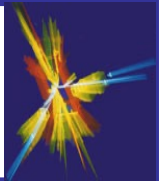
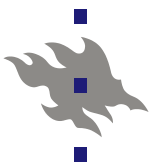


Electroweak unification

- ◆ **Experimental facts**
- ◆ **$SU(2)_L \otimes U(1)_Y$ gauge theory**
- ◆ **Charged current interaction**
- ◆ **Neutral current interaction**
- ◆ **Gauge self-interactions**



EXPERIMENTAL FACTS

♠ Family Structure

$$\begin{bmatrix} \nu_l & q_u \\ l^- & q_d \end{bmatrix} \equiv \begin{cases} \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L, & (\nu_l)_R, & l_R^- \\ \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L, & (q_u)_R, & (q_d)_R \end{cases}$$

♠ Three Families

$$\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix}, \quad \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix}, \quad \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}$$

♠ Charged Currents

$W^\pm$

- Left-handed fermions only
- Flavour Changing: $\nu_l \Leftrightarrow l_l^-$, $q_u \Leftrightarrow q_d$

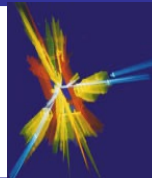
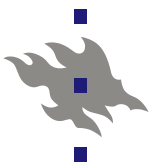
♠ Neutral Currents

γ, Z

- Flavour Conserving

♠ Universality (Family-Independent Couplings)

♠ $(\nu)_R$?



$SU(2)_L \otimes U(1)_Y$ GAUGE THEORY

Fields	$\psi_1(x)$	$\psi_2(x)$	$\psi_3(x)$
Quarks	$\begin{pmatrix} q_u \\ q_d \end{pmatrix}_L$	$(q_u)_R$	$(q_d)_R$
Leptons	$\begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L$	$(\nu_l)_R$	$(l^-)_R$

Free Lagrangian for Massless Fermions:

$$\mathcal{L}_0 = \sum_j i \bar{\psi}_j \gamma^\mu \partial_\mu \psi_j$$

$\vec{\tau} =$ Pauli matrices

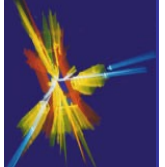
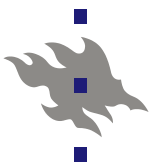
$SU(2)_L \otimes U(1)_Y$ Flavour Symmetry:

$$\psi_1 \rightarrow \exp\{i \vec{\tau} \cdot \vec{\alpha}/2\} \exp\{iy_1\beta/2\} \psi_1$$

$$\bar{\psi}_1 \rightarrow \exp\{-i \vec{\tau} \cdot \vec{\alpha}/2\} \exp\{-iy_1\beta/2\} \bar{\psi}_1$$

$$\psi_2 \rightarrow \exp\{iy_2\beta/2\} \psi_2 \quad \bar{\psi}_2 \rightarrow \exp\{-iy_2\beta/2\} \bar{\psi}_2$$

$$\psi_3 \rightarrow \exp\{iy_3\beta/2\} \psi_3 \quad \bar{\psi}_3 \rightarrow \exp\{-iy_3\beta/2\} \bar{\psi}_3$$



Gauge Principle: $\vec{\alpha} = \vec{\alpha}(x), \beta = \beta(x)$

$$D_\mu \psi_1(x) \equiv [\partial_\mu - igW_\mu(x) - ig'(y_1/2)B_\mu(x)]\psi_1(x)$$

$$D_\mu \psi_1(x) \rightarrow U(x) \exp\{i(y_1/2)\beta(x)\} D_\mu \psi_1(x)$$

$$B_\mu(x) \rightarrow B_\mu(x) + (1/g')\partial_\mu\beta(x)$$

$$W_\mu(x) \rightarrow U(x)W_\mu(x)U^\dagger(x) + (1/g)U(x)\partial_\mu U^\dagger(x)$$

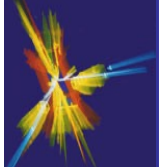
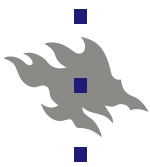
g (g') coupling connected to $SU(2)_L$ ($U(1)_Y$) interaction

$$W_\mu(x) \equiv \vec{\tau}/2 \cdot \vec{W}_\mu(x) ; \quad U(x) \equiv \exp\{i\vec{\tau}/2 \cdot \vec{\alpha}(x)\}$$

$$W_\mu^i \rightarrow W_\mu^i + \frac{1}{g}\partial_\mu\alpha^i - \varepsilon^{ijk}\alpha^j W_\mu^k + O(\alpha^j\alpha^k)$$

4 Massless Gauge Bosons

$$W_\mu^\pm, \quad W_\mu^3, \quad B_\mu^0$$



CHARGED CURRENTS

$$\sum_j i \bar{\psi}_j \gamma^\mu \mathbf{D}_\mu \psi_j$$
$$\longrightarrow g \bar{\psi}_1 \gamma^\mu \mathbf{W}_\mu \psi_1 + g' B_\mu \sum_j y_j/2 \bar{\psi}_j \gamma^\mu \psi_j$$

$$\mathbf{W}_\mu \equiv \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu = \frac{1}{2} \begin{pmatrix} W_\mu^3 & \sqrt{2} W_\mu^\dagger \\ \sqrt{2} W_\mu & -W_\mu^3 \end{pmatrix}$$

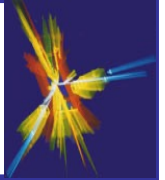
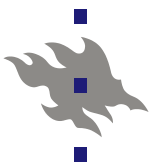
$$W_\mu \equiv W_\mu^1 + i W_\mu^2$$

$(1 - \gamma_5)$ takes only left-handed (right-handed) component of the fermion (antifermion) field

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} W_\mu^\dagger [\bar{q}_u \gamma^\mu (1 - \gamma_5) q_d + \bar{\nu}_l \gamma^\mu (1 - \gamma_5) l]$$
$$+ \text{h.c.}$$

Quark / Lepton Universality

Left-Handed Interaction



NEUTRAL CURRENTS

$$\mathcal{L}_{NC} = \sum_j \bar{\psi}_j \gamma^\mu \left[g \frac{\tau_3}{2} W_\mu^3 + g' y_j/2 B_\mu \right] \psi_j$$

Massless Fields $\rightarrow$ Arbitrary Combination

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

Impose condition: existence of QED with photon exchange

A_μ has the **QED** Interaction **IF**

$$g \sin \theta_W = g' \cos \theta_W = e \quad ; \quad y = 2 (Q - T_3)$$

$$(y_1/2 = Q_u - \frac{1}{2} = Q_d + \frac{1}{2} , \quad y_2/2 = Q_u , \quad y_3/2 = Q_d)$$



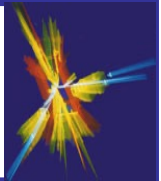
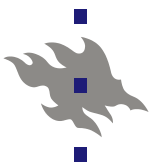
Electroweak Unification

