matter}} = 0.315 \pm 0.007$$

Baryonic matter :

$$\Omega_{\text{b}} = 0.0492 \pm 0.0008$$

stars, gas, brown & white dwarfs

Neutrinos:

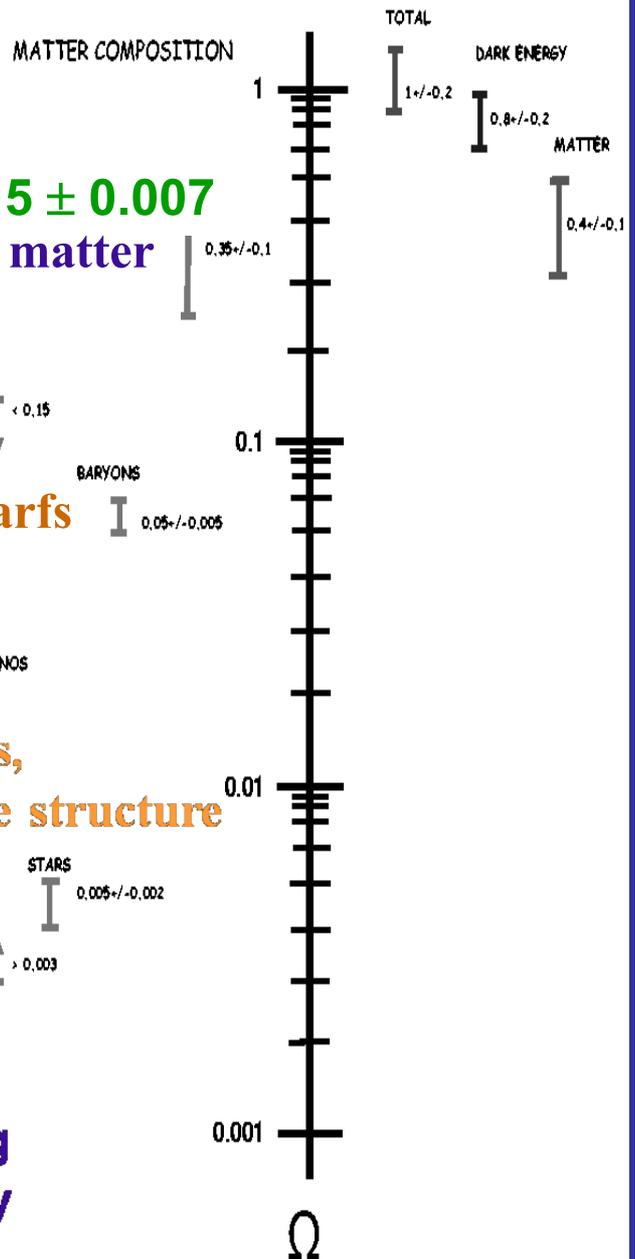
$$0.001 < \Omega_{\text{v}} < 0.004$$

lower bound from oscillations,
upper bound from large-scale structure

Cold Dark Matter :

$$\Omega_{\text{CDM}} \sim 0.264 \pm 0.005$$

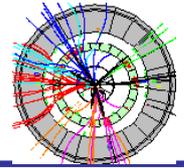
WIMPs (Weakly Interacting Massive Particles), usually neutralinos, or axions



NB! If Planck & BAO is combined, Ω_{Λ} & h (Ω_{CDM} & Ω_{M}) increase (decrease) slightly but overall picture looks similar.



DM generation



- Freeze-out (DM = dark matter, OM = ordinary matter):

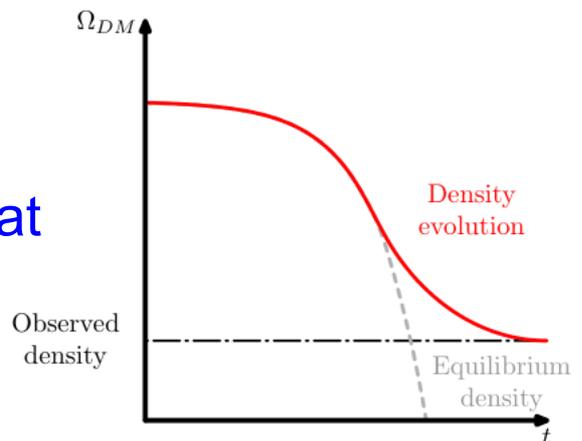
First equilibrium $DM + \overline{DM} \leftrightarrow OM + \overline{OM}$,

then $DM + \overline{DM} \rightarrow OM + \overline{OM}$

(when T of universe $> m_{DM}$),

finally expansion of space dilutes density of DM 's so that

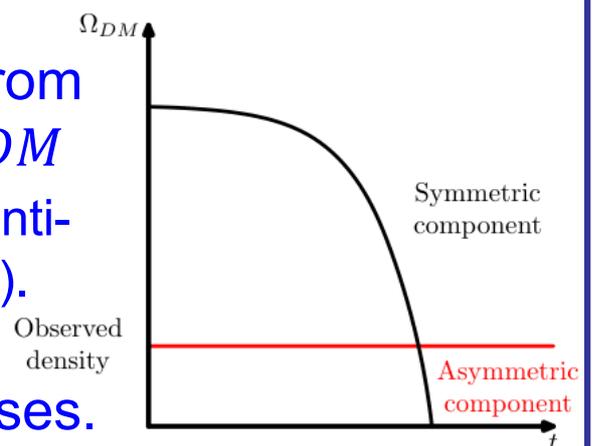
DM collisions happen too rarely and the total number of DM 's stops changing.



- Asymmetric DM:

Relic DM abundance arise from asymmetric probabilities of DM & \overline{DM} processes (\leftrightarrow baryon-anti-baryon asymmetry in universe?).

Relic density directly related to these CP violating processes.

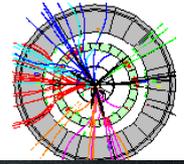


- Freeze-in:

Collision processes produce DM 's that from start are diluted so that they can gradually accumulate at cosmic times. E.g. the lightest observable-sector particle decays to DM with a relatively long lifetime.

- Non-thermal production:

DM production process is out of thermal equilibrium ("non-thermal"). E.g. via decay of "mother" particle, topological defects, moduli or gravitational effects.

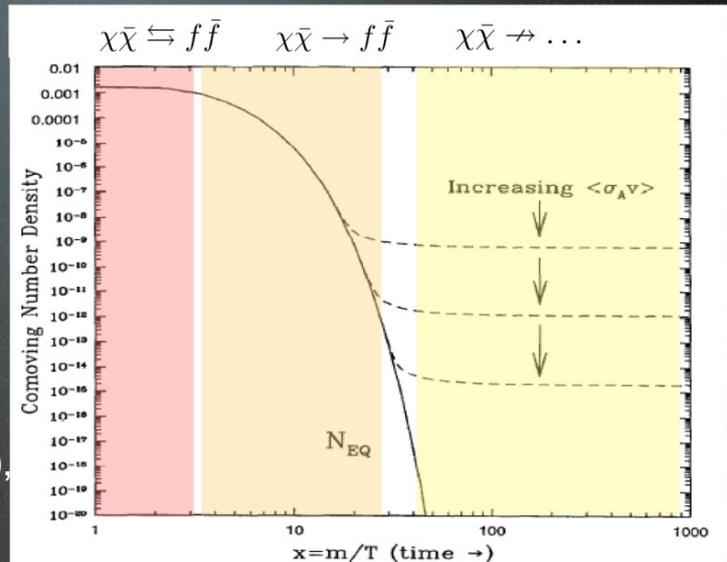


A thermal relic from the Early Universe

Consider a particle χ :

- subject to $\chi\bar{\chi} \rightarrow \dots$
- 'heavy' (e.g. 100 GeV)
- 'stable'
- in an expanding Universe
- symmetric abundance

"neutral", very long lived (life time \sim cosmological scale), weakly interacting particle, limited self-interactions



Kolb, Turner, The Early Universe, 1995

A thermal relic from the Early Universe

Boltzmann equation in the Early Universe:

$$\Omega_X \approx \frac{6 \cdot 10^{-27} \text{cm}^3 \text{s}^{-1}}{\langle\sigma_{\text{ann}} v\rangle}$$

Relic $\Omega_{\text{DM}} \simeq 0.23$ for

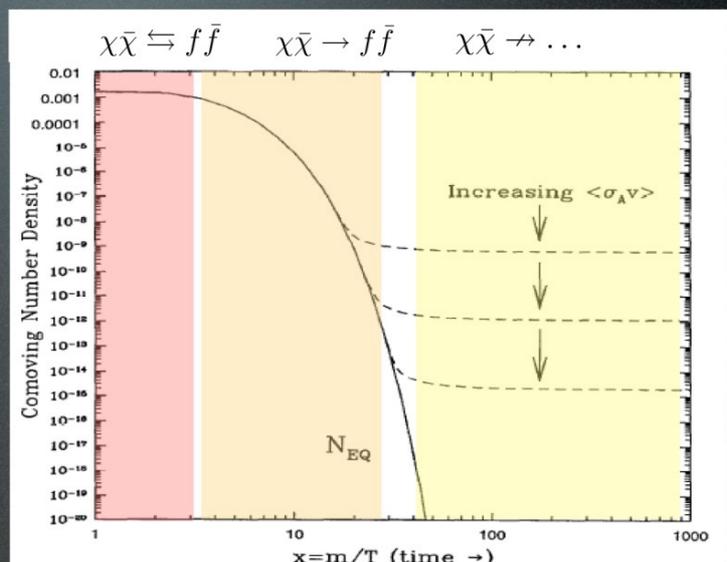
$$\langle\sigma_{\text{ann}} v\rangle = 3 \cdot 10^{-26} \text{cm}^3/\text{sec}$$

Weak cross section:



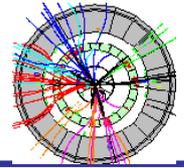
$$\langle\sigma_{\text{ann}} v\rangle \approx \frac{(g_w^2/4\pi)^2}{M^2} \approx 3 \cdot 10^{-26} \text{cm}^3/\text{sec}$$

WIMP miracle!



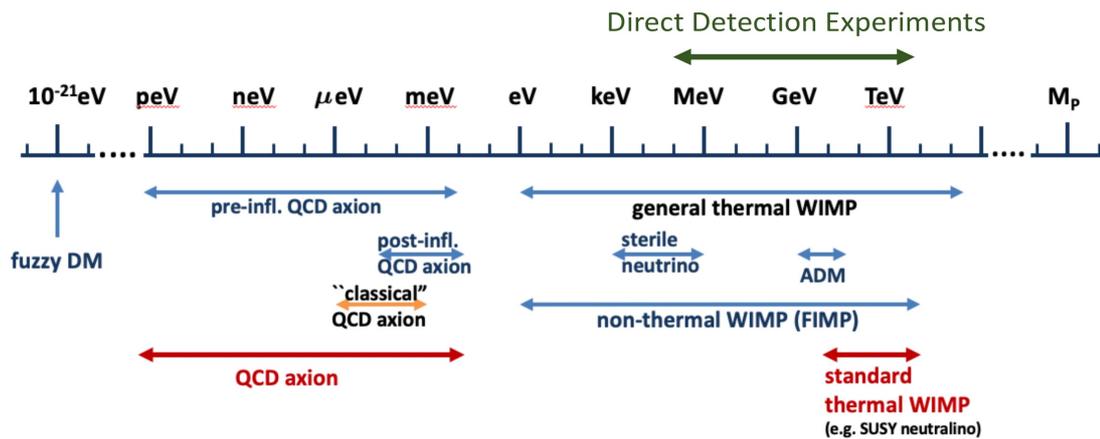


WIMP searches

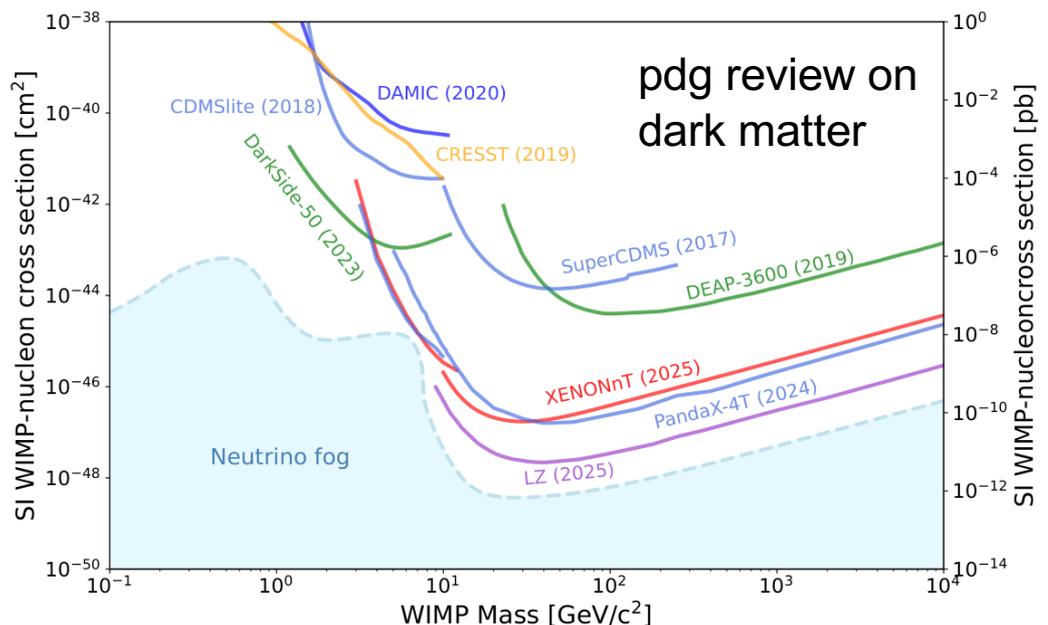


Searches for WIMPs (or other dark matter candidates):

- accelerator-based: (a) missing (transverse) momentum due to WIMP production; (b) excess or bump of jet or lepton pairs produced by a dark mediator
- direct detection of WIMPs from galactic WIMP halo in terrestrial detectors; (a) (in)elastic scattering off a target nucleus giving rise to measurable nuclear recoil (b) scattering off bound electrons (for WIMP masses $< \text{GeV}$ giving WIMP-nucleus scattering energy transfers below detection threshold) or absorption via “axioelectric” effect (for axions or dark photons, analogous to photoelectric effect)

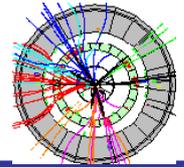


No WIMP signal observed \Rightarrow WIMP direct detection limits





Axions



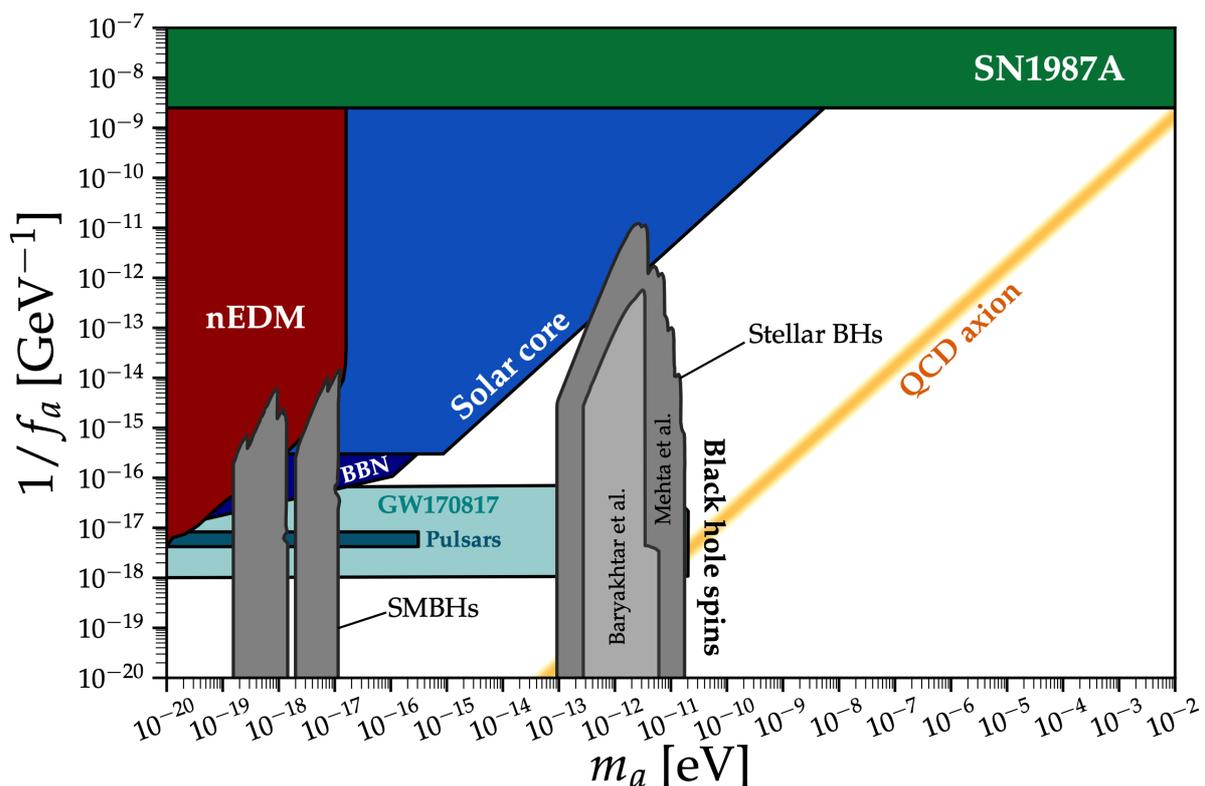
Axion

A very light neutral scalar boson, originally proposed by Peccei & Quinn: pseudo-Goldstone boson from a broken U(1) symmetry introduced to cure CP problem of QCD.

Motivation: CP violation in QCD not observed, stringent limits ($< 10^{-10}$) from neutron electric dipole moment. Axion field term in the QCD Lagrangian would compensate the CP violating term Θ_i . Coupling $\propto f_A^{-1}$ very small to matter due to the high scale of the U(1) symmetry breaking.

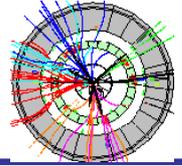
$$\mathcal{L}_{\text{QCD,CPviol}} = (\alpha_s / 8\pi) (\Theta_i - \phi_A / f_A) G^{\mu\nu a} \tilde{G}_{\mu\nu}^a \approx 0$$

Axion density contribution (pre-inflation U(1) symmetry breaking) $\Omega_A h^2 \sim 0.12 \cdot (6 \mu\text{eV} / m_A)^{1.165} \Rightarrow$ masses 10^{-5} to 10^{-3} eV most interesting as dark matter (assuming $\Theta_i \sim 1$ in axion potential); Axions can constitute CDM due to their non-thermal production.





Axions



axion would interact weakly with matter
(coupling $\propto f_A^{-1}$; very small if $f_A \gg v \rightarrow m_A \ll m_\pi$).

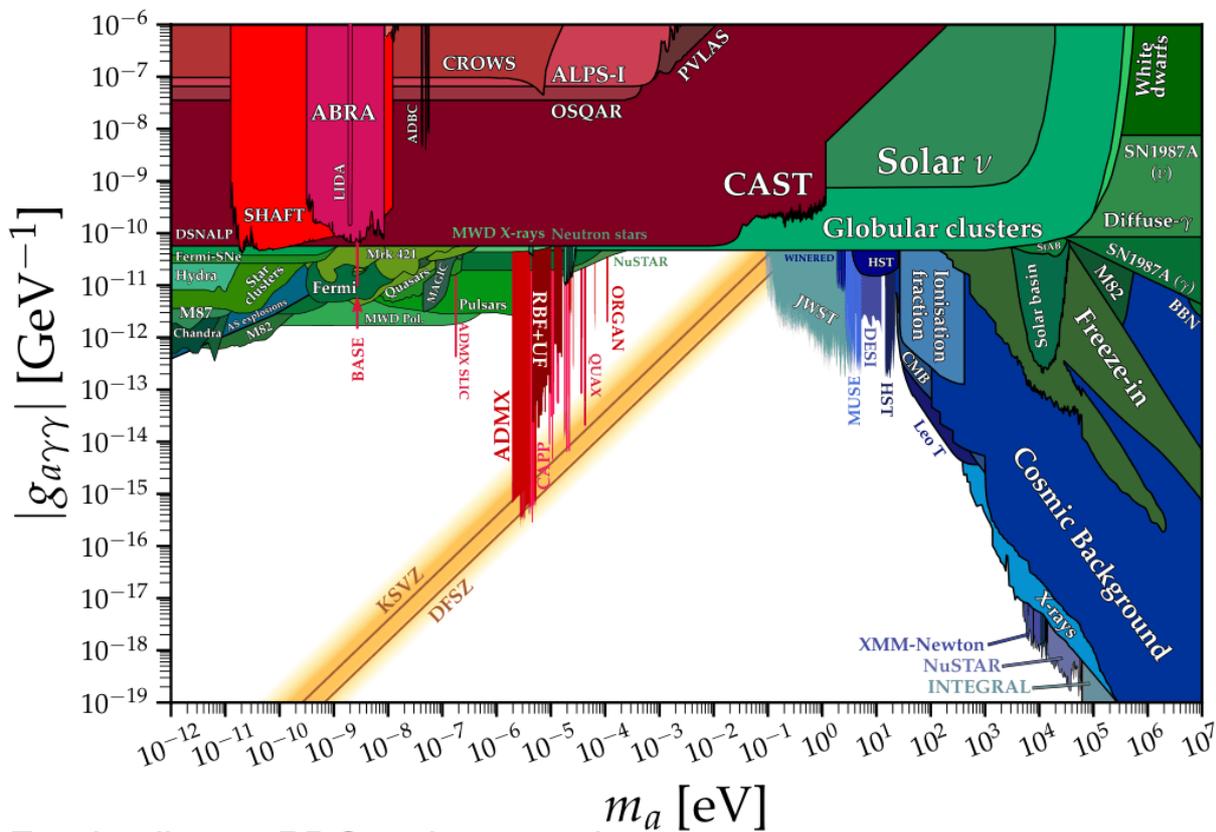
$$\mathcal{L}_{\text{QCD,CPviol}} \propto (\Theta_i - \phi_A / f_A) G^{\mu\nu a} \tilde{G}_{\mu\nu}^a \approx 0 \xrightarrow{\Theta_i \sim 1} m_A f_A \approx m_\pi f_\pi$$

Next-to-next-to-leading (NNLO) order correction in chiral perturbation theory gives: $m_A = 5.691 \left(\frac{10^9 \text{ GeV}}{f_A} \right) \text{ meV}$

Predicted decay $A \rightarrow \gamma\gamma$ in external E/B field presence
(coupling $g_{A\gamma\gamma}$ very model dependent).

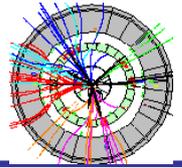
$$\mathcal{L}_{A\gamma\gamma} = (g_{A\gamma\gamma} / 4) F_{\mu\nu} \tilde{F}^{\mu\nu} \phi_A = -g_{A\gamma\gamma} \mathbf{E} \cdot \mathbf{B} \phi_A$$

Also very small fermion (i.e. electron) coupling g_{Aff} possible



For details see PDG review on axions:

<https://pdg.lbl.gov/2025/reviews/rpp2025-rev-axions.pdf>



Physical Energy Scales & Couplings

Ex// scalar field (particle) φ

- most general Lagrangian

$$\mathcal{L} = \partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2 + \lambda_3 \varphi^3 + \lambda_4 \varphi^4 + \lambda_5 \varphi^5 + \lambda_6 \varphi^6 + \dots$$

dimensions

$$\left. \begin{aligned} [\mathcal{L}] &= \frac{E}{L^3} = E^4 \\ [\partial_\mu] &= \frac{1}{L} = E \end{aligned} \right\} \Rightarrow [\varphi] = E$$

$$[m^2] = E^2$$

$$[\lambda_3] = E$$

couplings

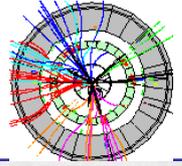
$$[\lambda_4] = E^0 \leftarrow \underline{\text{dimensionless}}$$

$$[\lambda_5] = E^{-1}$$

$$[\lambda_6] = E^{-2}$$



Energy scales & couplings



Energy dependence of amplitudes:



$$A(2 \rightarrow 2) \propto \lambda_4$$

$$\Rightarrow \sigma_{2 \rightarrow 2} \sim \frac{\lambda_4^2}{s}$$



$$A(2 \rightarrow 4) \sim \lambda_6^2$$

$$\Rightarrow \sigma_{2 \rightarrow 4} \sim \lambda_6^4 s$$

... more generally

$$\sigma \propto |A|^2 / F,$$

where $F \propto s$

$$[\lambda_i] = E^{d_i} \Rightarrow d_i = 4 - i \text{ (here)}$$

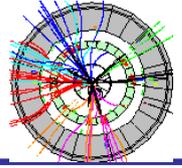
dimensionless quantity ruling perturbative expansion is

$$\bar{\lambda}_i = \lambda_i E^{-d_i}$$

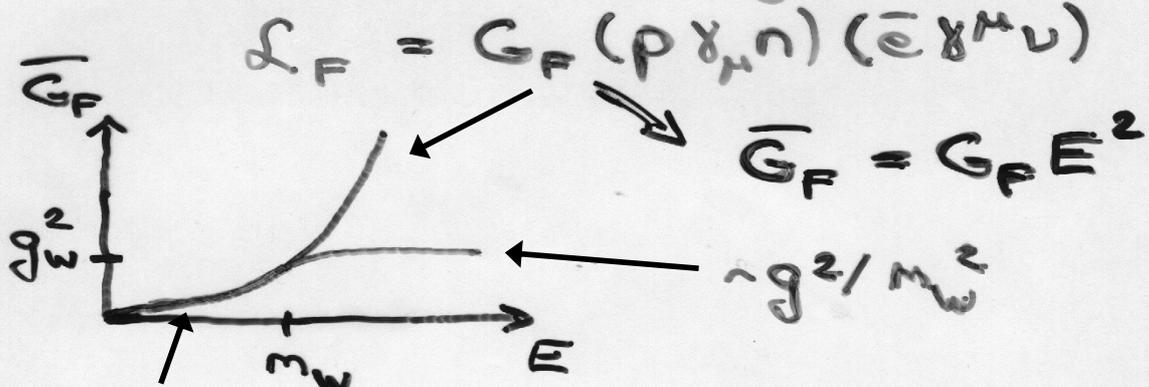
- weak coupling $\Leftrightarrow \bar{\lambda}_i \ll 1$
- * $d_i > 0 \Rightarrow$ relevant at small $E \ll \frac{E_{\text{UV}}}{m}$
- * $d_i = 0 \Rightarrow$ relevant at all E
- * $d_i < 0 \Rightarrow$ suppressed at small E
perturbative expansion breaks down at high E



Energy scales & couplings



Ex// Fermi Lagrangian



both work at low E

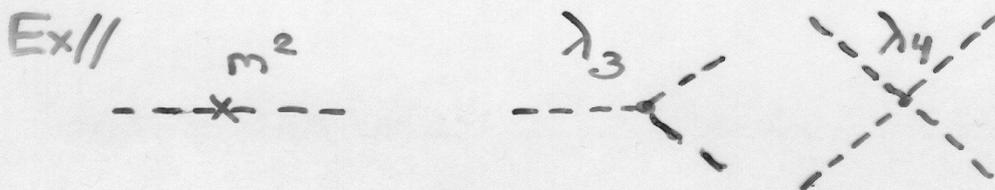
Imagine all couplings with $d_i < 0$ to scale like

$$\lambda_i \sim \Lambda^{d_i}$$

Ex// $\lambda_5 \sim \frac{1}{\Lambda}, \lambda_6 \sim \frac{1}{\Lambda^2} \dots$



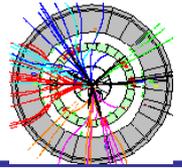
at $E \ll \Lambda$ the dynamics is accurately described by a finite set of couplings with $d_i \geq 0$



- $(m^2, \lambda_3, \lambda_4)$ fully describe an elementary (pointlike) particle...



Energy scales & couplings



- $\lambda_5, \lambda_6 \dots$ corresponds to inner structure
- to probe structure $E \sim \Lambda$ is needed
- * Ex// ψ is bound state of size $\sim \frac{1}{\Lambda}$

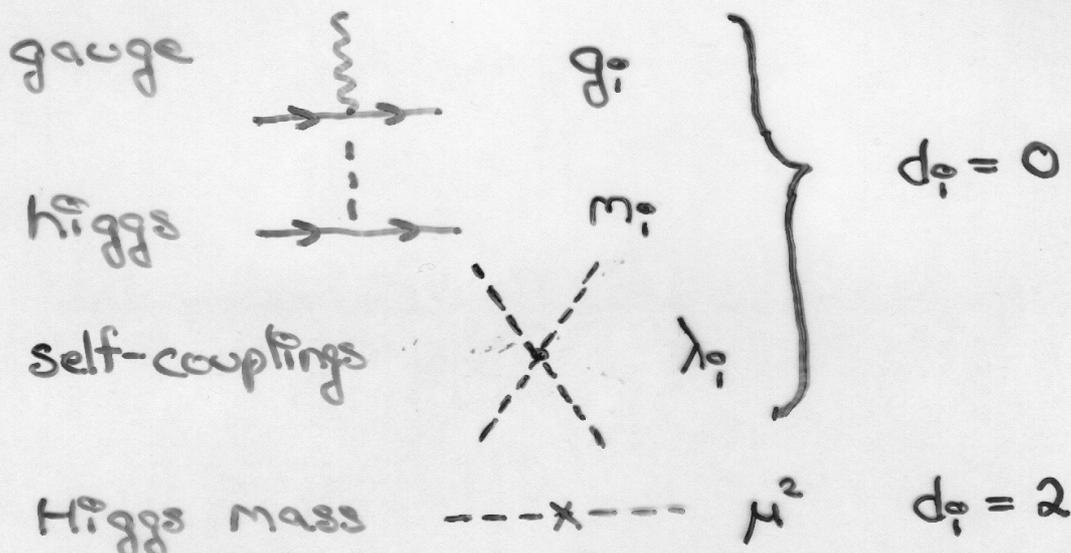
NB! Customary to assume New Physics couplings to be g_{SM}^n / Λ^n , where n depends on order of term

▲ $\lambda_i = 0$ for $d_i < 0 \Rightarrow$

Theory renormalizable (divergencies can be dealt with)

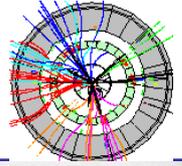
Physical meaning: particles have the minimal amount of internal structure

Standard Model couplings:





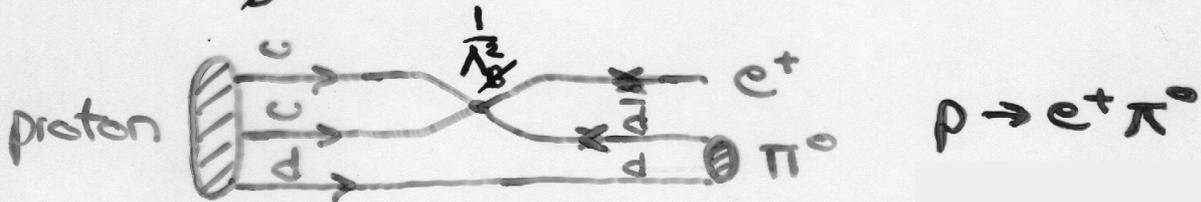
Energy scales & couplings



▣ Allowing $\delta_i < 0$ we would have

↳ B- & L-number violation

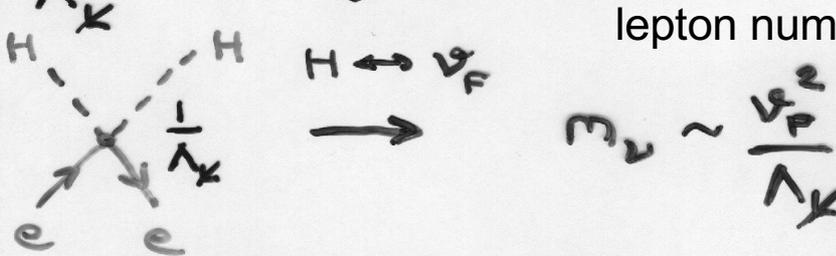
$$* \frac{1}{\Lambda_B^2} (\bar{u}_\alpha \gamma_\mu \nu_\beta) (\bar{e} \gamma^\mu d_\delta) \epsilon^{\alpha\beta\delta}$$



$$\tau_p \geq 24 \cdot 10^{33} \text{ years} \Rightarrow \Lambda_B \geq 10^{16} \text{ GeV}$$

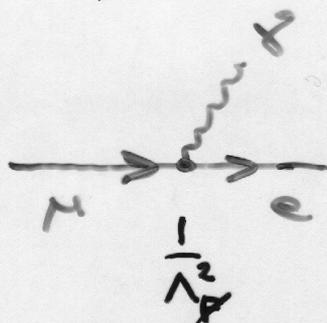
tiny neutrino mass can be generated with extended Higgs sector if allowing lepton number violation

$$* \frac{1}{\Lambda_\nu} (\bar{e}_i^A c e_j^B) H_A H_B$$



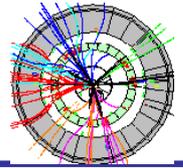
$$\nu\text{-oscillations: } m_\nu \sim 0.1 \text{ eV} \Rightarrow \Lambda_\nu \sim 10^{14} \text{ GeV}$$

$$* \frac{3}{\Lambda_F^2} (\bar{e} \gamma_\mu \nu_\mu) F^{\mu\nu}$$



$$\text{Br}(\mu \rightarrow e\gamma) < 3.1 \cdot 10^{-13}$$

$$\Rightarrow \Lambda_F \geq 10^7 \text{ GeV}$$



▲ It is tempting to conclude that the scale of "compositeness" Λ in the SM is extremely high ... but can we

- $\langle H \rangle \sim 125 \text{ GeV} \Rightarrow \mu^2 \sim (125 \text{ GeV})^2$
- can $|\mu|$ be $\ll \Lambda$
- have to consider quantum corrections leading:

$$M_{\text{eff}}^2 = -\frac{\mu^2}{\Lambda^2} + \text{[loop diagram with } \lambda_h \text{]} + \text{[loop diagram with } \lambda_t \text{]}$$

cut-off $\int d^4p$ at $p \sim \Lambda$

$$+ \frac{\lambda_h}{16\pi^2} \Lambda^2 \quad - \frac{\lambda_t}{16\pi^2} \Lambda^2$$

$$\Rightarrow M_{\text{eff}}^2 = \mu^2 + c\Lambda^2$$

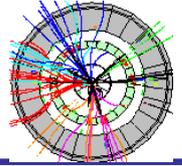
☐ M_{eff}^2 does not like to stay small when $\Lambda \rightarrow \infty$!!

large $\Lambda \Rightarrow \mu^2$ must be tuned to make M_{eff}^2 (fine-tuning $\sim 10^{-34}$)

This is the hierarchy problem!



Energy scales & couplings



$p \rightarrow e \pi^0$
 $\mu \rightarrow e \gamma$

$$\sim \frac{1}{\Lambda^2}$$

effective
Higgs mass

$$\sim \mu^2 + c\Lambda^2$$

2 possibilities

1) $\Lambda \gg \mu$ \rightarrow B, L conservation naturally follows
 \rightarrow separation of mass scales mystery

2) SM is not valid for energy $\geq \mu$:
it is replaced by more fundamental theory

In New Theory

- no Λ^2 corrections to Higgs mass
- must preserve as much as possible good features of SM

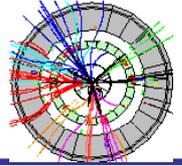
How solve Λ^2 corrections to Higgs mass?

additional loops cancel if $\lambda_t^2 = \lambda_{\tilde{t}}^2$

Need symmetry relating boson to fermions

SUPERSYMMETRY

Top quark



- top quark (t) discovered by the CDF & DØ experiments at the Tevatron in 1995 (SU(2)_L partner of the b quark)
- a most intriguing fermion : $m_{\text{top}} \approx 172.5 \text{ GeV}$ (heaviest known fundamental particle, $\times 40$ heavier than b quark) \rightarrow clues about origin of particle masses ($h_{\text{top}} = m_{\text{top}}/v_{\text{EW}} \sim 1$)
- top decays instantaneously and almost exclusively to W + b quark ($|V_{tb}| \sim 1$). $\Gamma(t \rightarrow Wb) \sim 1.4 \text{ GeV} \gg \Lambda_{\text{QCD}} \Rightarrow$
 - no hadronization (no toponium or T mesons)?
 - top decay purely an electroweak process (to first order)