$\theta_W \equiv$ Weinberg angle

$$\mathcal{L}_{NC} = e A_\mu \sum_j \bar{\psi}_j \gamma^\mu Q_j \psi_j + \mathcal{L}_{NC}^Z$$

$$Q_1 = \begin{pmatrix} Q_u & 0 \\ 0 & Q_d \end{pmatrix} \quad ; \quad Q_2 = Q_u \quad ; \quad Q_3 = Q_d$$

Nobel prize in physics 1979: Glashow, Salam & Weinberg



$$\begin{aligned}\mathcal{L}_{NC}^Z &= \frac{e}{\sin\theta_W \cos\theta_W} Z_\mu \sum_j \bar{\psi}_j \gamma^\mu \left[\frac{\tau_3}{2} - \sin^2\theta_W Q_j \right] \psi_j \\ &= \frac{e}{2 \sin\theta_W \cos\theta_W} Z_\mu \sum_f \bar{f} \gamma^\mu \left[v_f - a_f \gamma_5 \right] f\end{aligned}$$

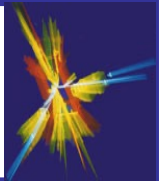
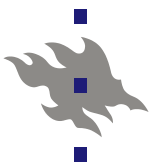
	$2v_f$	$2a_f$
qu	$1 - \frac{8}{3} \sin^2\theta_W$	1
qd	$-1 + \frac{4}{3} \sin^2\theta_W$	-1
ν_l	1	1
l^-	$-1 + 4 \sin^2\theta_W$	-1

$$v_f = T_f^3 - 2Q_f \sin^2\theta_W \quad a_f = T_f^3, \quad T_f^3 \text{ is the weak isospin}$$

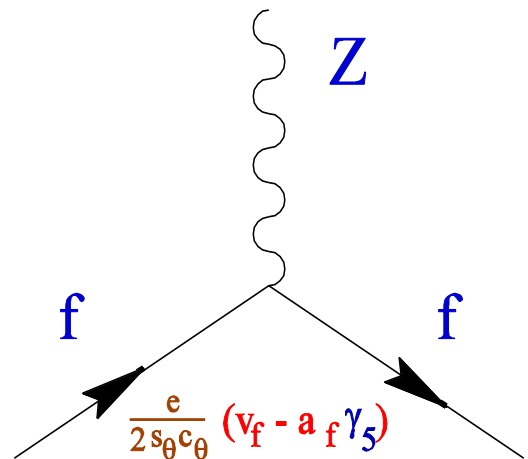
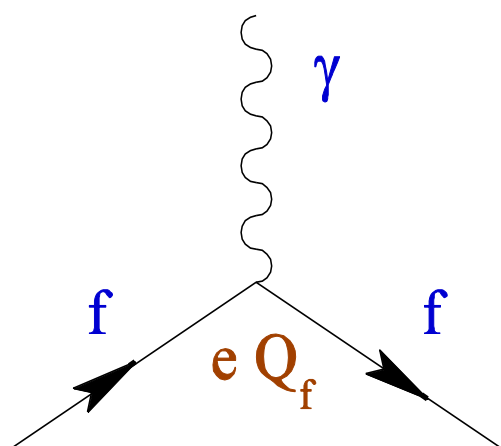
♠ IF ν_R do exist :

$$y(\nu_R) = Q_\nu = 0 \quad \longrightarrow \quad \text{No } \nu_R \text{ Interactions}$$

Sterile Neutrinos

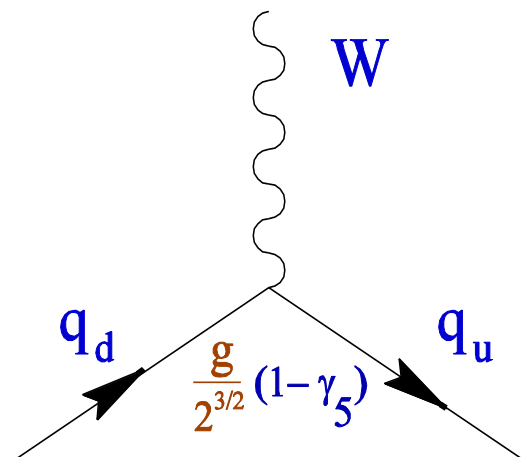
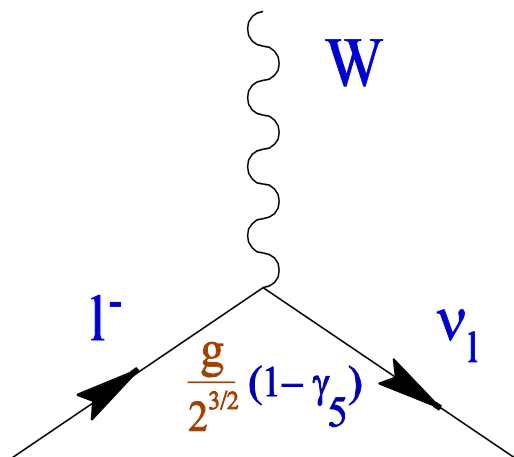


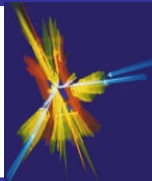
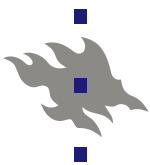
NEUTRAL CURRENTS



$$s_\theta = \sin \theta_W \quad c_\theta = \cos \theta_W$$

CHARGED CURRENTS





$$\mathbf{W}_{\mu\nu} \equiv \frac{i}{g} [\mathbf{D}_\mu, \mathbf{D}_\nu] \equiv \frac{\vec{\tau}}{2} \cdot \vec{W}_{\mu\nu} \longrightarrow \mathbf{U} \mathbf{W}_{\mu\nu} \mathbf{U}^\dagger$$

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g \epsilon^{ijk} W_\mu^j W_\nu^k$$

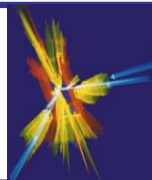
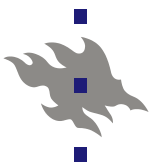
$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \longrightarrow B_{\mu\nu}$$

$$\begin{aligned} \mathcal{L}_K &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \text{Tr}(\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}) \\ &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} \vec{W}_{\mu\nu} \cdot \vec{W}_{\mu\nu} \\ &= \mathcal{L}_{\text{kin}} + \mathcal{L}_3 + \mathcal{L}_4 \end{aligned}$$

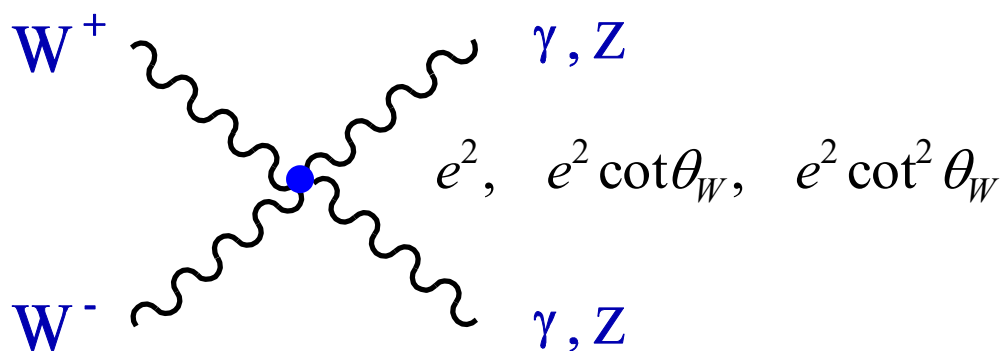
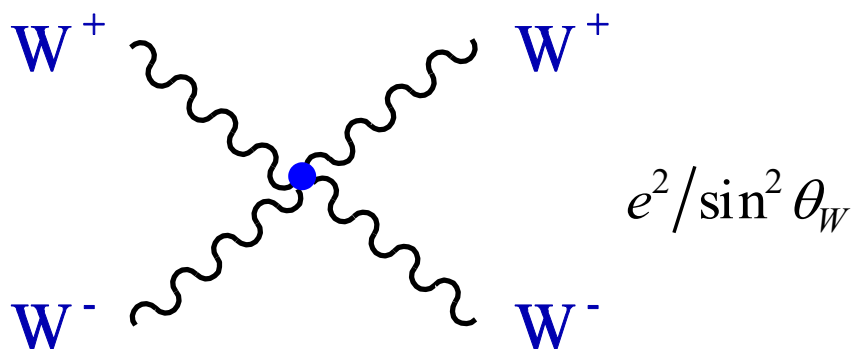
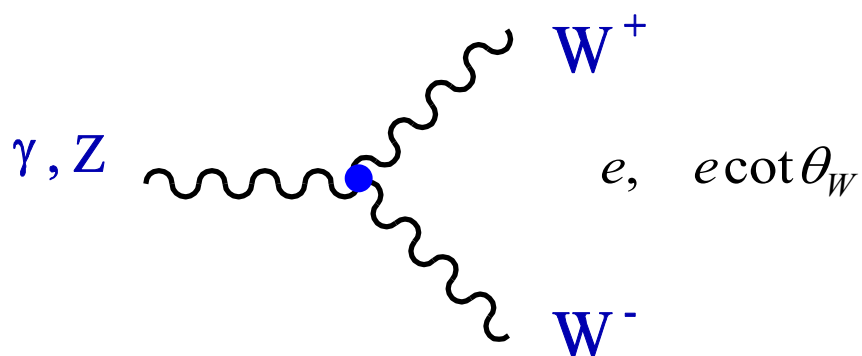
$$\begin{aligned} \mathcal{L}_3 = -ie \cot \theta_W \{ &(\partial^\mu W^\nu - \partial^\nu W^\mu) W_\mu^\dagger Z_\nu - (\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}) W_\mu Z_\nu \\ &+ W_\mu W_\nu^\dagger (\partial^\mu Z^\nu - \partial^\nu Z^\mu) \} \end{aligned}$$

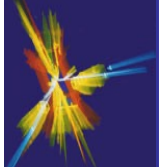
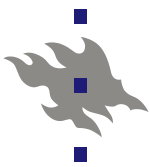
$$\begin{aligned} -ie \{ &(\partial^\mu W^\nu - \partial^\nu W^\mu) W_\mu^\dagger A_\nu - (\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}) W_\mu A_\nu \\ &+ W_\mu W_\nu^\dagger (\partial^\mu A^\nu - \partial^\nu A^\mu) \} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_4 = &-\frac{e^2}{2 \sin^2 \theta_W} \{ (W_\mu^\dagger W^\mu)^2 - W_\mu^\dagger W^{\mu\dagger} W_\nu W^\nu \} \\ &- e^2 \cot^2 \theta_W \{ W_\mu^\dagger W^\mu Z_\nu Z^\nu - W_\mu^\dagger Z^\mu W_\nu Z^\nu \} \\ &- e^2 \cot \theta_W \{ 2 W_\mu^\dagger W^\mu Z_\nu A^\nu - W_\mu^\dagger Z^\mu W_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu Z^\nu \} \\ &- e^2 \{ W_\mu^\dagger W^\mu A_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu A^\nu \} \end{aligned}$$



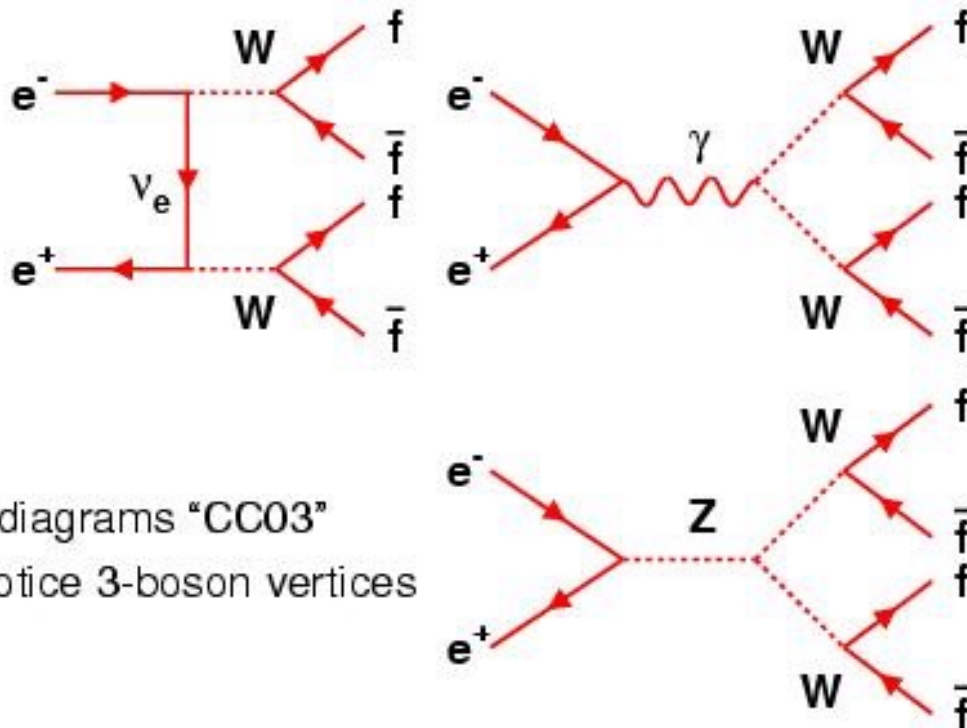
GAUGE SELF-INTERACTIONS





$$e^+e^- \rightarrow W^+W^-$$

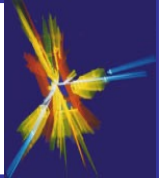
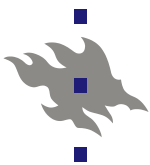
W pair production in e^+e^- annihilation:



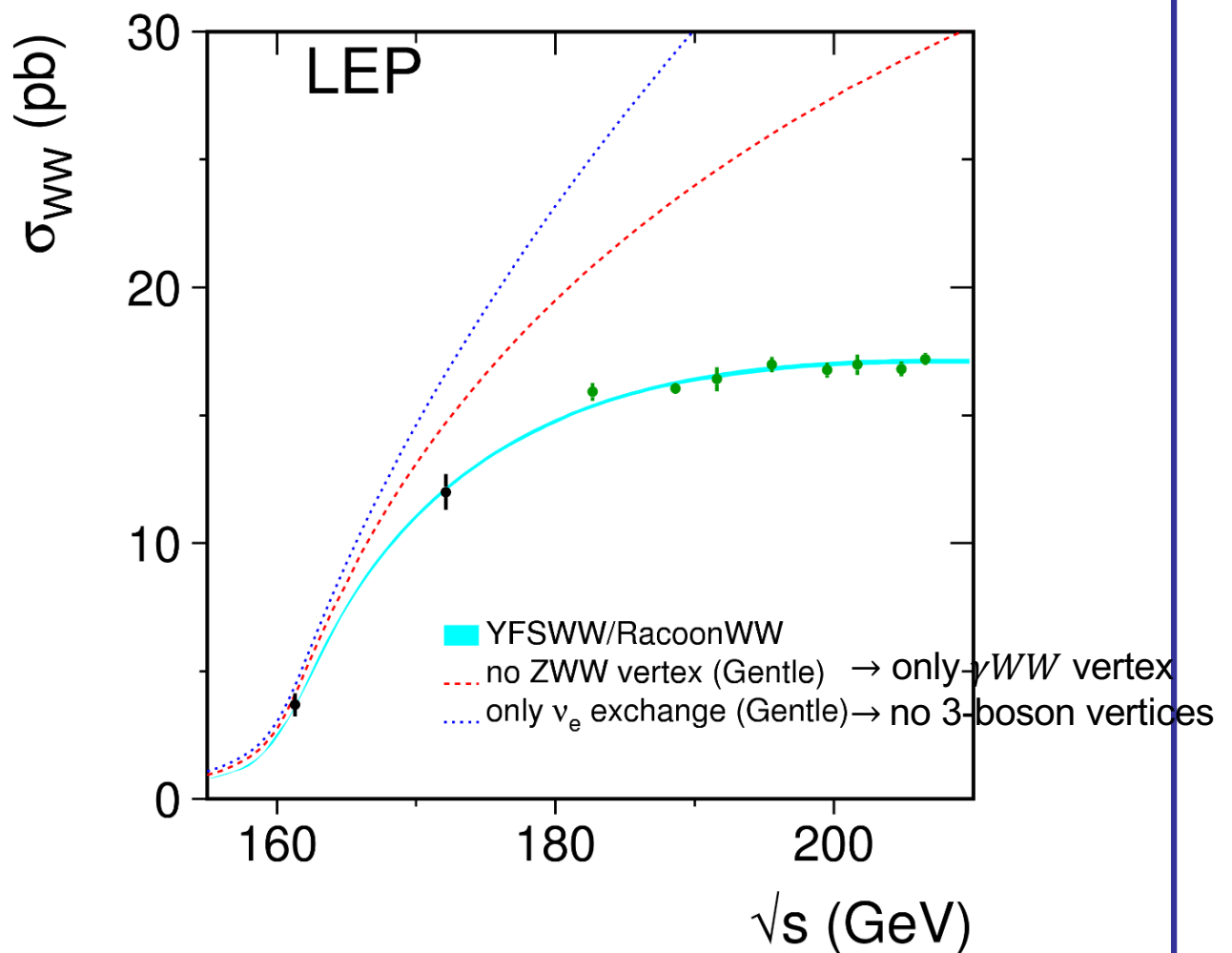
W decays to **68% $q\bar{q}$** , **32% $\ell\nu$** , so WW events are:

- 46% $q\bar{q}q\bar{q}$ – typically 4 jets
efficiency/purity $\sim 90\%/80\%$
- 44% $q\bar{q}\ell\nu$ – 2 jets, one charged lepton, missing momentum
efficiency/purity $\sim 80\%/90\%$
- 10% $\ell\nu\ell\nu$ – two charged leptons, missing momentum
efficiency/purity $\sim 60\text{--}80\%/90\%$

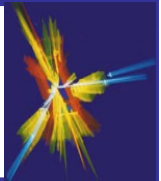
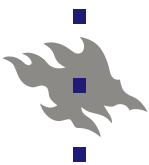
12 000 WW produced/experiment ($\sim 17 \text{ pb} \times 700 \text{ pb}^{-1}$).



W production cross section

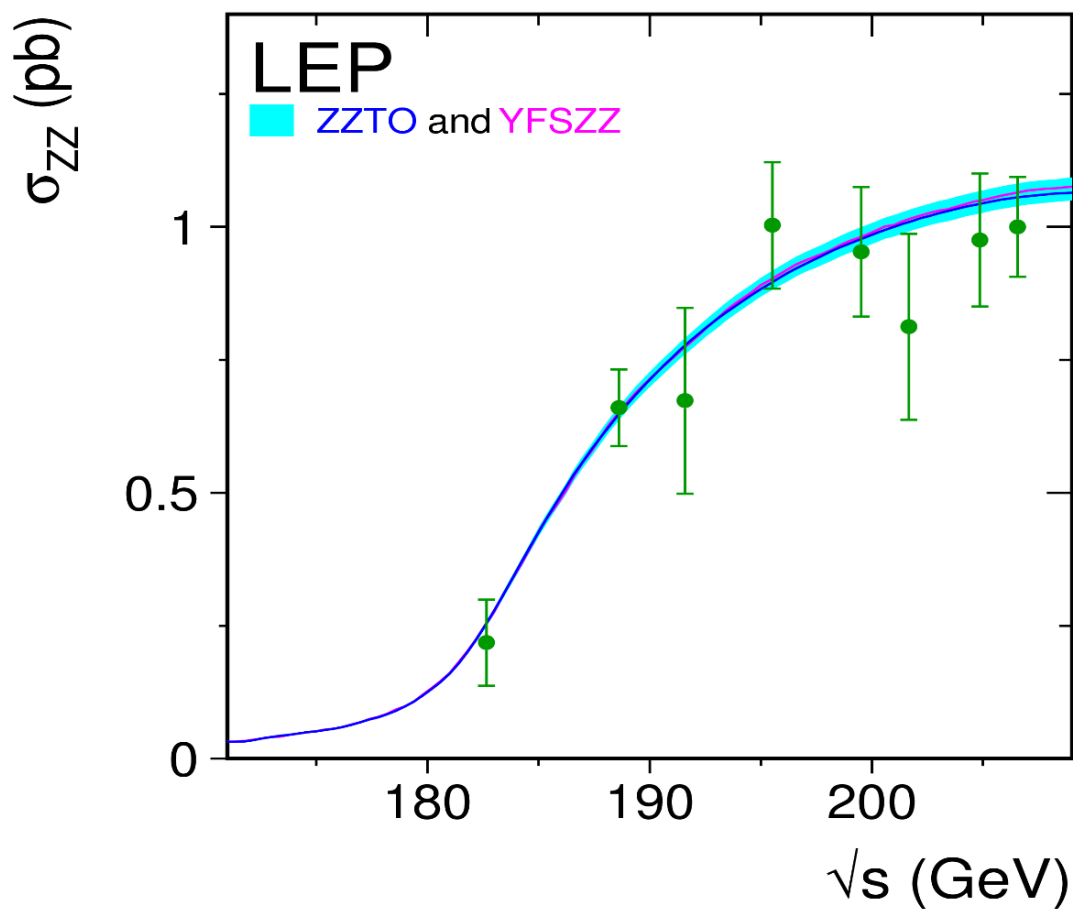
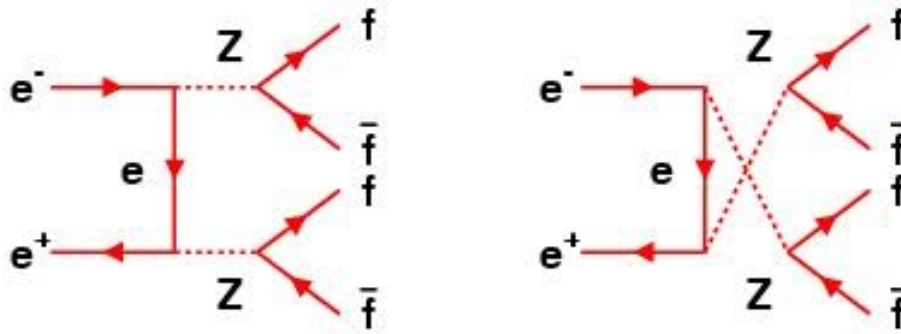