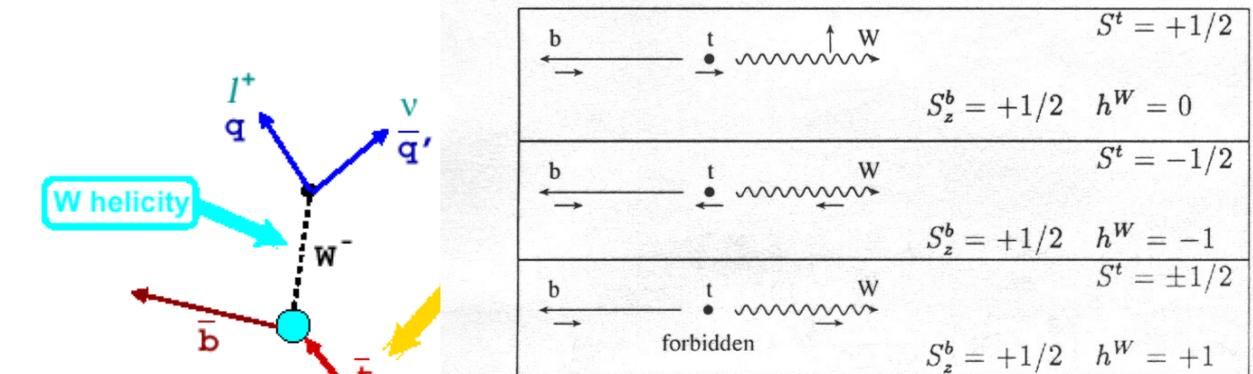
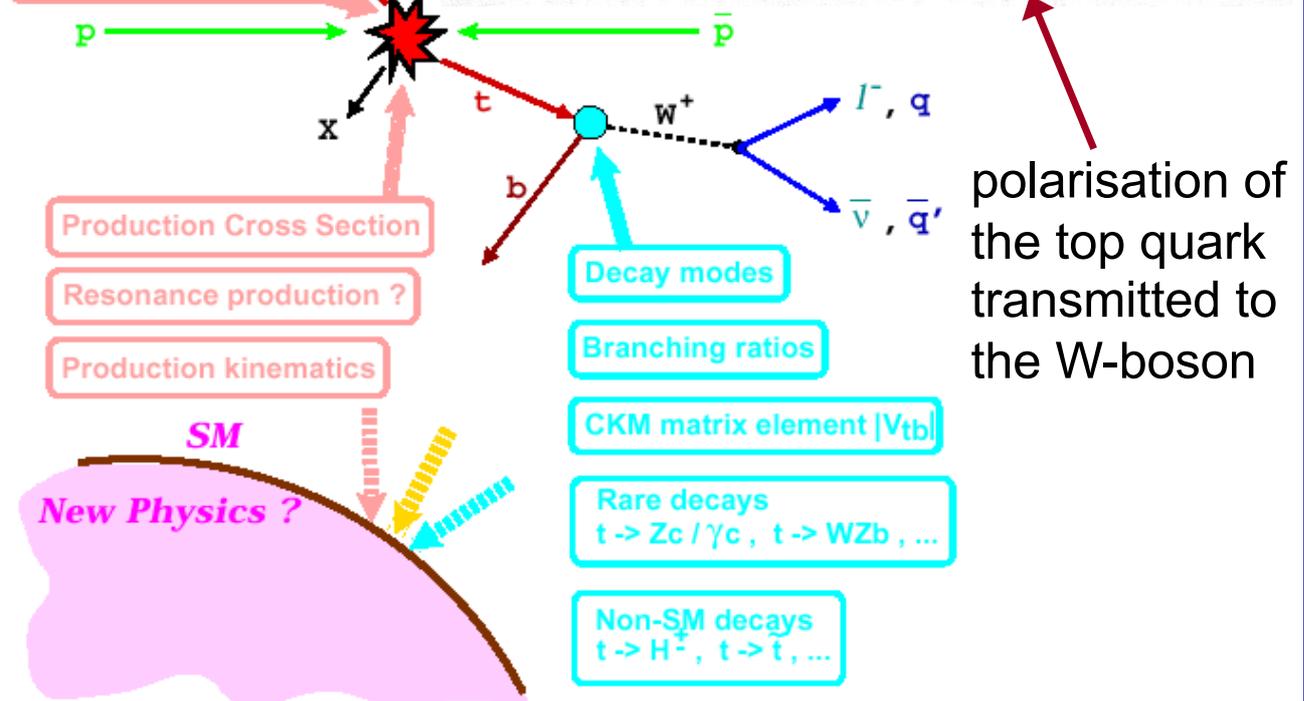
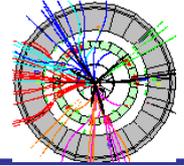


Figure 1.6: Top decays: angular momentum conservation





Electroweak symmetry breaking



In Standard Model we have the Higgs mechanism to explain the masses of W & Z

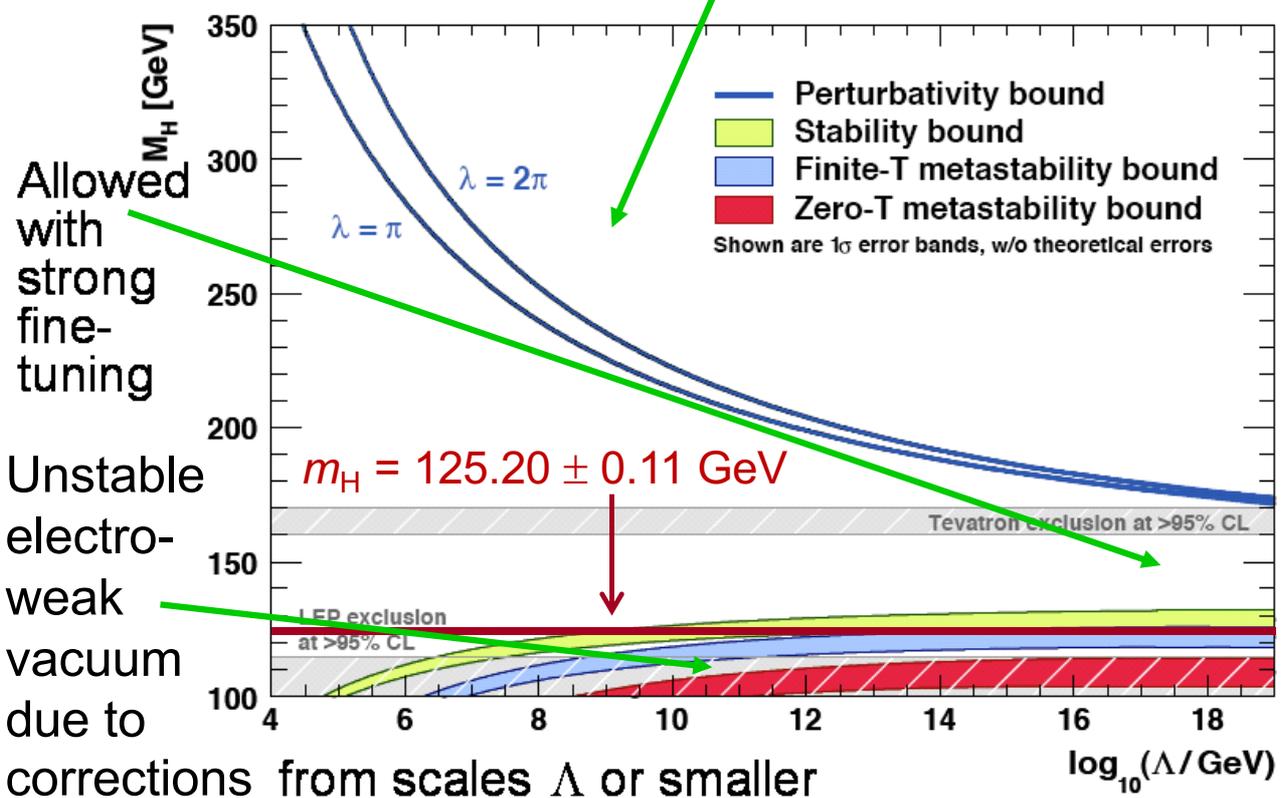
Direct observation of Higgs-like particle by ATLAS & CMS experiments @LHC; $m_H = 125.20 \pm 0.11$ GeV

Current wisdom: Electroweak symmetry breaking generates longitudinal degrees of freedom for W & Z

Assume there is nothing beyond SM; will that work?

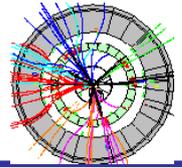
Possibly not due to metastable electroweak vacuum.

Unsatisfactory high-energy behaviour of Higgs quartic coupling λ if M_H is too large.





Electroweak symmetry breaking

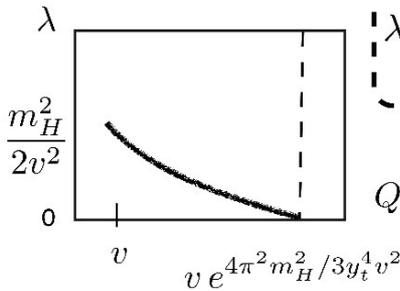


Stability of Higgs potential: $V(\phi) = \frac{1}{2} \mu^2 \phi^* \phi + \frac{1}{4} \lambda \phi^{*2} \phi^2$

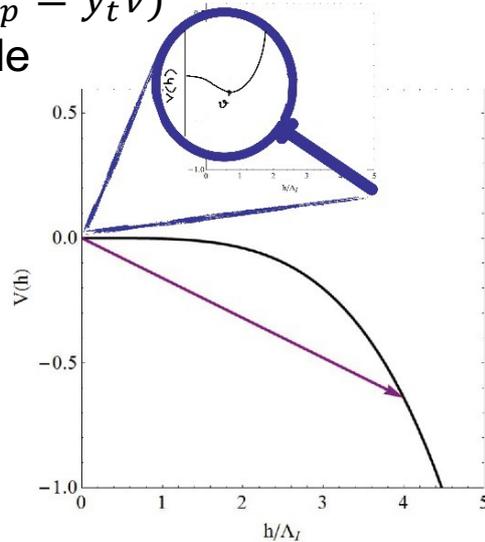
Small mass (y_t dominated RGE)

$$\lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln \frac{Q}{Q_0}}{1 - \frac{9}{16\pi^2} y_0^2 \ln \frac{Q}{Q_0}}$$

Linde '76, '80
Weinberg '76
Maini et al '78, '79
Politzer, Wolfram '79
Lindner '86
+...



top mass ($m_{top} = y_t v$)
 Q plays a key role



$\lambda < 0 \Rightarrow$ potential unbounded from below

$$\Lambda \leq v e^{4\pi^2 m_H^2 / 3y_t^4 v^2} \propto e^{m_H^2 / m_{top}^2}$$

New physics should appear before that point to restore stability

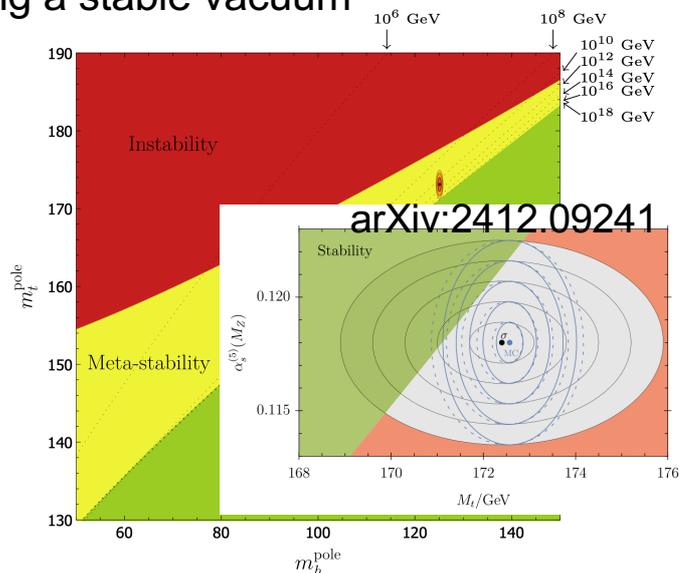
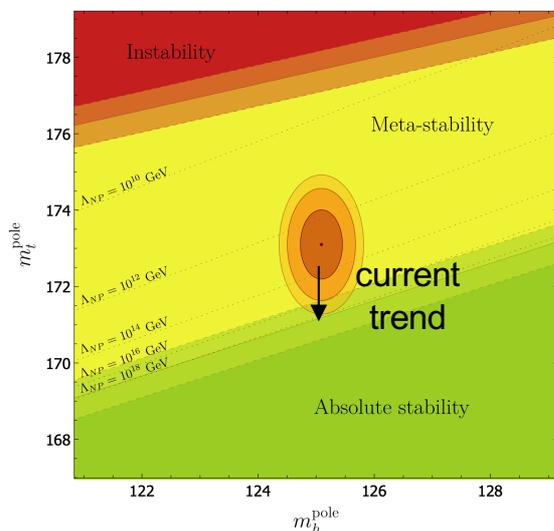
Small mass (y_t dominated RGE)

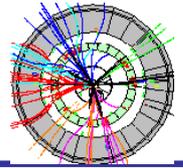
$$\lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln \frac{Q}{Q_0}}{1 - \frac{9}{16\pi^2} y_0^2 \ln \frac{Q}{Q_0}}$$

Linde '76, '80
Weinberg '76
Maini et al '78, '79
Politzer, Wolfram '79
Lindner '86
+...

A. Andreassen
et al., PRD 97
(2018) 056006

Λ_{NP} = new physics scale enabling a stable vacuum





GRAND UNIFICATION

- Unify gauge forces
- Simplify SM structure
- Predict gauge couplings

electroweak & strong \Rightarrow
described by a single gauge group

$$G \supset SU(3)_c \times SU(2)_L \times U(1)_Y$$

- Strength of force depends on energy scale \Rightarrow experimentally seen

minimal group to fit in all: $SU(5)$

Matter

$$\bar{5} = (\bar{3}, 1, +\frac{2}{3}) + (1, 2, -1)$$

$$\bar{d}_R \quad \quad \quad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

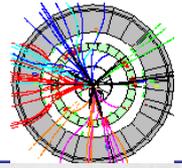
$$10 = (\bar{3}, 1, -\frac{4}{3}) + (3, 2, +\frac{1}{3}) + (1, 1, +2)$$

$$\bar{u}_R \quad \quad \quad q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \quad \bar{e}_R$$

Y quantized !! $(\pm \frac{5}{3})$

Beyond SM

VI/16



Gauge:

$$24 = \overset{g}{(8, 1, 0)} + \overset{W^\pm, W^0}{(1, 3, 0)} + \overset{B^0}{(1, 1, 0)} \\ + (3, 2, -\frac{5}{3}) + (\bar{3}, 2, \frac{5}{3}) \\ X, Y \quad Y, X$$

X, Y new gauge boson with colour and weak charge?

- Unification of forces and (partial) unification of matter
- Evidence: purely suggestive + gauge couplings

New symmetry \Rightarrow relation between couplings

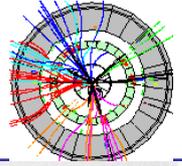
▲ SU(5) Higgs mechanism

SU(5) $\xrightarrow{\text{broken}}$ SU(3)_C × SU(2)_L × U(1)

\Rightarrow X and Y get (large) mass



Grand unified theories

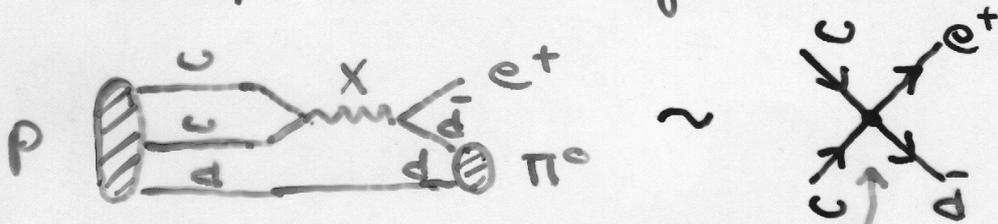


- Quarks & leptons belong to same representation \Rightarrow
 X and Y violate B -, L -number

New Phenomena not in SM

\Rightarrow p -decay, n - \bar{n} oscillations,
 ν masses ...

▲ proton decay



$$\tau_p \sim \frac{M_X^4}{g_5^4 m_p^5} \quad \frac{1}{\Lambda^2} = \frac{1}{2} \frac{g_5^2}{M_X^2}$$

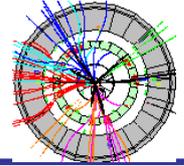
$$\tau_p \geq 24 \cdot 10^{33} \text{ years} \Rightarrow M_X \geq 10^{16} \text{ GeV}$$

NB! $M_X \sim 10^{16} \text{ GeV} \equiv$ "GUT" scale

┌ X, Y might also help baryogenesis
 if they slightly violate CP └



Grand unified theories



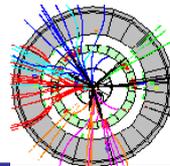
GUTs can also be constructed with larger groups e.g. $SO(10)$. In $SO(10)$, all SM fermions fit into one **16-dimensional** spinor along with a singlet that naturally plays the role of the right handed neutrino.

If $SO(10) \rightarrow SU(5) \times U(1)_X$ then $X = 2Y - 5(B - L)$. Then neutrino masses ~ 0.1 eV would most likely be generated through the seesaw mechanism: i.e. after GUT symmetry breaking $m_\nu \approx (m_\nu^D)^2 / M_{GUT}$ gives $m_\nu \sim 0.1$ eV if $m_\nu^D \approx v_{SM}$ & $M_{GUT} \approx 10^{16}$ GeV.

Table 92.1: Quantum numbers in the **16** representation of $SO(10)$

State	$U(1)_Y$	Color	Weak	$SU(5)$	$U(1)_X$	$SO(10)$
ν^c	0	---	--	1	-5	16
e^c	2	---	++	10	-1	
u^r	1/3	+- -	-+			
d^r	1/3	+- -	+-			
u^g	1/3	-+-	-+			
d^g	1/3	-+-	+-			
u^b	1/3	--+	-+			
d^b	1/3	--+	+-			
u_r^c	-4/3	+++	--			
u_g^c	-4/3	+-+	--			
u_b^c	-4/3	++-	--			
d_r^c	2/3	-++	++			
d_g^c	2/3	+-+	++			
d_b^c	2/3	++-	++			
ν	-1	+++	-+			
e	-1	+++	+-			

For more details see PDG review on Grand unified theories:
<https://pdg.lbl.gov/2025/reviews/rpp2025-rev-guts.pdf>



Gauge couplings

▲ unbroken SU(5) ⇒

$$\begin{array}{c}
 g_3 = g_2 = \sqrt{\frac{5}{3}} g_Y = g_5 \\
 = \quad = \quad \underbrace{\quad}_{g_1} \quad = g_5
 \end{array}$$

$$\sin^2 \Theta_w (M_{GUT}) = \frac{g_Y}{g_2 + g_Y} = \frac{3/5}{1 + 3/5} = 0.375$$

but couplings depend on energy

▲ must compare SU(5) prediction at $E \sim M_X$ with what we observe at $E \sim m_Z$

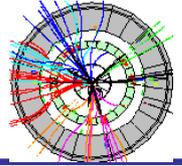
- Standard Model

$$\text{SM (p.)} \Rightarrow g_i(E)$$

$$* \left. \begin{array}{l}
 \alpha_{em}^{-1}(m_Z) = 127.9 \pm 0.1 \\
 g_3^2(m_Z) = 1.50 \pm 0.05
 \end{array} \right\} \Rightarrow$$

$$\sin^2 \Theta_w = 0.210 \pm 0.003 \neq$$

$$\sin^2 \Theta_w^{exp} = 0.2315 \pm 0.005 \quad \text{VI/19}$$

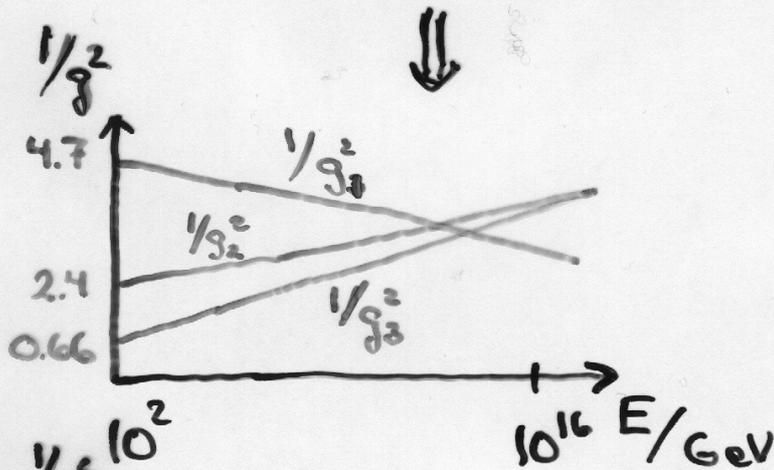


- Supersymmetric Standard Model

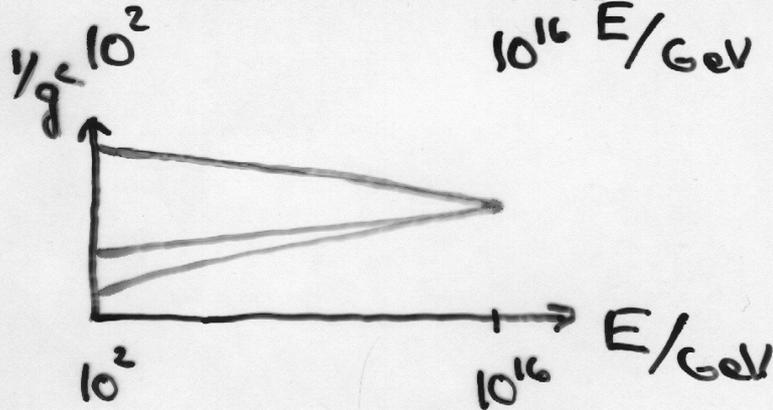


$$\left. \begin{matrix} \alpha_{em}(m_Z) \\ g_s^2(m_Z) \end{matrix} \right\}$$

$$\sin^2 \theta_w = 0.2334 \pm 0.005 \approx \sin^2 \theta_w^{exp}$$

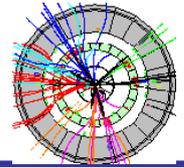


SM



SSM

$$\frac{1}{g^2(Q^2)} - \frac{1}{g^2(m_Z^2)} \propto b \log \frac{Q^2}{m_Z^2}$$



MOTIVATION

Why to go Beyond the Standard Model?