- Sensitivity to mass at threshold (only 10 pb^{-1}).
- Sensitivity to gauge couplings - beautiful demonstration of non-abelian nature of electroweak theory.
- $\text{BR}(W \rightarrow e\nu, \mu\nu, \tau\nu)$ measured. Test of lepton universality at 2.0% level. $\text{Br}(W \rightarrow q\bar{q}) = (67.41 \pm 0.27) \%$ (PDG)

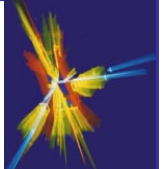
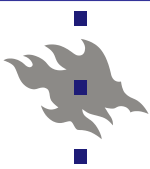


$$e^+e^- \rightarrow ZZ$$

2 diagrams "NC02"



NB! No $e^+e^- \rightarrow Z/\gamma \rightarrow ZZ$ vertex in the Standard Model



PROBLEM WITH MASS SCALES

Gauge Symmetry



$$m_\gamma = 0$$

Good

$$m_W = m_Z = 0$$

Bad!



Experimentally:

$$m_W = 80.37 \text{ GeV (PDG)}$$

$$m_Z = 91.19 \text{ GeV (PDG)}$$

Moreover

$$\mathcal{L}_{m_f} \equiv -m_f \bar{f} f = -m_f (\bar{f}_L f_R + \bar{f}_R f_L)$$

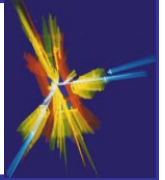
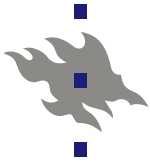
Also Forbidden by Gauge Symmetry



$$m_f = 0$$

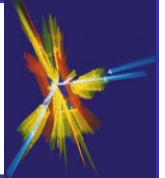
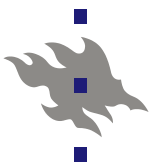
$$\forall f$$

All Particles Massless



Higgs mechanism

- ◆ Spontaneous symmetry breaking
- ◆ Higgs mechanism
- ◆ The Higgs boson
- ◆ Fermion masses
- ◆ Fermion mixing



We have been able to generate electromagnetism and the strong interactions through local gauge invariance. We would like to do the same for the weak interactions, but we immediately face a problem. Unlike the photon and the gluons, the W and Z have non-zero masses. A mass term in the Lagrangian, $\frac{1}{2} m^2 A_\mu A^\mu$, would not be gauge invariant.

Suppose we forget about gauge invariance and just put it in anyway. Then theory becomes non-renormalizable, i.e. the counterterms required to cancel all divergences become infinite and theory loses all predictive power.

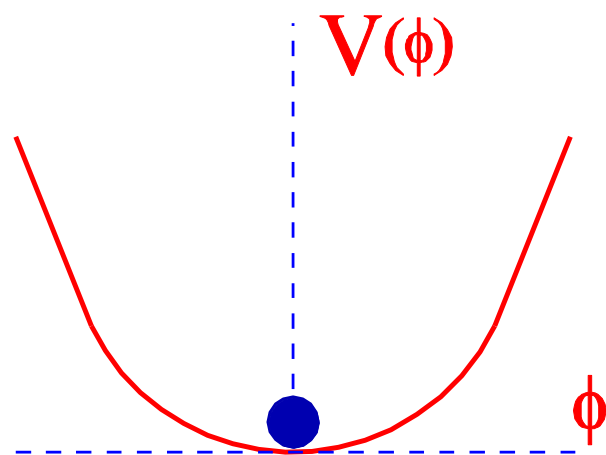
There is a more subtle way to generate a mass for the W and Z, called **spontaneous symmetry breaking**. Let us first consider a real scalar field ϕ with an arbitrary "potential".

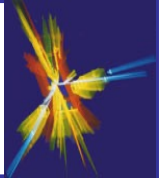
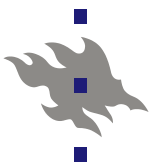
$$L = \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi)$$

We require the L to be invariant under $\phi \rightarrow \phi'$.

Then, if we expand V , it will have only even values of ϕ :

$$L = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} \mu^2 \phi^2 - \frac{1}{4} \lambda \phi^4 \dots$$

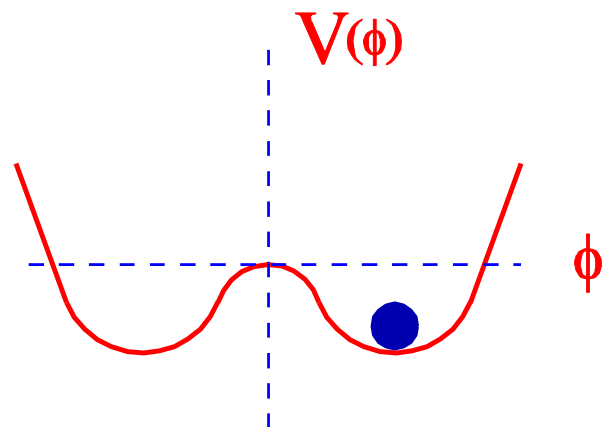




But what happens if μ^2 is allowed to be negative

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} \mu^2 \phi^2 - \frac{1}{4} \lambda \phi^4 \dots$$

The sign of the mass term has changed giving an imaginary mass. This makes no sense. The potential looks like in the figure. The minimum is now at $\phi = \pm \sqrt{-\mu^2/\lambda}$.

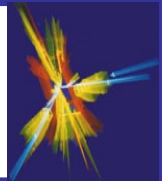
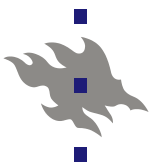


Feynman diagrams represent a perturbation serie. The serie would not converge if we expand about $\phi = 0$, since this is a local maximum. We need to expand about the global minimum, say $+\sqrt{-\mu^2/\lambda}$. $\phi(x^\mu) = +\sqrt{-\mu^2/\lambda} + \eta(x^\mu)$, and then $\eta = 0$ corresponds to the minimum. Then

$$\mathcal{L} = \frac{1}{2} [\partial_\mu (\sqrt{-\mu^2/\lambda} + \eta(x^\mu))]^2 - \frac{1}{2} \mu^2 (\sqrt{-\mu^2/\lambda} + \eta(x^\mu))^2 - \frac{1}{4} \lambda (\sqrt{-\mu^2/\lambda} + \eta(x^\mu))^4$$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \eta(x^\mu))^2 + \frac{1}{4} \mu^4/\lambda + \frac{1}{2} \mu^2 \eta^2(x^\mu) - \sqrt{-\mu^2 \lambda} \eta^3(x^\mu) - \frac{1}{4} \lambda \eta^4(x^\mu)$$

~~$\frac{1}{4} \mu^4/\lambda$~~ $= \text{constant}$ $m = \sqrt{-2\mu^2}$ ~~$\frac{1}{4} \lambda \eta^4(x^\mu)$~~



The η and ϕ fields are the same fields, but we now see that the η has a mass of $\sqrt{-\mu^2}$, and does not have reflection symmetry $\eta \Leftrightarrow -\eta$. This symmetry has been "spontaneously broken".

A more interesting case occurs, when we consider a complex field, $\phi = (1/\sqrt{2})(\phi_1 + i\phi_2)$

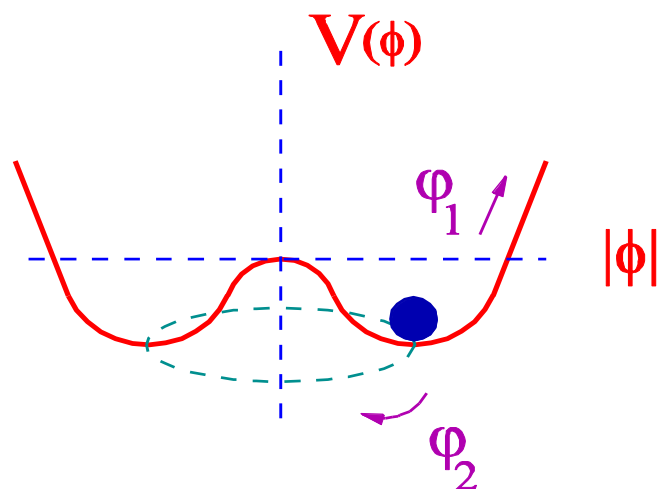
$$L = (\partial_\mu \phi)^*(\partial^\mu \phi) - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2$$

$$L = \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2$$

The minimum is now
a circle: $\phi_1^2 + \phi_2^2 = -\mu^2/\lambda$

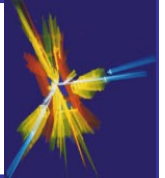
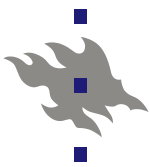
We choose to expand
about $\phi_1 = \sqrt{-\mu^2/\lambda}$, $\phi_2 = 0$.

$$\phi(x^\mu) = \frac{1}{\sqrt{2}}[\sqrt{-\mu^2/\lambda} + \eta(x^\mu) + i\xi(x^\mu)]$$



We get:
$$L = \frac{1}{2}(\partial_\mu \eta(x^\mu))^2 + \frac{1}{2}(\partial_\mu \xi(x^\mu))^2 + \frac{1}{4}\mu^4/\lambda + \mu^2\eta^2(x^\mu) + \sqrt{-\mu^2\lambda} [\eta^3(x^\mu) + \eta(x^\mu)\xi^2(x^\mu)] - \frac{1}{4}\lambda[\eta^4(x^\mu) + \xi^4(x^\mu) + 2\eta^2(x^\mu)\xi^2(x^\mu)]$$

The η field gets a mass $\sqrt{-\mu^2}$, as before, but the ξ field is massless. This always happens whenever a continuous symmetry is spontaneously broken. The massless particle is called a **Goldstone boson**.



Let us consider a complex scalar field which is invariant under a local gauge transformation

$$\phi \rightarrow \exp(i\alpha(x^\mu)) \phi$$

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu - iqA_\mu \quad A_\mu \rightarrow A_\mu + (1/q) \partial_\mu \alpha(x^\mu)$$

$$\mathcal{L} = (\partial^\mu + iqA^\mu)\phi^*(\partial_\mu - iqA_\mu)\phi - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

As before $\phi = (1/\sqrt{2})(\phi_1 + i\phi_2)$, we choose

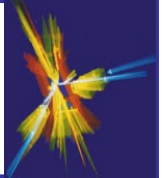
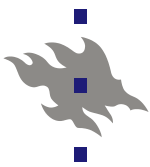
$$\phi_1(x^\mu) = \sqrt{-\mu^2/\lambda} + \eta(x^\mu); \quad \phi_2(x^\mu) = \xi(x^\mu) \text{ and we get}$$

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}(\partial_\mu \eta)^2 + \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{4}\mu^4/\lambda + \mu^2\eta^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & - \frac{1}{2}q^2 (\sqrt{-\mu^2/\lambda})^2 A_\mu A^\mu + q(\sqrt{-\mu^2/\lambda})(\partial_\mu \xi)A^\mu \\ & (+ \text{interaction terms in } \eta, \xi \text{ and } A.) \end{aligned}$$

Note: (a) The A field has acquired a mass $q(\sqrt{-\mu^2/\lambda})$
(b) There is still a Goldstone boson, ξ .
(c) There is a strange term $q(\sqrt{-\mu^2/\lambda})(\partial_\mu \xi)A^\mu$

$$\xi \text{ --- } \bullet \text{ --- } A$$

This indicates that we haven't chosen the correct fields.



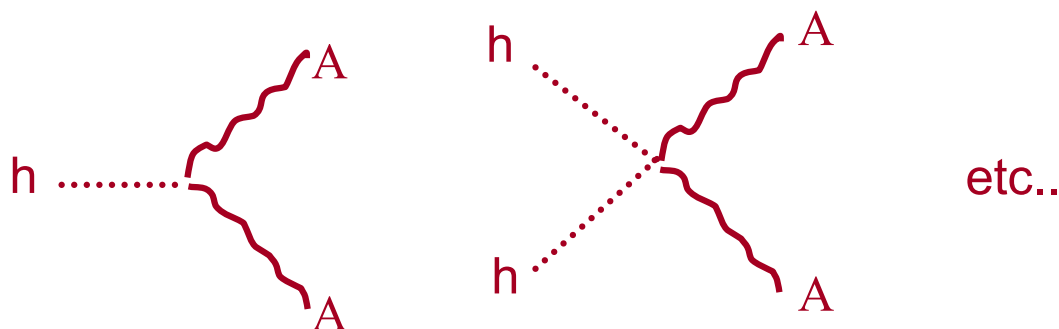
We can use our freedom to choose a gauge so that

$$\phi(x^\mu) \rightarrow (1/\sqrt{2})[\sqrt{-\mu^2/\lambda} + h(x^\mu)] \exp\{i\theta(x^\mu)/(\sqrt{-\mu^2/\lambda})\}$$