- Standard Model is an effective theory:
 ~ 20 parameters to be fixed by experiments.

- SM includes only part of the fundamental interactions: Gravity is missing.



Hierarchy problem: $m_W/m_{Planck} \sim 10^{-17}$.

- Quantum corrections to particle masses:

Fermions

E.g. QED:

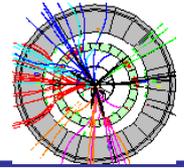
$$L_{electron} = \bar{e}\not{D}e + m_e\bar{e}_L e_R + m_e\bar{e}_R e_L.$$

- $m_e = 0 \rightarrow$ chiral symmetry:

$$e_L \rightarrow e_L, e_R \rightarrow e^{i\alpha} e_R$$

- $m_e \neq 0$, quantum corrections:

$$\delta m_e = 3\alpha_{em} \log(E/\Lambda) / 4\pi \cdot m_e \text{ (small even if } \Lambda = M_P)$$



Photon

Mass term $m_\gamma^2 A_\mu A^\mu$ not invariant under gauge symmetry.

$$\implies m_\gamma = 0$$

Symmetries keep fermions and γ light.

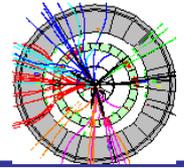
Scalars

BSM physics comes to play at $\Lambda =$ physical upper limit in the quantum corrections.

For scalar particles:

$$\begin{aligned} \delta m^2 &\sim \int^\Lambda \frac{d^4 k}{(2\pi)^4} \left\{ \text{---} \text{---} \text{---} \text{---} \right\} \\ &\sim \frac{\lambda_B}{16\pi^2} \Lambda^2 - \frac{h_f^2}{16\pi^2} \Lambda^2 \sim \Lambda^2. \end{aligned}$$

Should be $m_H \sim$ electroweak scale.



- With suitable symmetry bosonic and fermionic contributions cancel!
⇒ Supersymmetry

- Volkov, Akulov, 1973;
Wess, Zumino, 1974

- In nature supersymmetric partners of the SM particles have not been seen ⇒ SUSY must be broken. To solve the naturalness problem, must be

$$|m_B^2 - m_F^2| < \mathcal{O}(1 \text{ TeV}^2). \quad \Rightarrow \text{low-energy supersymmetry}$$

- Generators of supersymmetry, translations and Lorentz-transformations satisfy a common algebra.

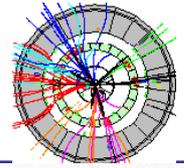
Consistency → supersymmetry is local.

⇒ Supergravity (which includes gravity)

- Important ingredient in superstring theories.

For more details see PDG review on Supersymmetry: theory:

<https://pdg.lbl.gov/2025/reviews/rpp2025-rev-susy-1-theory.pdf>



SYPERSYMMETRY MODELS

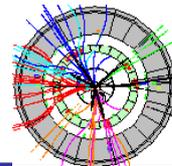
Bosons: commutation relations

Fermions: anticommutation relations.

- An indefinite number of bosons can exist at the same place at the same time, whereas only one fermion can be in any given place at a given time.
- The matter is made of fermions, while the forces are associated with bosons.

Symmetries come in two types: external (or space-time) and internal symmetries.

- Internal symmetries include the Standard Model symmetry.
- External symmetries include invariance under Lorentz transformations.
- Particle spin is an external symmetry, while isospin is not based on Lorentz invariance and is an internal symmetry.



Translations

$$x^\mu \rightarrow x^\mu + a^\mu \implies$$

$$\Phi(x) \rightarrow \Phi(x + a); \quad \delta\Phi = a^\mu \partial_\mu \Phi$$

Lorentz

$$x^\mu \rightarrow A^\mu_\nu x^\nu; \quad x^\mu x_\mu \text{ invariant} \implies$$

$$\delta\Phi = A^\mu_\nu x^\nu \partial_\mu \Phi$$

- SUSY is a space-time symmetry.
- A supersymmetry operation alters particle spin by 1/2, changing bosons into fermions and vice versa.
- Supersymmetry is the first symmetry that can unify matter and force.

Supersymmetry

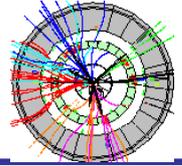
$$\delta\phi = \bar{\xi}(1 - \gamma_5)\psi$$

$$\delta\psi = -i\gamma^\mu(1 + \gamma_5)\xi\partial_\mu\phi$$

ξ is fermionic, analogue of a^μ and A^μ_ν

$$\delta_2\delta_1\phi = \underbrace{\bar{\xi}_1\gamma^\mu(1 + \gamma_5)\xi_2}_{\equiv a^\mu} \partial^\mu\phi$$

$$\equiv a^\mu \implies (\delta_{\text{susy}})^2 \sim \text{translation}$$



MSSM (Minimal Supersymmetric Standard Model)

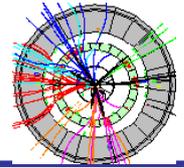
The particle content:

Two Higgs doublets

$$H_{1,Y=-1} = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_{2,Y=+1} = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}.$$

Field Content of the MSSM						
Super-multiplets	Super-field	Bosonic fields	Fermionic partners	SU(3)	SU(2)	U(1)
gluon/gluino	\widehat{V}_8	g	\widetilde{g}	8	1	0
gauge/	\widehat{V}	W^\pm, W^0	$\widetilde{W}^\pm, \widetilde{W}^0$	1	3	0
gaugino	\widehat{V}'	B	\widetilde{B}	1	1	0
slepton/	\widehat{L}	$(\widetilde{\nu}_L, \widetilde{e}_L^-)$	$(\nu, e^-)_L$	1	2	-1
lepton	\widehat{E}^c	\widetilde{e}_R^-	e_R^-	1	1	-2
squark/	\widehat{Q}	$(\widetilde{u}_L, \widetilde{d}_L)$	$(u, d)_L$	3	2	1/3
quark	\widehat{U}^c	\widetilde{u}_R	u_R	3	1	4/3
	\widehat{D}^c	\widetilde{d}_R	d_R	3	1	-2/3
Higgs/	\widehat{H}_d	(H_d^0, H_d^-)	$(\widetilde{H}_d^0, \widetilde{H}_d^-)$	1	2	-1
higgsino	\widehat{H}_u	(H_u^+, H_u^0)	$(\widetilde{H}_u^+, \widetilde{H}_u^0)$	1	2	1

Two Higgs doublets needed to cancel gauge anomalies generated by higgsinos. Also without second Higgs doublet, one cannot generate mass for both “up”- and “down”-type quarks (and charged leptons) in a way consistent with the underlying SUSY.



MSSM (Minimal Supersymmetric Standard Model)

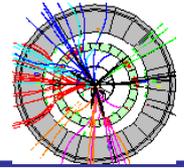
The particle content:

$$H_{1,Y=-1} = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_{2,Y=+1} = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}.$$

particle	sparticle	weak interaction eigenstate	mass eigenstate	
$q = u, d, c, s, t, b$	\tilde{q}_L, \tilde{q}_R	squark	\tilde{q}_1, \tilde{q}_2	squark
$l = e, \mu, \tau$	\tilde{l}_L, \tilde{l}_R	slepton	\tilde{l}_1, \tilde{l}_2	slepton
$\nu = \nu_e, \nu_\mu, \nu_\tau$	$\tilde{\nu}$	sneutrino	$\tilde{\nu}$	sneutrino
g	\tilde{g}	gluino	\tilde{g}	gluino
W^\pm	\tilde{W}^\pm	wino	$\tilde{\chi}_{1,2}^\pm$	chargino ($\chi_{1,2}^\pm$ lightest, next lightest ...)
H_1^+	\tilde{H}_1^+	higgsino		
H_2^-	\tilde{H}_2^-	higgsino		
γ	$\tilde{\gamma}$	photino		
Z	\tilde{Z}	zino	$\tilde{\chi}_{1,2,3,4}^0$	neutralino ($\chi_{1,2,3,4}^0$ lightest, next lightest ...)
H_1^0	\tilde{H}_1^0	higgsino		
H_2^0	\tilde{H}_2^0	higgsino		
\mathbf{g}_2	$\tilde{\mathbf{g}}_{3/2}$	gravitino	(only in supergravity)	

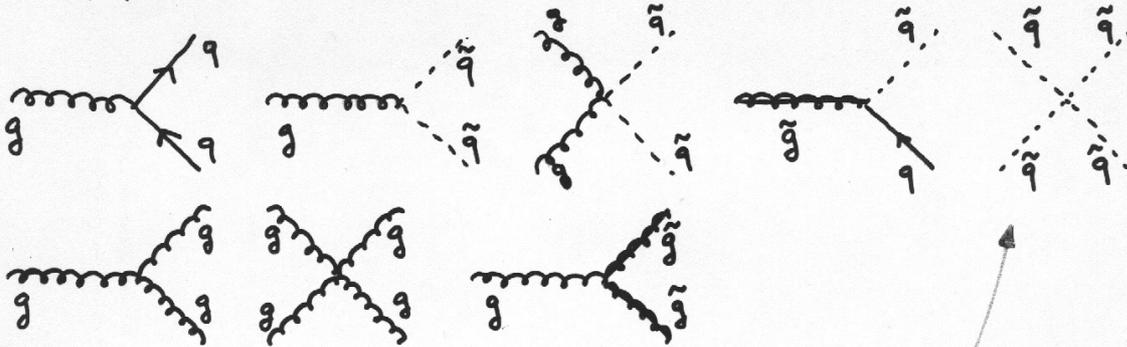


Supersymmetric gauge interactions



Supersymmetric gauge interactions

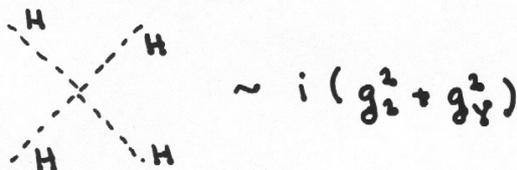
SU(3):



- all vertices controlled by g_s

- quartic scalar vertex $\propto i g_s^2$

* Higgs quartic coupling determined by gauge interactions

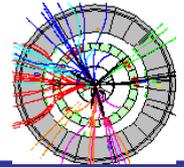


- In the Standard Model $\text{Higgs quartic vertex} = i\lambda_H = \text{free parameter}$

$$m_H = \sqrt{\lambda_H} v_F$$

* in SUSY $m_H \propto \sqrt{g_2^2 + g_Y^2} v = m_Z$

Both vertex couplings & supersymmetric diagrams (to first order) identical to Standard Model ones, only "dressed" by supersymmetric partners NB! Higgs vertex couplings modified.



Parameters of the models

Supersymmetric parameters:

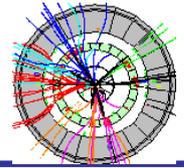
- $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge couplings g_3, g_2, g_1
- Higgs-fermion coupling matrices h_u, h_d, h_e .
- Higgs mixing parameter μ

Soft breaking parameters (these do not bring back the quadratic divergences):

- gaugino masses M_3, M_2, M_1
(= M of SU(3), SU(2), U(1) gauginos i.e. gluinos, wino/zino, bino)
- \tilde{q}, \tilde{l} masses $M_{\tilde{Q}}^2, M_{\tilde{U}}^2, M_{\tilde{D}}^2, M_{\tilde{L}}^2, M_{\tilde{E}}^2$
- Higgs masses m_1^2, m_2^2, b (b = soft Higgs mixing parameter)
- $H - \tilde{q} - \tilde{q}, H - \tilde{l} - \tilde{l}$ interaction parameters A_U, A_D, A_E

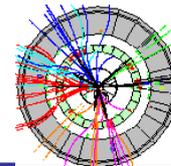
Instead of m_i^2 , use Higgs VEVs v_1, v_2 and mass of one neutral Higgs (m_A).

From the known $m_W, v_1^2 + v_2^2 = (246 \text{ GeV})^2$
 \implies the free parameter is $\tan \beta = v_2/v_1$.



- General count of parameters is 124!
 - The models can be experimentally constrained by
 - direct searches of sparticles or by the
 - quantum corrections to precision tests.
 - One can also try to constrain possible more fundamental theories by their low energy limits.
 - The dimensionless parameters should remain perturbative in the energy range where the MSSM is valid:
quantum corrections change the value of the dimensionless parameters when using different energy scales (RGE).
This way e.g. the mass of the top quark bounds the values of $\tan \beta$ to certain range, which depends on the scale up to which the MSSM is valid.

RGE = renormalization group equations



allowed mass range for top quark as function of $\tan\beta$ assuming that SUSY valid up to $\Lambda = 10^{16}$ GeV

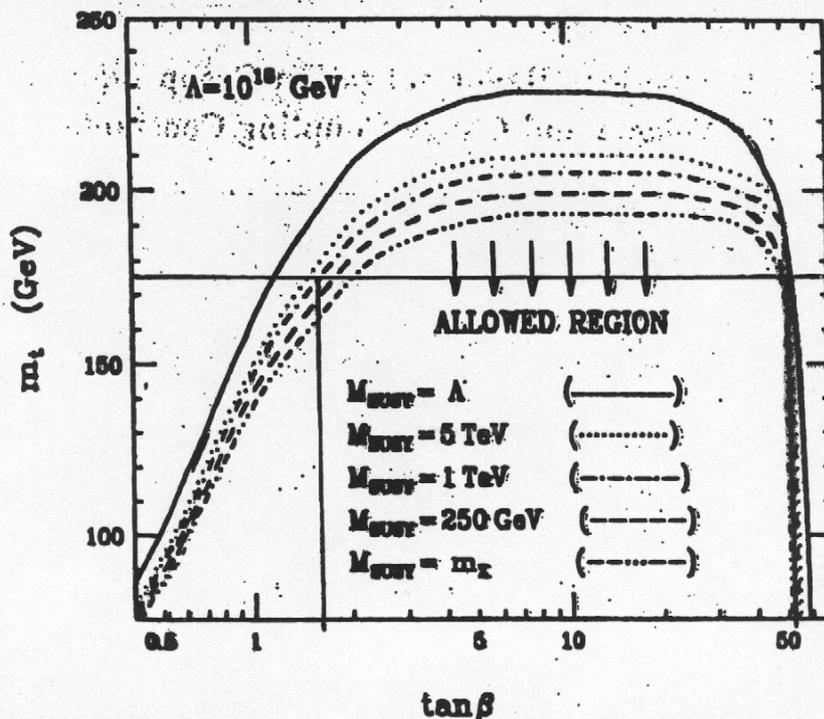
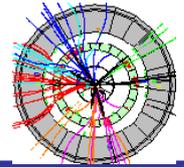


Fig. 1. The region of $\tan\beta$ - m_t parameter space in which all running Higgs-fermion Yukawa couplings remain finite at all energy scales, μ , from m_Z to $\Lambda = 10^{16}$ GeV [79]. Non-supersymmetric two-Higgs-doublet (one-loop) renormalization group equations (RGEs) are used for $m_Z \leq \mu \leq M_{\text{SUSY}}$ and the RGEs of the minimal supersymmetric model are used for $M_{\text{SUSY}} \leq \mu \leq \Lambda$ (see table 2). Five different values of M_{SUSY} are shown; the allowed parameter space lies below the respective curves.



- Interestingly in the MSSM the gauge couplings change in such a way that they seem to unite at certain energy scale. This does not happen in the SM.

- Assume GUT, + measured $\sin \theta_W (m_Z)$ & $\alpha_i(m_Z)$:

$$M_{SUSY} \approx \text{few TeV}$$

$$M_{GUT} \approx 2 \cdot 10^{16} \text{ GeV}$$

$$\alpha_{GUT}^{-1}(M_{GUT}) \approx 24.3$$

- It is often assumed that in addition to the gauge coupling constants also the gaugino masses, scalar masses and trilinear A -terms unify at the GUT scale or Planck scale:

"constrained" MSSM (CMSSM):

$$g_1(M_X) = g_2(M_X) = g_3(M_X) = g_U, \text{ couplings}$$

$$M_1(M_X) = M_2(M_X) = M_3(M_X) = m_{1/2}, \text{ gaugino masses}$$

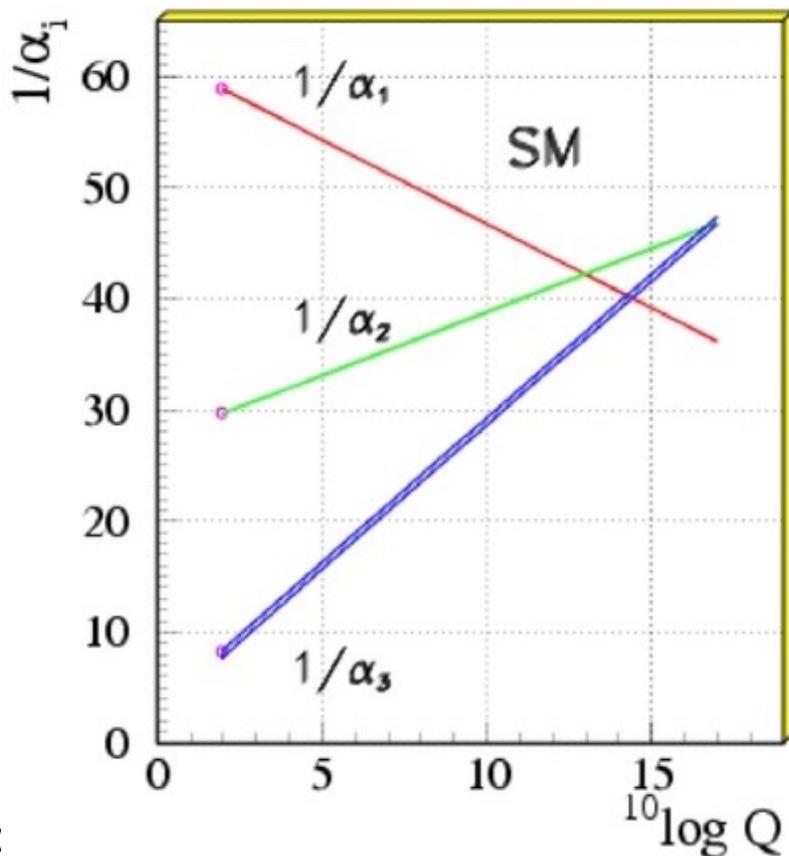
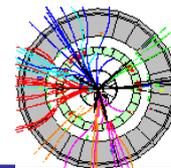
$$M_{\tilde{Q}}^2(M_X) = \dots = M_{\tilde{E}}^2(M_X) = m_0^2, \text{ scalar masses}$$

$$m_1^2(M_X) = m_2^2(M_X) = m_0^2, \text{ Higgs-}$$

$$A_U(M_X) = A_D(M_X) = A_E(M_X) = A_0, \text{ sfermion couplings}$$

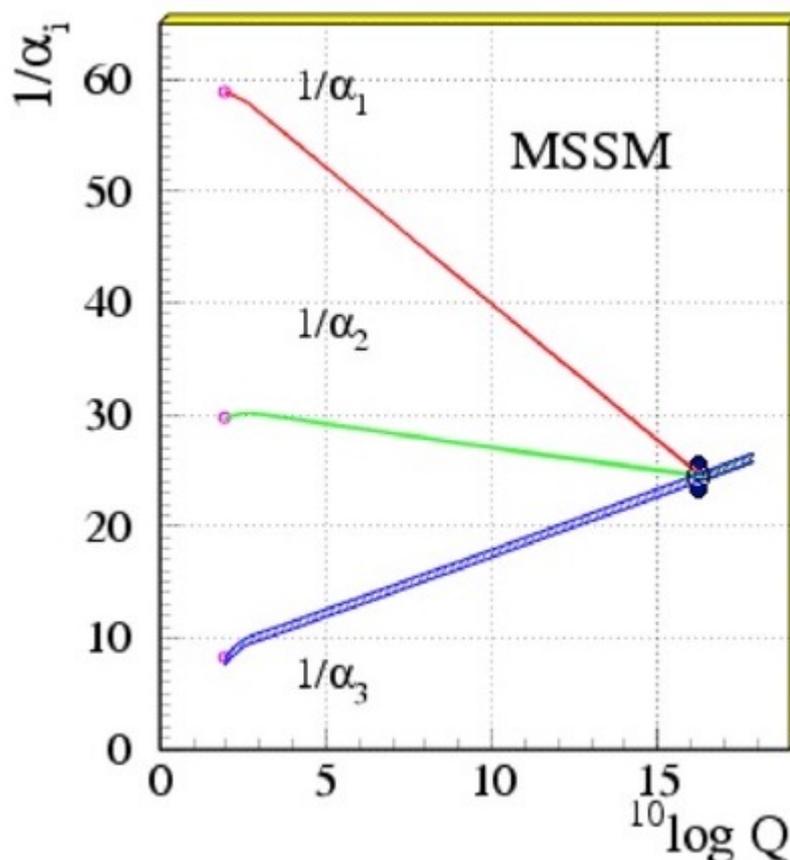


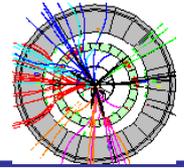
Supersymmetric model parameters



All couplings unite at a specific m_{GUT} in Minimal Supersymmetric Standard Model (MSSM) but not in the Standard Model (SM)

Predicted m_{GUT} in MSSM larger than in SM and therefore more consistent with the lower limits on the proton lifetime from experiments.

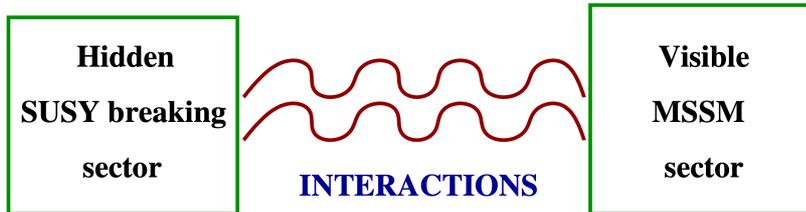




SUPERSYMMETRY BREAKING

- spin-3/2 fermion gravitino, whose mass is related to supersymmetry breaking scale $\Lambda_S = \sqrt{3m_{gravitino}M_P}$

- breaking in hidden sector, messenger transmission with coupling λ_I



Gravity mediated supersymmetry breaking

- Supersymmetry breaking is mediated from the hidden sector by interactions which are of gravitational strength.

- Appealing: Gravity exists anyway.

superpartner mass splitting: $\Delta m^2 \sim \lambda_I \Lambda_S^2$

- $m_{gravitino} \sim$ electroweak scale, but interactions suppressed by M_{Planck} . \Rightarrow large Λ_S + very small λ_I

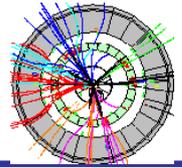
- mSUGRA (Best studied SUSY model):

- Radiative symmetry breaking assumed

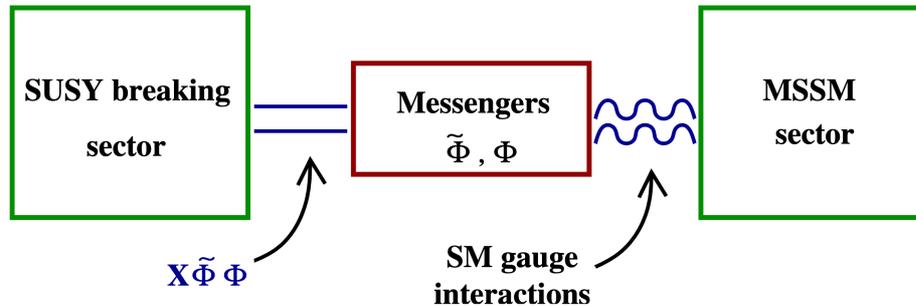
$\Rightarrow |\mu|$ determined. (through radiative corrections)

$$\{sgn(\mu), m_0, M_{1/2}, A_0, \tan(\beta)\}$$

- The lightest neutralino (or sneutrino) is the LSP.
LSP = lightest supersymmetric particle



Gauge mediated supersymmetry breaking



- Supersymmetry breaking is transmitted to MSSM via gauge interactions.

- Messenger sector has particles with SM quantum numbers. Gaugino masses:

black (green) lines MSSM (messenger) fields



- The parameters are chosen so that the gauge mediated effects dominate over the gravity mediated effects

$$\implies m_{\text{gravitino}} \sim \text{eV-keV} \implies \lambda_I \sim 1$$

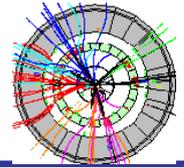
- Parameters: (m_{GMSB}) SUSY breaking scale

$$\{\Lambda, M_m, \tan(\beta), n_5, \text{sign}(\mu)\}.$$

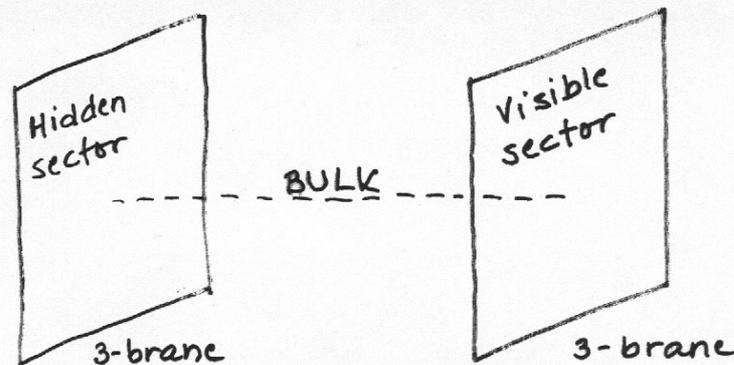
⚡
messenger scale

⚡
of complete reps. of SU(5) in messenger sector

- The gravitino is the LSP.



Scenarios with higher dimensional space time



- Assume that the visible fields are in one 3+1 dimensional brane, while the hidden sector is on another brane.

⇒ Anomaly mediated and gaugino mediated models. (supersymmetry breaking transmitted through fields that live in the "bulk")

Anomaly mediation:

- gravity generates soft terms even if no direct coupling between hidden and observable sectors
 - soft terms depend only on ew scale coupling constants
 - RGE invariant
 - high predictability → negative slepton masses
→ need something more
- heavy gravitino** ⇒
large Λ_S + very small λ_I



Supersymmetry breaking

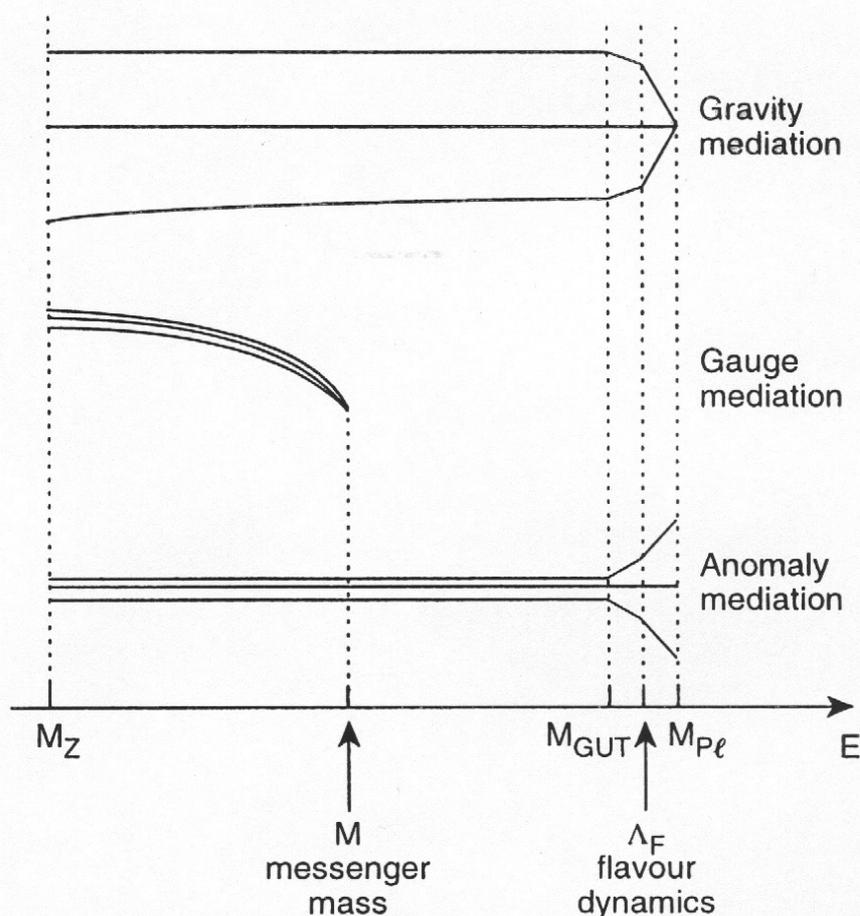
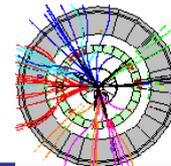
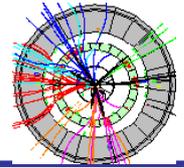


Figure 1: A schematic illustration of the energy dependence of the running squark masses belonging to the three different generations, in the context of the various supersymmetric scenarios discussed in the text. In gravity mediation, new dynamics at the scale Λ_F and GUT physics tend to induce large flavour-breaking effects in the squark spectrum, even if we start from a universality assumption at M_{Pl} . In the case of gauge mediation, the squark masses can be generated at scales sufficiently low to ensure a super-GIM mechanism. In anomaly mediation, the squark spectrum is determined by the low-energy theory and it is insensitive to flavour violations occurring at large scales.



R-parity



Consistency with proton decay lifetime limits require (global) **B-L conservation** \Rightarrow **R-parity invariance**

$$R_P = (-1)^{3B + L + 2S}$$

Baryon Number
Spin
Lepton number

+1 for Standard Particles

-1 for Supersymmetric Partners

R_p Conserved

- ◆ **SUSY particles are pair-produced**
- ◆ **The LSP is stable**
(\rightarrow neutral, colourless
 \rightarrow good dark-matter candidate)
- ◆ **All SUSY particles decay into the LSP**

R_p Violated

- ◆ **The LSP decay into standard particles**
(no candidate for dark matter)
- ◆ **And so do all other SUSY particles**
- NB! R_p conservation not required by neither SUSY or gauge invariance**

Experimental

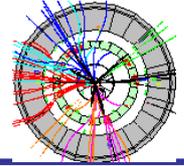
- ◆ **The LSP (neutral, colourless) interacts only weakly with matter: it is invisible.**
 \rightarrow **MISSING ENERGY**

Signature

- ◆ **SUSY particles decay into quarks, leptons, neutrinos.**
 \rightarrow **Multi-jet, multi-leptons final state, not (necessarily) missing energy!!**



Supersymmetric particle searches



Supersymmetric Particle Searches

Tchi1chi1C: electroweak pair production of charginos $\tilde{\chi}_1^\pm$, where $\tilde{\chi}_1^\pm$ decays through an intermediate slepton or sneutrino to $l\nu\tilde{\chi}_1^0$ and where $m_{\tilde{\ell},\tilde{\nu}} = (m_{\tilde{\chi}_1^\pm} + m_{\tilde{\chi}_1^0})/2$.

All supersymmetric mass bounds here are model dependent.

The limits assume:

1) $\tilde{\chi}_1^0$ is the lightest supersymmetric particle; 2) R -parity is conserved, unless stated otherwise;

See the Particle Listings for a Note giving details of supersymmetry.

$\tilde{\chi}_i^0$ — neutralinos (mixtures of $\tilde{\gamma}$, \tilde{Z}^0 , and \tilde{H}_i^0)

Mass $m_{\tilde{\chi}_1^0} > 0$ GeV, CL = 95%

[general MSSM, non-universal gaugino masses]

Mass $m_{\tilde{\chi}_1^0} > 46$ GeV, CL = 95%

[all $\tan\beta$, all m_0 , all $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$]

Mass $m_{\tilde{\chi}_2^0} > 62.4$ GeV, CL = 95%

[$1 < \tan\beta < 40$, all m_0 , all $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$]

Mass $m_{\tilde{\chi}_3^0} > 99.9$ GeV, CL = 95%

[$1 < \tan\beta < 40$, all m_0 , all $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$]

Mass $m_{\tilde{\chi}_4^0} > 116$ GeV, CL = 95%

[$1 < \tan\beta < 40$, all m_0 , all $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$]

Mass $m_{\tilde{\chi}} \text{ none } 200\text{--}670$ GeV, CL = 95% [R-Parity Violating]

[wino production, $\tilde{\chi} \rightarrow b + \ell/\nu + t/b$ via λ'_{i33} coupling]

$\tilde{\chi}_i^\pm$ — charginos (mixtures of \tilde{W}^\pm and \tilde{H}_i^\pm)

Mass $m_{\tilde{\chi}_1^\pm} > 94$ GeV, CL = 95%

[$\tan\beta < 40$, $m_{\tilde{\nu}^\pm} - m_{\tilde{\nu}^0} > 3$ GeV, all m_0]

Mass $m_{\tilde{\chi}_1^\pm} > 1000$ GeV, CL = 95%

[$2\ell + \cancel{E}_T$, Tchi1chi1C, $m_{\tilde{\chi}_1^0} = 0$ GeV]

Mass $m_{\tilde{\chi}_1^\pm} > 1600$ GeV, CL = 95% [R-Parity Violating]

[Tchi1n2l, $\tilde{\chi}_1^0 \rightarrow \ell^\pm \ell^\mp \nu$, $\lambda_{12k} \neq 0$, $m_{\tilde{\chi}_1^0} = 1200$ GeV]

$\tilde{\chi}^\pm$ — long-lived chargino

Mass $m_{\tilde{\chi}^\pm} > 1050$ GeV, CL = 95%

[$\tilde{\chi}^\pm \rightarrow \tilde{\chi}_1^0 \pi^\pm$, wino LSP, stable]

Mass $m_{\tilde{\chi}^\pm} > 1050$ GeV, CL = 95%

[$\tilde{\chi}^\pm \rightarrow \tilde{\chi}_1^0 \pi^\pm$, wino LSP, $\tau = 20$ ns]

$\tilde{\nu}$ — sneutrino

Mass $m > 41$ GeV, CL = 95% [model independent]

Mass $m > 94$ GeV, CL = 95%

[CMSSM, $1 \leq \tan\beta \leq 40$, $m_{\tilde{e}_R} - m_{\tilde{\chi}_1^0} > 10$ GeV]

Mass $m > 4200$ GeV, CL = 95% [R-Parity Violating]

[$1e + 1\mu, \nu_\tau \rightarrow e\mu, \lambda = \lambda' = 0.1$]

\tilde{e} — scalar electron (selectron)

Mass $m > 107$ GeV, CL = 95% [all $m_{\tilde{e}_L} - m_{\tilde{\chi}_1^0}$]

Mass $m > 700$ GeV, CL = 95%

[$2\ell + \cancel{E}_T$, $m_{\tilde{\ell}_R} = m_{\tilde{\ell}_L}$ and $\tilde{\ell} = \tilde{e}, \tilde{\mu}$, $m_{\tilde{\chi}_1^0} = 0$ GeV]

Mass $m > 250$ GeV, CL = 95%

[$\ell^\pm \ell^\mp + \cancel{E}_T$, \tilde{e}_R , $m_{\tilde{\chi}_1^0} = 0$ GeV]

Tchi1n2l: electroweak associated production of mass-degenerate charginos $\tilde{\chi}_1^\pm$ and neutralinos $\tilde{\chi}_2^0$, where $\tilde{\chi}_1^\pm$ decays to $W^\pm + \tilde{\chi}_1^0$ and where $\tilde{\chi}_2^0$ decays 50% of the time to $Z + \tilde{\chi}_1^0$ and 50% of the time to $H + \tilde{\chi}_1^0$.

Mass $m > 410$ GeV, CL = 95% [R-Parity Violating]

[$\geq 4\ell^\pm, \tilde{\ell} \rightarrow l\tilde{\chi}_1^0, \tilde{\chi}_1^0 \rightarrow \ell^\pm \ell^\mp \nu$]

Mass $m > 1200$ GeV, CL = 95% [R-Parity Violating]

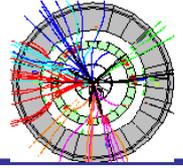
[$\geq 4\ell, \lambda_{12k} \neq 0, m_{\tilde{\nu}^0} = 900$ GeV (m-degenerate $\tilde{\ell}_L, \tilde{\nu}$)]

weakly coupling sparticles ($\tilde{\chi}, \tilde{\ell}, \tilde{\nu}$ etc ...): difficult to search for at LHC \Rightarrow most general limits from e^+e^- collisions (LEP) \Rightarrow lower mass limits typically ~ 100 GeV.

LHC provides much higher limits for specific decay modes that are only valid in more limited regions of the SUSY parameter space.