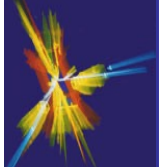
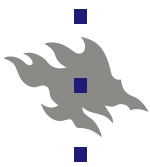
($h(x^\mu)$ and $\theta(x^\mu)$ are real)

$$A_\mu \rightarrow A_\mu + (1/q(\sqrt{-\mu^2/\lambda})) \partial_\mu \theta(x^\mu)$$

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}(\partial_\mu h(x^\mu))^2 + \mu^2 h^2(x^\mu) + \frac{1}{4}\mu^4/\lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & - \frac{1}{2}q^2(\sqrt{-\mu^2/\lambda})A_\mu A^\mu + (\sqrt{-\mu^2/\lambda})q^2 h A_\mu A^\mu + \frac{1}{2}q^2 h^2 A_\mu A^\mu \\ & - \sqrt{-\mu^2/\lambda} h^3 - \frac{1}{4}\lambda h^4 \end{aligned}$$



We have 1 massive scalar h and 1 massive vector A . Note that we started with two scalars (η and ξ) or (h and θ) and one massless vector A . This is 4 spin degrees of freedom, ($2 \times 1 + 1 \times 2$). We end with one scalar and one massive vector. This is still 4 spin degrees of freedom ($1 \times 1 + 1 \times 3$). The gauge field has eaten up the Goldstone boson. This is the **Higgs mechanism**, and h is called the Higgs boson.



ELECTROWEAK SSB

New Scalar Doublet

$$\phi(x) \equiv \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix} \quad ; \quad y_\phi = 1$$

$$\mathcal{L}(\phi) = (\mathbf{D}_\mu \phi)^\dagger \mathbf{D}^\mu \phi - \mu^2 \phi^\dagger \phi - h (\phi^\dagger \phi)^2$$

$$\mathbf{D}^\mu \phi = \left[\partial^\mu - i g \mathbf{W}^\mu - i g' y_\phi / 2 B^\mu \right] \phi \quad ; \quad \mathbf{W}^\mu = \vec{\tau} \cdot \vec{W}^\mu$$

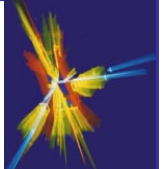
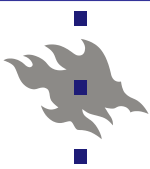
SU(2)_L ⊗ U(1)_Y Symmetry

Degenerate Vacuum States: $\mu^2 < 0$, $h > 0$

$$|\langle 0 | \phi_0 | 0 \rangle| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}} \quad v \equiv \text{vacuum expectation value}$$

Spontaneous Symmetry Breaking:

$$\phi(x) = \exp \left\{ i \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x) \right\} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$



HIGGS MECHANISM

$$\phi(x) = \exp \left\{ i \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x) \right\} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

$SU(2)_L$ Invariance $\rightarrow$ $\vec{\theta}(x)$ Unphysical

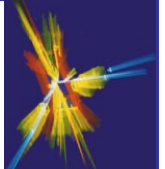
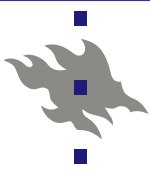
Unitary Gauge: $\vec{\theta}(x) = 0$

$$(\mathbf{D}_\mu \phi)^\dagger \mathbf{D}^\mu \phi \rightarrow \frac{1}{2} \partial_\mu H \partial^\mu H + \frac{g^2}{4} (v + H)^2 \left\{ W_\mu^\dagger W^\mu + \frac{1}{2 \cos^2 \theta_W} Z_\mu Z^\mu \right\}$$



$$M_Z \cos \theta_W = M_W = \frac{1}{2} v g$$

Massive Gauge Bosons



Bosonic Degrees of Freedom

Massless $W^\pm, Z$

$$3 \times 2 \text{ helicities} = 6$$

+

3 Goldstones $\vec{\theta}$

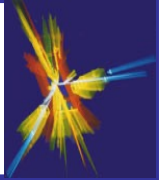
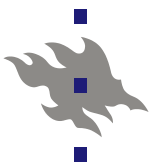


SSB

Massive $W^\pm, Z$

$$3 \times 3 \text{ helicities} = 9$$

SAME PHYSICS



THE HIGGS BOSON

$$\mathcal{L}_S = \frac{h v^4}{4} + \mathcal{L}_H + \mathcal{L}_{HG^2}$$

$$\mathcal{L}_H = \frac{1}{2} \partial_\mu H \partial^\mu H - \frac{1}{2} M_H^2 H^2 - \frac{M_H^2}{2v} H^3 - \frac{M_H^2}{8v^2} H^4$$

$$\begin{aligned} \mathcal{L}_{HG^2} = & M_W^2 W_\mu^\dagger W^\mu \left\{ 1 + \frac{2}{v} H + \frac{H^2}{v^2} \right\} \\ & + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \left\{ 1 + \frac{2}{v} H + \frac{H^2}{v^2} \right\} \end{aligned}$$

1 Scalar Particle H^0 should exist !!!

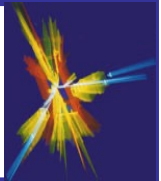
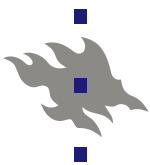
$$M_H = \sqrt{-2\mu^2} = \sqrt{2h} v \quad \text{Free Parameter}$$

LHC:

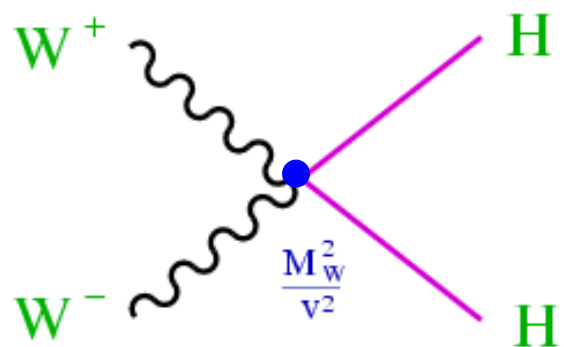
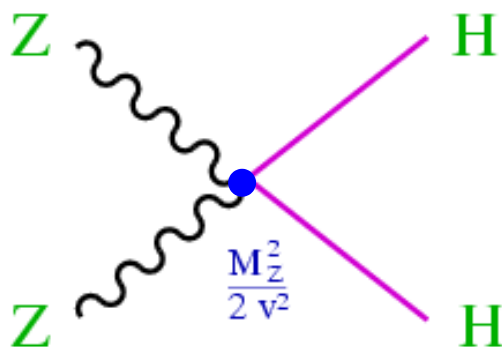
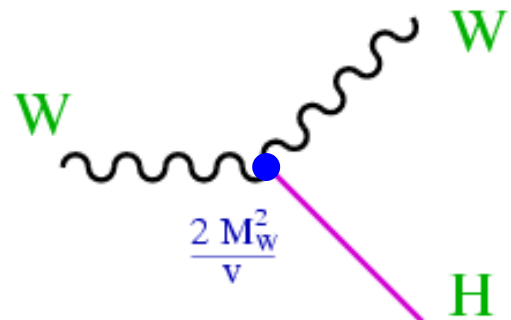
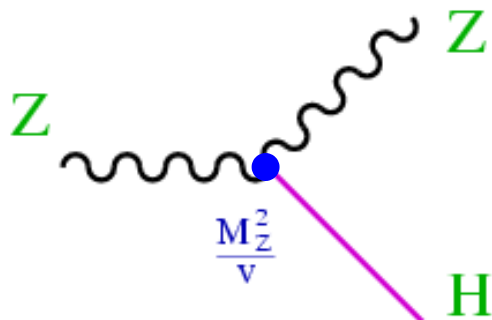
discovery of a Higgs-like particle 4.7.2012 !!!