Supersymmetric particle searches



$\tilde{\mu}$ — scalar muon (smuon)

- Mass $m > 700$ GeV, CL = 95%
[$2\ell + \cancel{E}_T, m_{\tilde{\ell}_R} = m_{\tilde{\ell}_L}$ and $\tilde{\ell} = \tilde{e}, \tilde{\mu}, m_{\tilde{\chi}_1^0} = 0$ GeV]
- Mass $m > 210$, CL = 95%
[$\ell^\pm \ell^\mp + \cancel{E}_T, \tilde{\mu}_R, m_{\tilde{\chi}_1^0} = 0$ GeV]
- Mass $m > 94$ GeV, CL = 95%
[CMSSM, $1 \leq \tan\beta \leq 40, m_{\tilde{\mu}_R} - m_{\tilde{\chi}_1^0} > 10$ GeV]
- Mass $m > 410$ GeV, CL = 95% [R-Parity Violating]
[$\geq 4\ell^\pm, \tilde{\ell} \rightarrow l\tilde{\chi}_1^0, \tilde{\chi}_1^0 \rightarrow \ell^\pm \ell^\mp \nu$]
- Mass $m > 1200$ GeV, CL = 95% [R-Parity Violating]
[$\geq 4\ell, \lambda_{12k} \neq 0, m_{\tilde{\chi}_1^0} = 900$ GeV (m-degenerate $\tilde{\ell}_L, \tilde{\nu}$)]

$\tilde{\tau}$ — scalar tau (stau)

- Mass $m > 81.9$ GeV, CL = 95%
[$m_{\tilde{\tau}_R} - m_{\tilde{\chi}_1^0} > 15$ GeV, all $\theta_\tau, B(\tilde{\tau} \rightarrow \tau\tilde{\chi}_1^0) = 100\%$]
- Mass $m > 500$ GeV, CL = 95%
[2 hadronic $\tau + \cancel{E}_T, \tilde{\tau}_{R,L} \rightarrow \tau\tilde{\chi}_1^0, m_{\tilde{\chi}_1^0} = 1$ GeV]
- Mass $m > 90$ GeV, CL = 95% [R-Parity Violating]
[$\tilde{\tau}_R$, indirect, $\Delta m > 5$ GeV]
- Mass $m > 1200$, CL = 95% [R-Parity Violating]
[$\geq 4\ell, \lambda_{12k} \neq 0, m_{\tilde{\chi}_1^0} = 900$ GeV (m-degenerate $\tilde{\ell}_L, \tilde{\nu}$)]
- Mass $m > 286$ GeV, CL = 95% [long-lived $\tilde{\tau}$]

\tilde{q} — squarks of the first two quark generations

- Mass $m > 1.220 \times 10^3$ GeV, CL = 95%
[jets + \cancel{E}_T , Tsqk1, 1 non-degenerate $\tilde{q}, m_{\tilde{\chi}_1^0} = 0$ GeV]
- Mass $m > 1.600 \times 10^3$ GeV, CL = 95% [R-Parity Violating]
[$\tilde{q} \rightarrow q\tilde{\chi}_1^0, \tilde{\chi}_1^0 \rightarrow \ell\ell\nu, \lambda_{121}, \lambda_{122} \neq 0, m_{\tilde{g}} = 2400$ GeV]
- Mass $m > 1000$ GeV, CL = 95% [R-Parity Violating]
[jets, $\tilde{q} \rightarrow q\tilde{\chi}_1^0, \tilde{\chi}_1^0 \rightarrow \ell q q, m_{\tilde{\chi}_1^0} = 108$ GeV and $2.5 < c\tau_{\tilde{\chi}_1^0} < 200$ mm]

\tilde{q} — long-lived squark

- Mass $m > 1340$ GeV, CL = 95% [\tilde{t} R-hadrons]
- Mass $m > 1250$ GeV, CL = 95% [\tilde{b} R-hadrons]
- Mass $m > 1350$ GeV, CL = 95%
[long-lived \tilde{t} , RPV, $\tilde{t} \rightarrow b\ell, 7 \text{ mm} < c\tau < 110 \text{ mm}$]

\tilde{b} — scalar bottom (sbottom)

- Mass $m > 1.270 \times 10^3$ GeV, CL = 95%
[b-jets + \cancel{E}_T , Tsbott1, $m_{\tilde{\chi}_1^0} = 0$ GeV]
- Mass $m > 307$ GeV, CL = 95% [R-Parity Violating]
[$\tilde{b} \rightarrow td$ or ts, λ_{332}' or λ_{331}' coupling]

\tilde{t} — scalar top (stop)

- Mass $m > 1.310 \times 10^3$ GeV, CL = 95%
[jets + \cancel{E}_T , Tstop1, $m_{\tilde{\chi}_1^0} < 300$ GeV]
- Mass $m > 1900$ GeV, CL = 95% [R-Parity Violating]
[$\tilde{t} \rightarrow be$, prompt, Tstop2RPV]
- Mass $m > 1800$ GeV, CL = 95% [R-Parity Violating]
[$\tilde{t} \rightarrow b\mu$, prompt, Tstop2RPV]
- Mass $m > 800$ GeV, CL = 95% [R-Parity Violating]
[$\tilde{t} \rightarrow b\tau$, prompt, Tstop2RPV]
- Mass $m > 460$ GeV, CL = 95%
[R-Parity Violating, long-lived $\tilde{t}, \tilde{t} \rightarrow d\tilde{\ell}, 0.01 \text{ cm} < c\tau < 1000 \text{ cm}$]

\tilde{g} — gluino

- Mass $m > 2.300 \times 10^3$ GeV, CL = 95%
[jets + \cancel{E}_T , Tglu1A, $m_{\tilde{\chi}_1^0} < 200$ GeV]
- Mass $m > 2.260 \times 10^3$ GeV, CL = 95% [R-Parity Violating]
[$\geq 4\ell, \lambda_{12k} \neq 0, m_{\tilde{\chi}_1^0} > 1000$ GeV]

strongly coupling sparticles (\tilde{q}, \tilde{g} , etc ...): easy to search for at LHC \Rightarrow lower mass limits $> \sim 1\text{-}2$ TeV.

For more details see PDG review on Supersymmetry: experiment:

<https://pdg.lbl.gov/2025/reviews/rpp2025-rev-susy-2-experiment.pdf>

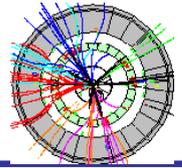
- Tsqk1:** squark pair production with $\tilde{q} \rightarrow q\tilde{\chi}_1^0$
- Tsbott1:** sbottom pair production with $\tilde{b} \rightarrow b\tilde{\chi}_1^0$
- Tstop1:** stop pair production with $\tilde{t} \rightarrow t\tilde{\chi}_1^0$
- Tglu1A:** gluino pair production with $\tilde{g} \rightarrow q\bar{q}\tilde{\chi}_1^0$

Tstop2RPV: stop pair production with $\tilde{t} \rightarrow b\ell$, via RPV coupling $\lambda_{i33}^{\tilde{t}}$.

$\lambda_{ijk}, \lambda''_{ijk}$ & λ'_{ijk} R-parity violating terms for lepton, quark & both superfields. i, j, k generation indices.



Lower bounds on the LSP mass

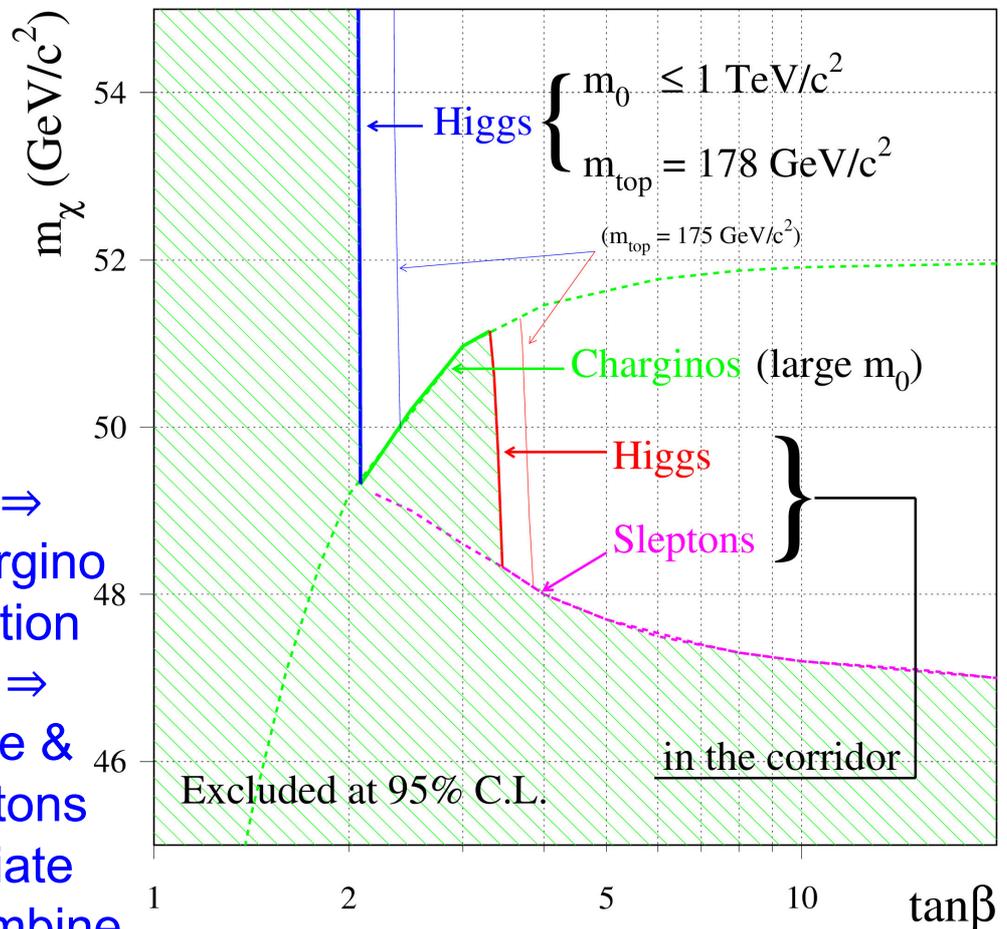


Lower bounds on the LSP mass can be extracted e.g. in constrained MSSM, where gaugino and sfermion masses separately unify at the GUT scale. Free parameters: $\tan\beta$, $M_{1/2}$ (gaugino masses at GUT scale), m_0 (sfermion & Higgs mass at GUT scale), μ (Higgs mass mixing term), A_t (trilinear coupling in the stop sector) & m_A (pseudoscalar Higgs mass)

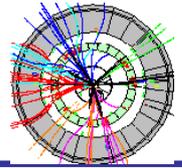
Combining slepton, Higgs & chargino searches to constrain lightest neutralino.

- large $m_0 \Rightarrow$ large chargino cross section
- small $m_0 \Rightarrow$ detectable & light sleptons
- Intermediate $m_0 \Rightarrow$ combine with Higgs search

with LEP Combined Results



A more elaborate analysis in mSUGRA (only $\tan\beta$, $\text{sign}(\mu)$, m_0 , $M_{1/2}$ & A_0 free) using also stable particle searches & electroweak parameter constraints give $m_{\text{LSP}} > 50 \text{ GeV}$.



HIGGS SECTOR IN MSSM

$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \quad H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}.$$

- Physical Higgs scalars: h, H, A, H^\pm
- Tree-level Higgs potential:

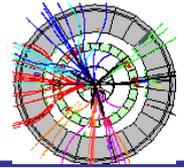
$$V = (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |\cancel{H_u^+}|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |\cancel{H_d^-}|^2) + b(\cancel{H_u^+ H_d^-} - H_u^0 H_d^0) + c.c.$$

Corresponds to λ term of SM

$$+ \frac{g^2 + g'^2}{8} (|H_u^0|^2 + |\cancel{H_u^+}|^2 - |H_d^0|^2 - |\cancel{H_d^-}|^2)^2 + \frac{1}{2} g^2 |\cancel{H_u^+ H_d^0} - H_u^0 \cancel{H_d^-}|^2$$

- $\langle \tilde{l} \rangle = \langle \tilde{q} \rangle = 0$
- $\langle H_u^+ \rangle = \langle H_d^- \rangle = 0$
- VEVs and couplings real

Two Higgs doublets are needed in SUSY to (1) cancel gauge anomalies (higgsino contributions in 3 gauge boson diagrams) (2) generate masses for both "up"- and "down"-type quarks



- The masses of physical particles are

$$m_A^2 = 2b / \sin 2\beta, \quad m_{H^\pm}^2 = m_A^2 + m_W^2,$$

$$m_{h,H}^2 = \frac{1}{2} \left[m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta} \right].$$

Here

$$h = (H_{\mathbf{2}_L}^{0r} - v_2) \cos \alpha - (H_{\mathbf{1}}^{0r} - v_1) \sin \alpha,$$

$$H = (H_{\mathbf{1}}^{0r} - v_1) \cos \alpha + (H_{\mathbf{2}_L}^{0r} - v_2) \sin \alpha,$$

where

$$\frac{\cos 2\alpha}{\cos 2\beta} = -\frac{m_A^2 - m_Z^2}{m_H^2 - m_h^2}, \quad \frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_H^2 + m_h^2}{m_H^2 - m_h^2}.$$

- At tree-level:

$$m_h^2 < m_{Z,A}^2, \quad m_H^2 > m_{Z,A}^2$$

$$m_h^2 + m_H^2 = m_A^2 + m_Z^2.$$

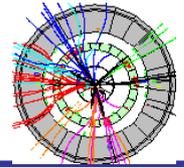
Final combined LEP limit:

- Experimentally: $m_{H^\pm} > 80 \text{ GeV}$

Eur. Phys. J. C73 (2013) 2463

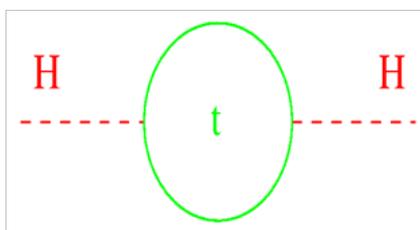


Higgs sector: SM vs MSSM



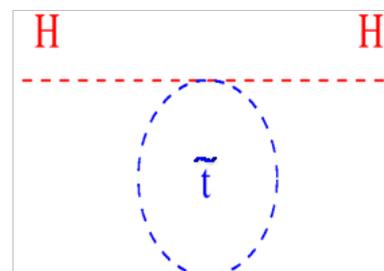
In the Standard Model

- One Higgs doublet
v.e.v. v
- One physical state
 H
- One parameter
 M_H
- Radiative corrections
to m_h
quadratically divergent



In the M.S.S.M

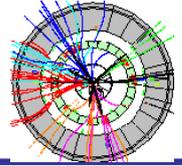
- Two Higgs doublets
v.e.v.'s v_1 and v_2
- Five physical states
 h, H, A, H^+, H^-
CP-even CP-odd Charged
- Two parameters
(at tree-level)
 $M_h, \tan\beta = v_2/v_1$
- Radiative corrections
to m_h, m_H
stabilized and finite



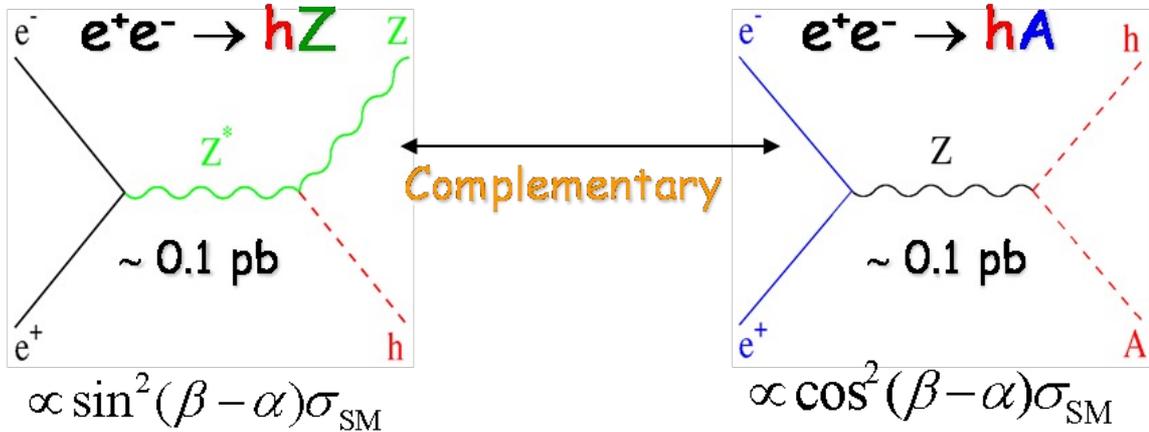
Depend on $M_{\text{top}}, M_{\text{stop(L,R)}} \dots$



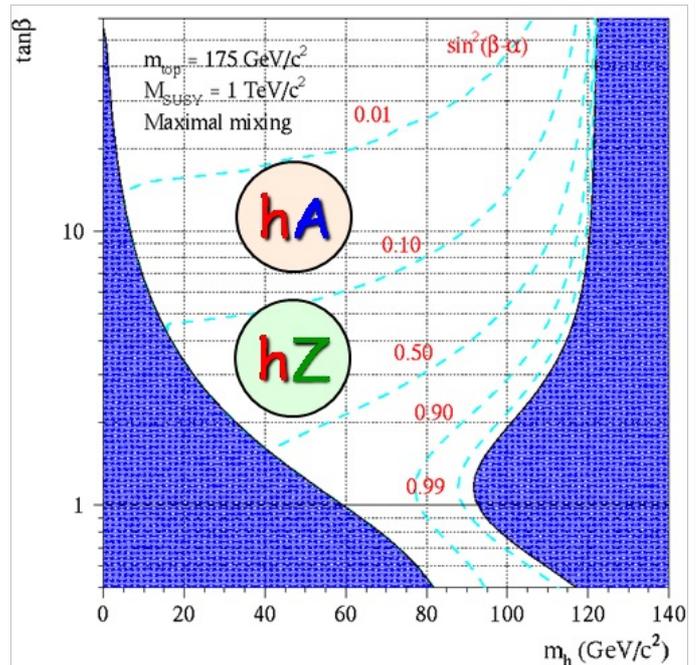
MSSM Higgs



• In SUSY processes very often complementary e.g.

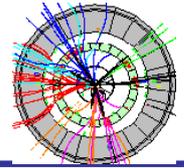


⇒ Look for both processes



Couplings between Higgses & the particles in MSSM:

	$u\bar{u}$	$d\bar{d}, l^+l^-$	VV ($v=Z,W$)
h	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\sin(\beta-\alpha)$
H	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos(\beta-\alpha)$
A	$\cot\beta$	$\tan\beta$	0



In exact Supersymmetry: $m_h \leq m_Z |\cos 2\beta|$

In broken Supersymmetry: $m_h^2 \leq m_Z^2 + \Delta m_h^2$

SUSY little hierarchy problem

SUSY needs new (super)particles that haven't been seen (yet?)
SUSY (at least MSSM) predicts a (very) light Higgs

$$V = (|\mu|^2 + m_{H_u}^2) |H_u^0|^2 + (|\mu|^2 + m_{H_d}^2) |H_d^0|^2 - B(H_u^0 H_d^0 + c.c.) + \frac{g^2 + g'^2}{8} (|H_u^0|^2 - |H_d^0|^2)^2$$

one-loop level

$$m_h^2 \approx m_Z^2 \cos^2 2\beta + \frac{3G_F m_t^4}{\sqrt{2}\pi^2} \log \frac{m_{\tilde{t}}^2}{m_t^2} \quad \text{terms with } A_t \text{ ignored}$$

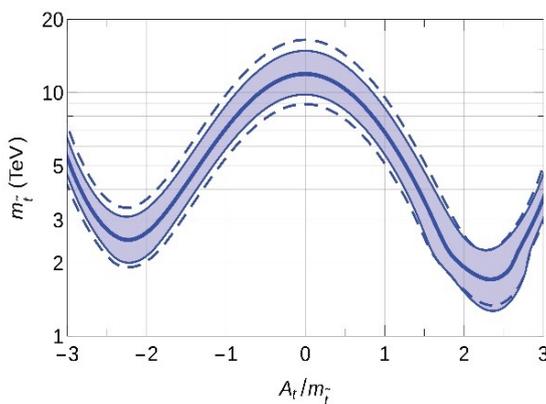
$$m_Z^2/2 = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1}$$

$$m_H \approx 125 \text{ GeV} \Rightarrow m_{\tilde{t}} \geq 2 \text{ TeV}$$

$$\delta m_{H_u}^2 = -\frac{3\sqrt{2}G_F m_t^2 m_{\tilde{t}}^2}{4\pi^2} \log \frac{\Lambda}{m_{\tilde{t}}}$$

requires fine-tuning at least O(1 %) to get m_Z (SUSY vs electroweak mass scale)

fine-tuned
susy little hierarchy problem



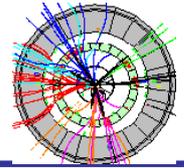
Pardo Vega, Villadoro '15 + many others

Figure 5: Allowed values of the OS stop mass reproducing $m_h = 125 \text{ GeV}$ as a function of the stop mixing, with $\tan \beta = 20$, $\mu = 300 \text{ GeV}$ and all the other sparticles at 2 TeV. The band reproduce the theoretical uncertainties while the dashed line the 2σ experimental uncertainty from the top mass. The wiggle around the positive maximal mixing point is due to the physical threshold when $m_{\tilde{t}}$ crosses $M_1 + m_t$.

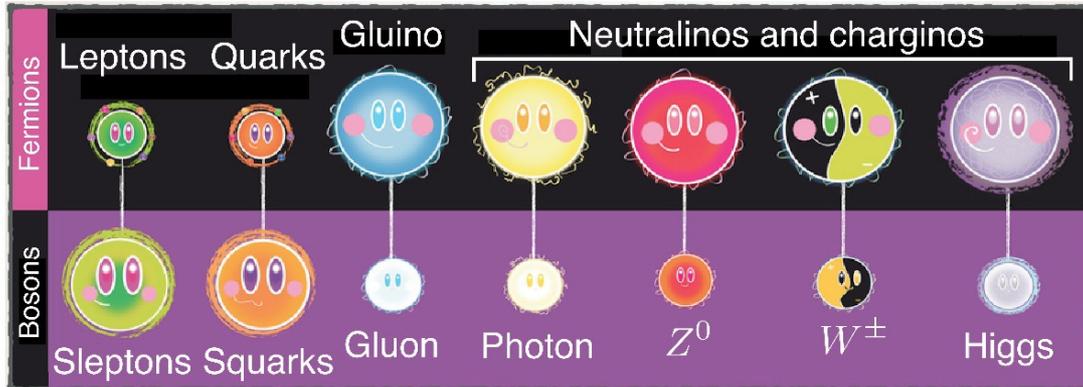


One needs heavy stop(s) to obtain a 125GeV Higgs (within the MSSM)

Current lower limits on stop mass: $\sim 1.3 \text{ TeV}$ (depends on stop decay & LSP mass)



Natural SUSY: where is everybody



Two-loops: gluinos $\propto \left(\frac{16 \pi^2}{\text{Log}} m_h \right)$

One-loop: stops $\propto \left(\sqrt{\frac{16 \pi^2}{\text{Log}}} m_h \right)$

Tree-level: Higgsinos $\propto (m_h)$

HIGGSINO
 $\mu \lesssim 200 \text{ GeV} \left(\frac{\Delta^{-1}}{20\%} \right)^{-1/2}$

STOP
 $m_{\text{stop}} \lesssim 500 \text{ GeV} \frac{\sin \beta}{\sqrt{1 + (A_t/m_{\text{stop}})^2}} \sqrt{\frac{3}{\log(\Lambda/\text{TeV})}} \left(\frac{\Delta^{-1}}{20\%} \right)^{-1/2}$

GLUINO
 $m_{\text{gluino}} \lesssim 1000 \text{ GeV} \sin \beta \frac{3}{\log(\Lambda/\text{TeV})} \left(\frac{\Delta^{-1}}{20\%} \right)^{-1/2}$

TIM COHEN [UNIVERSITY OF OREGON]

NMSSM (next-to-MSSM) includes additional H singlet requiring much less fine tuning to get $m_h \approx 125 \text{ GeV}$

$$\text{Tuning: } \Delta \equiv \frac{2 \delta m_H^2}{m_h^2}$$

BSM

SO

CERN, July 2017

LHC_{300fb⁻¹} will tell!

Good coverage of hidden natural susy

- ▶ mono-top searches (DM, flavored naturalness - mixing among different squark flavors-, stop-higgsino mixings)
- ▶ mono-jet searches with ISR recoil (compressed spectra)
- ▶ precise tt inclusive measurement+ spin correlations (stop \rightarrow top + very soft neutralino)
- ▶ multi-hard-jets (RPV, hidden valleys, long decay chains)

SUSY is Natural but not plain vanilla

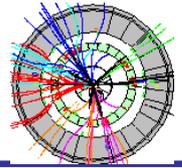
- ❌ CMSSM
- ❌ pMSSM
- ❌ NMSSM
- ❌ Hide SUSY, e.g. smaller phase space

pMSSM treats 3rd generation SUSY parameters independently from 1st & 2nd

- ▶ reduce production (eg. split families) Mahbubani et al
- ▶ reduce MET (e.g. R-parity, compressed spectrum) Csaki et al
- ▶ dilute MET (decay to invisible particles with more invisible particles)
- ▶ soften MET (stealth susy, stop-top degeneracy) Fan et al



Gravity

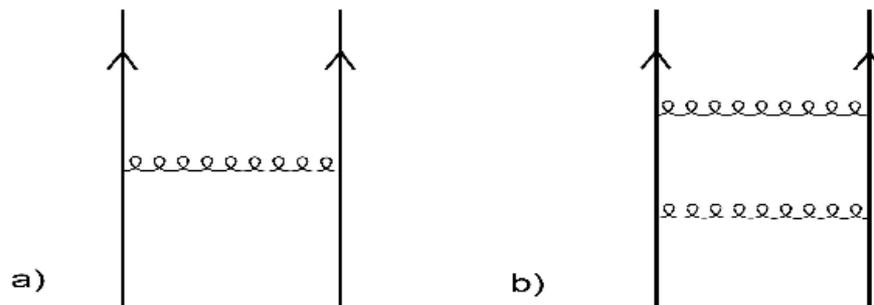


Gravity and the SM

The existence of gravity is the most solid argument that the SM is not the final theory.

- Gravity interacts with SM fields.
- At some high energy scale, Λ_P gravity will become strong, and quantum effects must be incorporated. This scale could be $M_P \sim 10^{19}$ GeV but (as we will see later) it could also be much lower.
- This fundamental theory, would look like classical gravity plus the SM at energies $E \ll \Lambda_P$.
- In this sense the SM is an effective theory, valid (at most) up to Λ_P .
- Things look bad, since classical gravity (general relativity) is a non-renormalizable theory.

Gravity at short distances?



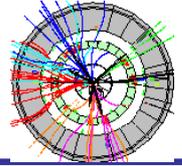
- The classical gravitational theory is non-renormalizable

$$(b) \sim \frac{E^2}{M_{\text{Planck}}^4} \int_0^\Lambda dp p \sim \frac{\Lambda^2 E^2}{M_{\text{Planck}}^4},$$

- At higher orders it gets worse and worse.
- No clue as to what the short distance theory is.
- This has been an open problem for more than 50 years.



Gravity and string theory



Gravity and String Theory

- **String theory** is a different framework for describing and unifying all interactions.
- It has become popular because it always includes quantum gravity, without UV problems (divergences)
- Moreover it also includes the other ingredients of the SM: Gauge interactions, chiral matter (fermions) and if needed, supersymmetry.
- It offers some conceptual features that are appealing to physicists:
 - (a) **String theory ALWAYS contains gravity**
 - (b) **The existence of fermions implies supersymmetry at high energy.**
 - (c) **It has a priori no fundamental parameters but only one dimensionfull scale: the size of the strings.** All dimensionless parameters of a given ground state of the theory are “dynamical” (expectation values of scalar fields).
 - (d) **It contains solitonic extended objects (known as branes) that provide an incredible richness to the theory as well as a deep link between gauge theories and gravity.**

What is String Theory?

Shift in paradigm: from point particle to a closed string.

- **In QFT fields are “point-like”. In string theory, they depend not on a point of space-time but a loop in space-time (the position of a closed string).**

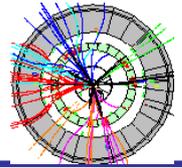
What is the difference between a closed “fundamental” string and a loop of wire?

- (A) **The fundamental string is much smaller: its size is definitely smaller than 10^{-18} m. This would explain why we have not seen one so far.**
- (B) **Apart from the usual degrees of freedom (their coordinates in space-time), fundamental strings have also fermionic degrees of freedom. There a kind of supersymmetry relating the coordinates to such fermionic degrees of freedom.**

Since the smallest length we can see today (with accelerators) is approximately 10^{-18} m strings would appear in experiments so far as point-like objects.



String theory



String Theory, Vol II

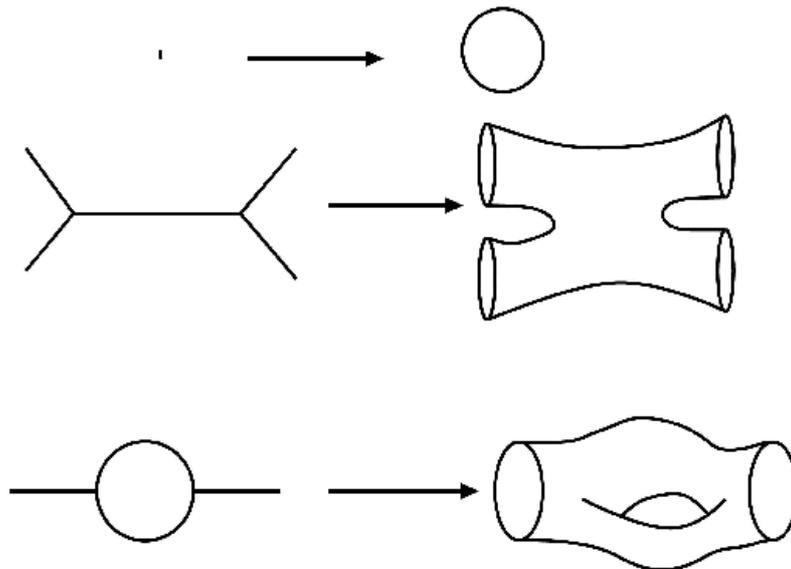
- Fundamental strings, like the analogous classical objects, can vibrate in an infinite possible number of harmonics.
- Upon quantization, these harmonics behave like different particles in space-time.

A single string upon quantization \Rightarrow an infinite number of particles with ever increasing mass.

- Infinity of particles is responsible for the unusual properties of string theory (and its complicated structure).
- Strings live in diverse dimensions. Lorentz invariances \Leftrightarrow 9+1 dimensions. Although this seems to contradict common experience it can be compatible under certain circumstances. **How do we see the extra dimensions?** More on this later

String Theory, Vol III

- In perturbation theory, standard QFT Feynman diagrams are replaced with string diagrams (two-dimensional surfaces)

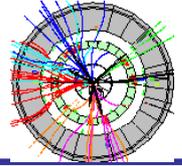


Beyond the Standard Model, E. Kiritsis

70

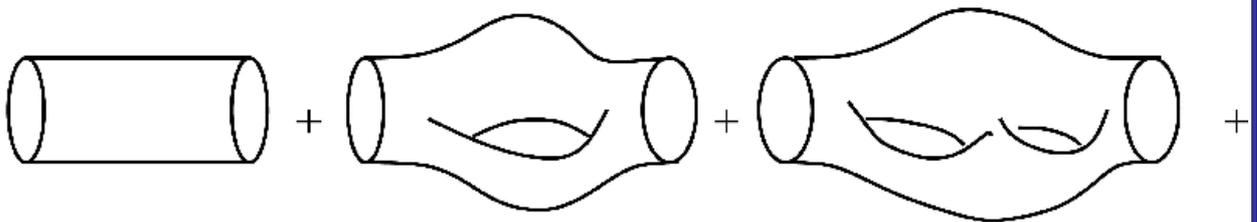


String theory



String perturbation theory

- ♣ In QFT perturbation theory is formulated using Feynman diagrams.
- ♠ In string theory we have Riemann surfaces. For closed strings, each order contains a single diagram. At low energy, they reduce to the (many) QFT Feynman diagrams.

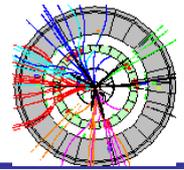


- String theory diagrams, when appropriately defined, give finite amplitudes in the UV. Quantum gravity, which is part of string theory is essentially finite.

Extra space dimensions

- The idea that space has extra, hitherto unobservable dimensions goes back to the beginning of the twentieth century, with Nordström (1914), Kaluza (1925) and Klein (1926).
- It comes naturally in string theory.
How come they are not visible today?

- (A) Because they compact and sufficiently small.
- (B) Because we are “stuck” on the four-dimensional world.
- (C) Because they are of a more bizarre kind (for example, they are discretized appropriately)



Extra dimensional models

- Problematic aspect of SM:

$$v_{EW} \ll M_{GUT}, M_{Pl}$$

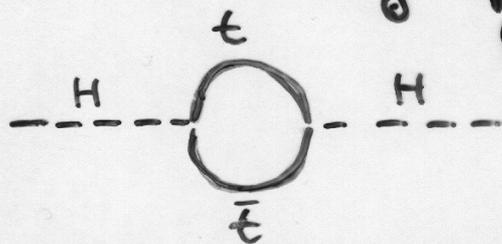
Electroweak scale: $v_{EW} \approx 246 \text{ GeV} (2 \cdot 10^{-18} \text{ m})$

Mass scale of (quantum) gravity:

$$M_{Pl} = \left(\frac{\hbar c}{G_N}\right)^{\frac{1}{2}} = 1.2 \cdot 10^{19} \text{ GeV} (2 \cdot 10^{-32} \text{ m})$$

This large separation leads to:

Hierarchy problem:



(how keep $m_H \sim v_{EW}$)

$$\delta m_H^2 \sim \Lambda^2$$

$$\Lambda \sim M_{GUT}$$

possible solutions:

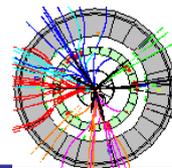
- (a) connect bosons & fermions by symmetry \Rightarrow SUPERSYMMETRY

boson + fermion loops cancel !

fundamental scale M_{Pl}



Large extra dimensions



(b) exploit geometry of space time
& fundamental scale v_{EW}

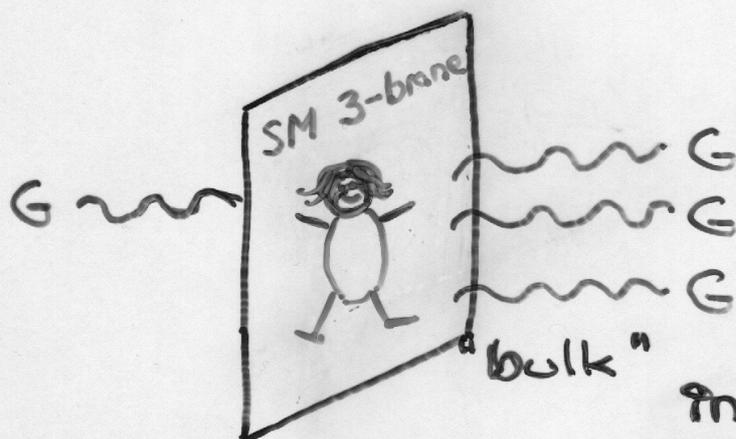
$$\Rightarrow \Lambda \sim v_{EW}$$

NB! here M_{Pl} only derived effect



- (large) extra spatial dimensions
- confinement of matter on subspace

Natural setting in string theory!



SM confined

to 3+1 dimensions
("3-brane")

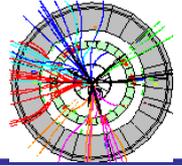
Gravity propagates
in D-dimensional
space-time ("bulk")

$$D \equiv (3 + d + 1)$$

extra dimensions!

(c) mere coincidence, corrections cancel!

$$M_a^2 - M_b^2 \sim v_{EW}^2; M_a, M_b \sim M_{GUT}$$



Compute Newtons Constant

- Einstein action in D -dimensions :

$$S_E^D = \frac{1}{16\pi \hat{G}_N} \int d^D x \sqrt{-\hat{g}} R(\hat{g})$$

("^" D -dimensional equivalents)

Assume factorizable geometry:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + h_{ij}(y) dy^i dy^j$$

$$\mu, \nu = (0 \dots 3) ; i, j = (1 \dots \delta)$$

and that the extra dimensions, δ , are compactified on circles with radius $R_c \rightarrow V_c = 2\pi R_c \rightarrow$

- Effective action in 4D :

$$S_E = \frac{V_c^\delta}{16\pi \hat{G}_N} \int d^4 x \sqrt{-g} R(g)$$

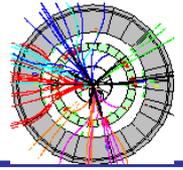
$$\Rightarrow \frac{1}{G_N} = \frac{V_c^\delta}{\hat{G}_N} ; \hat{G}_N = \frac{1}{M_D^{D-2}}$$

$$M_{Pl} \equiv \sqrt{\hbar c / G_N}$$

$$\Rightarrow M_{Pl} = M_D^{\frac{\delta+2}{2}} V_c^{\delta/2} \approx M_D^{\frac{\delta+2}{2}} R_c^{\frac{\delta}{2}}$$



Large extra dimensions



M_D fundamental scale?

Possible for $M_D \sim \text{TeV}$? \rightarrow

M_{Pl} large if $V_c(R_c)$ large

Arkani Hamed - Dimopoulos - Dvali 98

NB! electroweak & strong tested to $\sim 10^{-18}$ m, gravity to $\sim 10^{-4}$ m

• R_c (radius of compactification):

{	10^{12} m $\sim (10^{-19} \text{ eV})^{-1}$	$\delta=1$	excluded
	$5 \cdot 10^{-4}$ m $\sim (10^{-4} \text{ eV})^{-1}$	$\delta=2$	not a priori excluded
	10^{-8} m $\sim (10 \text{ eV})^{-1}$	$\delta=3$	
	10^{-14} m $\sim (10^7 \text{ eV})^{-1}$	$\delta=6$	

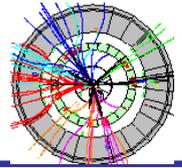
if $M_D \sim 1 \text{ TeV}$

Gravity weak because it is diluted by a large space!

NB! No convincing explanation why $R_c \gg 1/M_D$, in fact leads to a new hierarchy problem that might require supersymmetry in the extradimensional bulk.



Large extra dimensions



- Gravitational interactions modified at small distances:

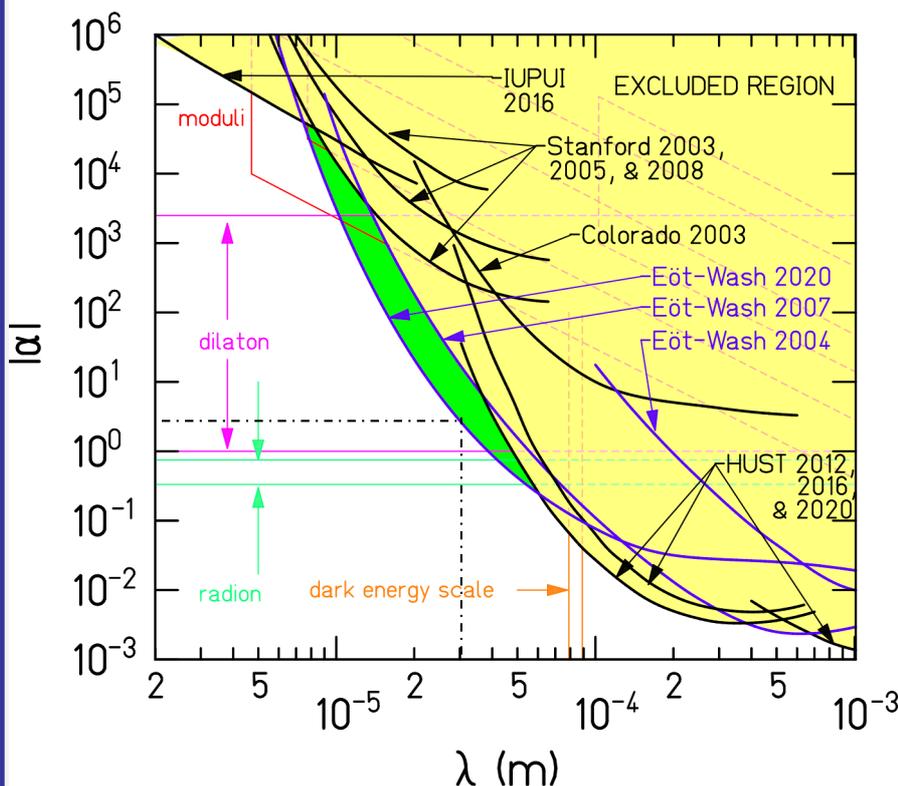
$$V(r) = G_N \frac{m_1 m_2}{r} \quad \text{at } r \gg R_c$$

$$V(r) = G_N \frac{m_1 m_2}{r^{1+\delta}} = G_N R_c^\delta \frac{m_1 m_2}{r^{1+\delta}}$$

at $r \ll R_c$

- Experimental tests of Newton's Law: (torsion experiments)

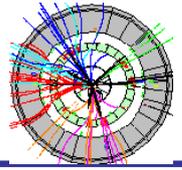
$$V(r) = -G_N \frac{m_1 m_2}{r} (1 + \alpha e^{-r/\lambda})$$



$R_c < 30 \mu\text{m}$
 for $\delta = 2$ ($\Rightarrow |\alpha| = 16/3$)
 leading to $M_D \geq 4.0 \text{ TeV}$.
 Bounds from astrophysics tighter but these limits from torsion experiments more general.



First ideas for extra dimensions



Nordström-Kaluza-Klein unification

<https://www.mv.helsinki.fi/home/eisakso/nordstrom/nordstrom.html>

1914 - Gunnar Nordström: Unification of scalar gravity with electromagn. in 5D:

$$A_M (M=0,1,2,3,5) \Rightarrow A_\mu (\mu=0,1,2,3) + A_5$$

where scalar field ϕ is a gravitational field coupled to T_μ^μ

1915 - Albert Einstein: General relativity gravitational field is a tensor $g_{\mu\nu}(x^M) T^{\mu\nu}$

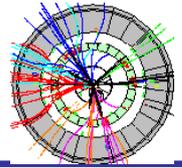
1921 - Theodor Kaluza: General relativity in 5D as an unified theory of gravity and electromagnetism

1926 - Oscar Klein: rediscovered Kaluza's theory and gave a geometrical interpretation of extra dimension (compactness)

4D gravity and electromagnetism unified in 5D! Charge & masses quantized? $R_c = \sqrt{\frac{4G_N}{\alpha}} \approx 10^{-31} \text{ cm}$

Doesn't describe the real world!

charge states of $m \sim M_{Pl}$, $F_{\mu\nu} F^{\mu\nu} = 0 \dots$



Manifestations in 4D

- example: massless free scalar $\varphi(x^M, y)$ field in 5D; $x^5 \equiv y$ is a circle of radius R_c , i.e. $y + 2\pi R_c \approx y$

Klein-Gordon equation:

$$\left. \begin{aligned} (\partial_\mu \partial^\mu - \partial_y^2) \varphi(x^M, y) &= 0 \\ \varphi(x^M, y + 2\pi R_c) &= \varphi(x^M, y) \end{aligned} \right\}$$

$$\varphi(x^M, y) = \sum_{n=-\infty}^{\infty} \phi_n(x^M) f_n(y)$$

$$f_n(y) = e^{iny/R_c}; \quad n=0, \pm 1, \pm 2, \dots$$

solution same as QMs "particle in a box"

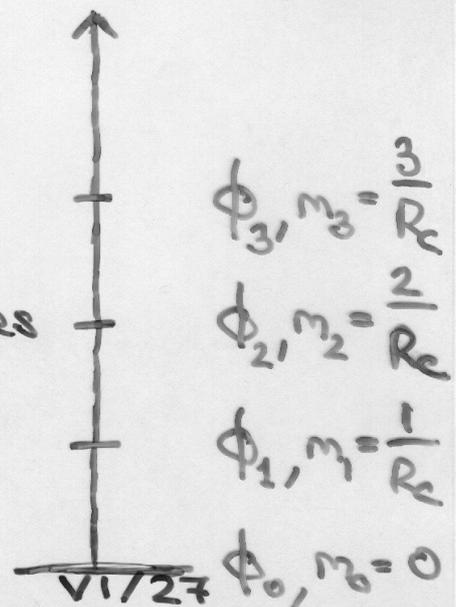
$$(\partial_\mu \partial^\mu + m_n^2) \phi_n(x^M) = 0;$$

$$m_n^2 = \frac{n^2}{R_c^2}$$

Get in 4D a tower of "Kaluza-Klein" (KK)-states with equal quantum numbers!

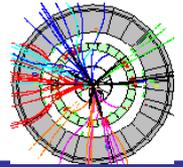
Beyond SM

KK





Predictions of extra dimensional models



- Mass splitting between gravitons

$$\Delta m \sim \frac{1}{R_e} = M_D \left(\frac{M_D}{M_{Pl}} \right)^{2/d}$$

$$\begin{cases} 5 \cdot 10^{-4} \text{ eV} & d=2 \\ 20 \text{ keV} & d=4 \end{cases} \quad \text{a continuum of states!}$$

- Probability for producing a KK graviton $\alpha_{\text{grav}} \sim E^2 / M_{Pl}^2$; impossible to see!

of KK states with $m_n < E$ LHC@13 TeV: $M_D > 5.9 - 11.2$ TeV

$$\sim E^d M_{Pl}^2 / M_D^{2+d} \rightarrow$$

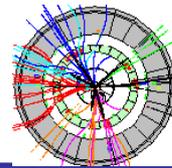
e.g. $\sum_n \sigma(pp \rightarrow G_n \text{ jet}) \approx \frac{\alpha_s}{\pi} \frac{E^d}{M_D^{2+d}}$

$\sigma(pp \rightarrow G_n \gamma) \Rightarrow$ LHC@13 TeV: $M_D > 2.9$ TeV

accessible for colliders if $M_D \sim O(\text{TeV})$

- will give supersymmetry-like missing energy signal when gravitons escape into the bulk
- also $2 \rightarrow 2$ scattering processes modified through virtual graviton exchange ("contact" interaction type)

Beyond SM $\times \sim 4/M_{TT}^4$ LHC @13 TeV: $M_{TT} > 6-9$ TeV



Mini black hole - production?

- Schwarzschild radius, R_s , (i.e. within which nothing escapes gravitation)

$$4D : R_s \sim \frac{2M_{BH}}{M_{Pl}^2}$$

Semi-classical BH ($\sqrt{s} \gg M_D$): $M_{BH} > 9.0-10.1 \text{ TeV}$

$$DD : R_s \sim \frac{1}{M_D} \left(\frac{M_{BH}}{M_D} \right)^{\frac{1}{\delta+1}}$$

Quantum BH ($\sqrt{s} \approx M_D$):
2-particle decay

$M_{QBH} > 2.3-9.4 \text{ TeV}$
(large dependence on model)

- If $M_D \sim 1 \text{ TeV}$ and $\sqrt{s} > M_D > R_s^{-1}$, tiny black holes ($M_{BH} \sim \text{TeV}$) can be produced if two partons pass at a distance, b , $< R_s$. Cross sections large!
 $\sim \pi R_s^2$

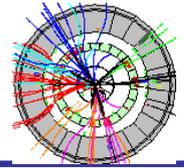
$$\sigma(pp \rightarrow BH) \sim 100 \text{ fb} \quad (M_D \sim 3 \text{ TeV}, \delta=4)$$

\Rightarrow 1000 events/year at low \sqrt{s} LHC

- Mini black holes decay immediately ($\tau \sim 10^{-26} \text{ s}$) by evaporation to q, l, \dots

expected signature: \sim spherical events with many high energy jets, leptons, γ 's

NB! BH's should be produced also in cosmic ray experiments



Most significant constraints
come from astrophysics!

- Copious emission of KK gravitons and ν compete in Supernova cooling

neutrinos $\sim G_F^2 T^2$; gravitons $\sim \frac{T^\delta}{M_D^{2+\delta}}$

SN1987A: $M_D > 27 \text{ TeV}$ for $\delta = 2$

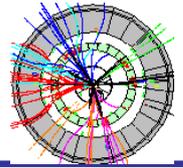
$M_D > 2.4 \text{ TeV}$ for $\delta = 3$

- $G \rightarrow \gamma\gamma$ decay distorts cosmic γ -ray background and leads to anomalous heating of neutron stars

$\tau_G \sim 3 \cdot 10^9 \left(\frac{100 \text{ MeV}}{m_G} \right)^3 \text{ years}$

Neutron stars: $M_D > 1700 \text{ TeV}$ for $\delta = 2$
 $M_D > 76 \text{ TeV}$ for $\delta = 3$ } reduced if KK gravitons decay mainly to non-SM particles

- Cosmology: Relic KK gravitons contribution to cosmic gamma radiation: $M_D > 100 \text{ TeV}$ for $\delta = 2$
- NB! Very weak limits from astrophysics & cosmology if $\delta \geq 4 \Rightarrow$ focus in collider searches



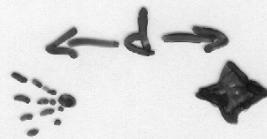
Non-factorizable geometry

Randall-Sundrum g_9

- A classical mechanism to make quanta softer



observer



emitter star

- Time independent metrics $g_{0\mu} = 0$

$\Rightarrow E \sqrt{|g_{00}|}$ conserved (proper time)
 $dz^2 = g_{00} dt^2$

Schwarzschild metric $g_{00} = 1 - \frac{2G_N M}{d}$

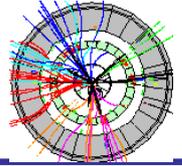
$$\frac{E_{obs} - E_{em}}{E_{em}} = \sqrt{|g_{00}|} - 1 \approx -\frac{G_N M}{d}$$

- In non-trivial metrics, we see far-away objects as red-shifted

("curved" or "warped" space time)



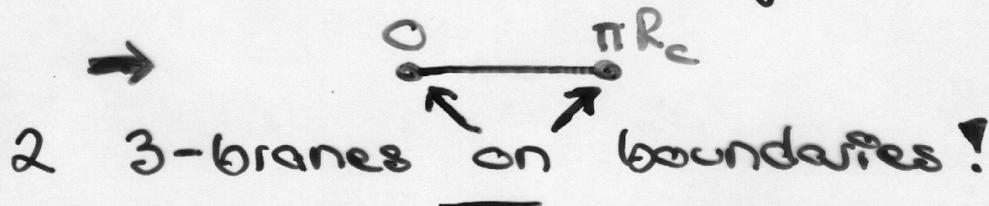
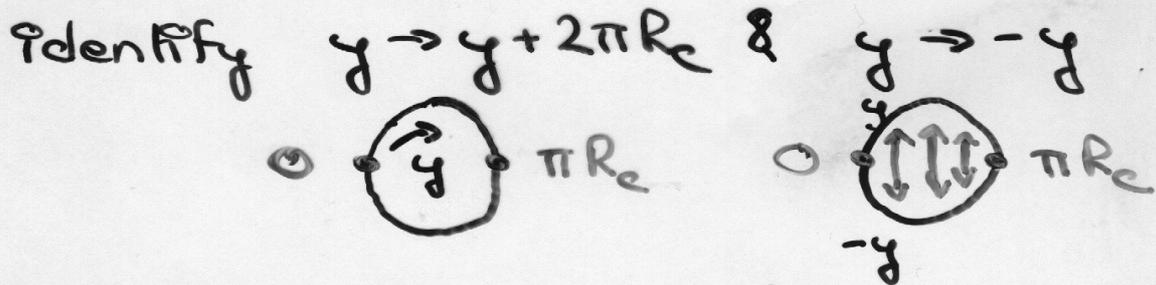
Non-factorizable geometry



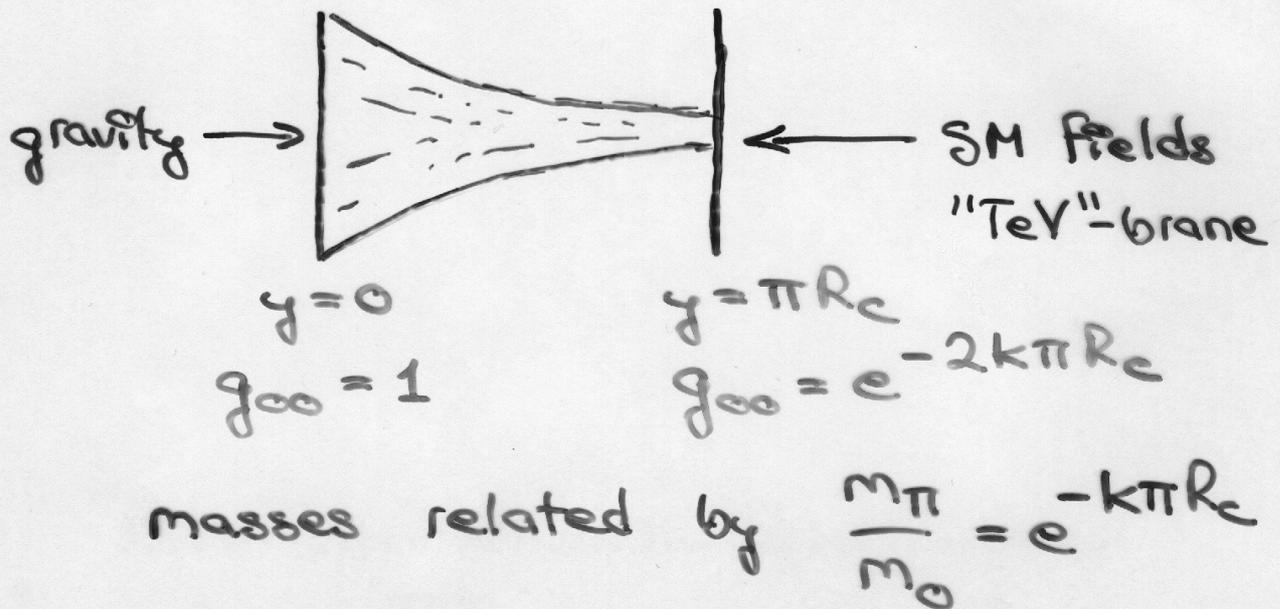
- Non-factorizable geometry in 5D:

$$ds^2 = e^{-2ky} g_{\mu\nu} dx^\mu dx^\nu - dy^2$$

- 5th dimension S_1 / \mathbb{Z}_2



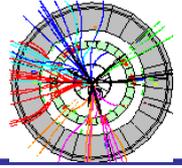
- Gravitational redshift!



$$R_c \approx 11/k \Rightarrow m_\pi/m_0 \approx 0(1 \text{ TeV})/M_{\text{Pl}}$$



Non-factorizable geometry



$$\Rightarrow M_{Pl}^2 = \frac{M_D^3}{k} (1 - e^{-2k\pi R_c})$$

$$\Rightarrow M_D \sim k \sim \frac{11}{R_c} \sim M_{Pl}$$

(not a model with large extra dimensions)

- Effective theory on TeV-brane:

$$\Lambda_\pi = M_{Pl} e^{-k\pi R_c} \sim 1 \text{ TeV}$$

- Masses of bulk graviton KK towers:

$$m_n = x_n k e^{-k\pi R_c} = x_n k \frac{\Lambda_\pi}{M_{Pl}}$$

x_n roots of first order

Bessel functions

$$\begin{cases} x_1 = 3.8 \\ x_2 = 7.0 \\ x_n \approx (n + \frac{1}{4})\pi \end{cases}$$

- KK states not evenly spaced
- characteristic mass $k \frac{\Lambda_\pi}{M_{Pl}} \sim \text{TeV}$
- couplings ("strong" for excitations)

$$\mathcal{L} = - \frac{h_{\mu\nu}^{(0)}}{M_{Pl}} T^{\mu\nu} - \frac{T^{\mu\nu}}{\Lambda_\pi} \sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}$$

Beyond SM

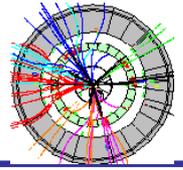
$k \sim 11/R_c$

Λ_π

$n=1$

$h_{\mu\nu}^{(n)}$

VI/33



Physical Interpretation:

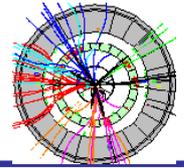
- Gravity concentrated at $y=0$, our world confined at $y=\pi R_c$
small overlap \Rightarrow gravity seems weak
- Graviton KK states are not equally spaced and couples strongly to matter; can be observed as resonances
- Lightest new state in this scenario radion (or graviscalar), r ,
$$O(10 \text{ GeV}) \lesssim m_r \lesssim \Lambda_\pi$$

Dominant decay mode $r \rightarrow gg$ if $m_r \lesssim 2M_w$, otherwise $r \rightarrow W^+W^-, ZZ, hh$

couplings $\approx v_{EW} / (\sqrt{24}\Lambda_\pi) \cdot \text{SM couplings}$

radion originates from quantum excitations of the distance between branes

radion can mix with Higgs and alter g_{Hff} and g_{HVV} !



Constraints on non-factorizable models

• most astrophysical limits weak

• best limits from LHC

$G \rightarrow \gamma\gamma/ee/\mu\mu$ $m_G > 2.2 - 5.6$ TeV

Radion (R) $\rightarrow WW/ZZ$ $m_R > 3.2$ TeV
(no SM fields in the bulk)

electroweak tests: $m_G > \sim 10$ TeV (if SM gauge bosons in the bulk)

$G \rightarrow WW/ZZ/tt$ $m_G > 1.8 - 3.7$ (VV) – 3.4-4.55 (tt) TeV

KK gluon $\rightarrow gR \rightarrow ggg/WW$ $m_{gKK}(m_R) > 3.5$ (2.2) TeV
(SM fields also in the bulk)

Open problems in extradimensional models

• no large mass scales to suppress violation of approximate symmetries (proton decay, flavour changing

neutral currents, neutrino mass)
exception warped space time & no SM fields in bulk

• unification of gauge couplings

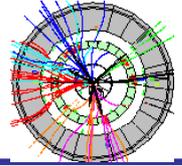
• cosmology, baryogenesis ...

• no theory at $\sqrt{s} \sim M_0$!

quantum gravity? string theory?

For more details see PDG review on Extra dimensions searches

<https://pdg.lbl.gov/2025/reviews/rpp2025-rev-extra-dimensions.pdf>



What is physics beyond the Standard Model?



I don't know. Nobody knows

If it were known, it would be part of the SM!

You ~~won't learn~~ ^{haven't learned} during these lectures what BSM is.

(maybe) ~~You'll learn~~ ^{you learned} what BSM could be.

" Looking and not finding is different than not looking "

We'll study the limitations/defaults of the SM as a guide towards BSM.

We want to learn from our failures

The hierarchy problem made easy

only a few electrons are enough to lift your hair ($\sim 10^{25}$ mass of e^-)
the electric force between 2 e^- is 10^{43} times larger than their gravitational interaction



we don't know why gravity is so weak?

we don't know why the masses of particles are so small?

Several theoretical hypothesis

new dynamics? new symmetries? new space-time structure?

modification of special relativity? of quantum mechanics?