$$m_H = 125.20 \pm 0.11 \text{ GeV} \quad (2025 \text{ CMS \& ATLAS average by PDG})$$

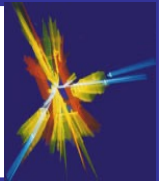
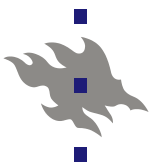
Up to now all measurements of the newly discovered particle consistent with the Standard Model Higgs boson !



Higgs Couplings $\propto$ Masses



$$v = (\sqrt{2} G_F)^{-1/2} = 246 \text{ GeV}$$



FERMION MASSES

Scalar–Fermion Couplings allowed by Gauge Symmetry

$$\mathcal{L}_Y = (\bar{q}_u, \bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right] \\ + (\bar{\nu}_l, \bar{l})_L c^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l_R + \text{h.c.}$$

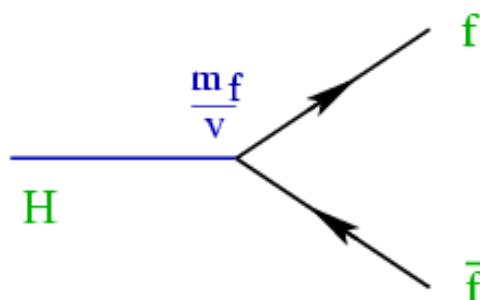


SSB

$$\mathcal{L}_Y = - \left(1 + \frac{H}{v} \right) \left\{ m_{q_d} \bar{q}_d q_d + m_{q_u} \bar{q}_u q_u + m_l \bar{l} l \right\}$$

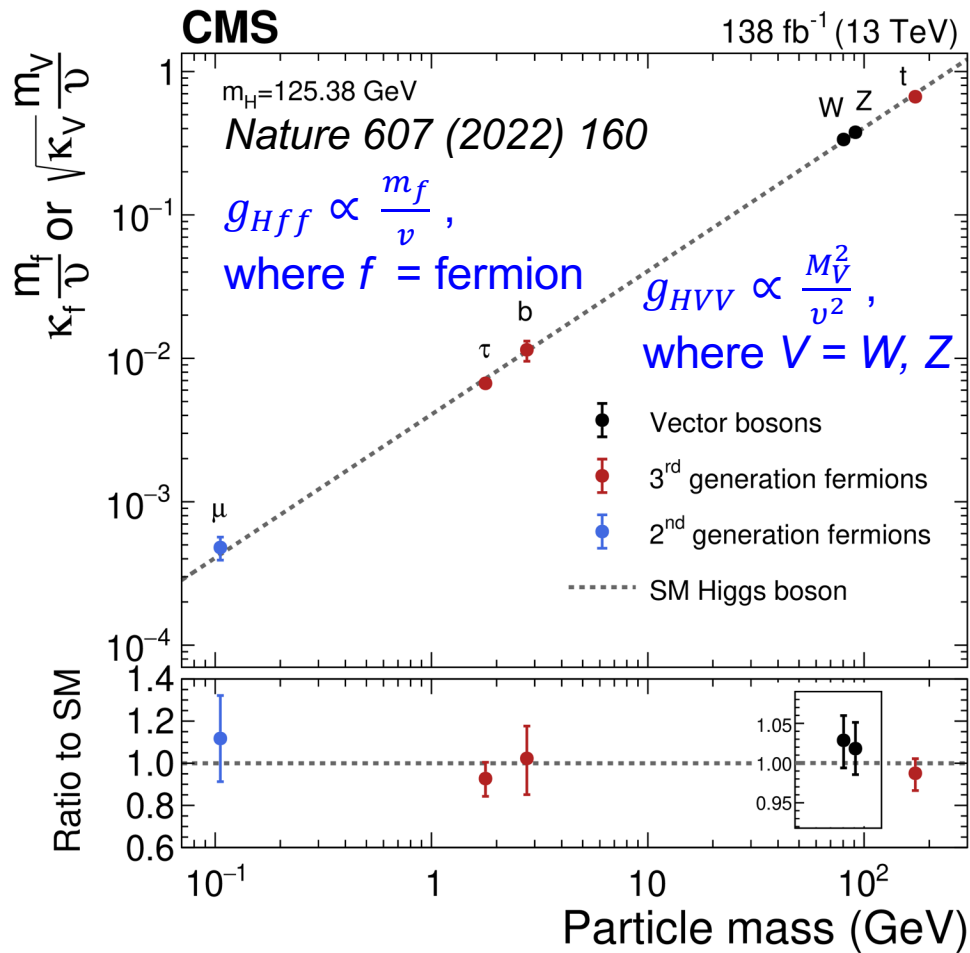
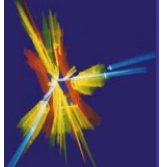
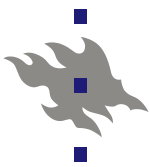
Fermion Masses are New Free Parameters

$$[m_{q_d}, m_{q_u}, m_l] = - [c^{(d)}, c^{(u)}, c^{(l)}] \frac{v}{\sqrt{2}}$$



Couplings Fixed

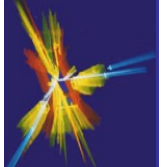
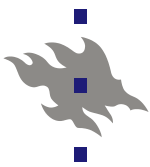
$$g_{Hf\bar{f}} = \frac{m_f}{v}$$



BR(%)	Higgs 125 GeV	Z boson
$q\bar{q}$		70
$b\bar{b}$	58	15
$c\bar{c}$	3	12
$g\bar{g}$	8	0
$\ell^+\ell^-$	0.02 (μ)	10
$\tau^+\tau^-$	6.4	3
$\gamma\gamma$	0.2	
W^*W^*	22	
Z^*Z^*	2.7	

at 1st order: $\Gamma(H \rightarrow f\bar{f}) = \frac{N_C^f G_F m_f^2}{4\sqrt{2}\pi} m_H \beta_f$,

where β_f is the fermion velocity after the decay.



FERMION GENERATIONS

$N_G = 3$ Identical Copies

WHY ?

$$\begin{array}{ll} Q = 0 & \left(\begin{array}{c|c} \nu'_j & u'_j \end{array} \right) & Q = +\frac{2}{3} \\ Q = -1 & \left(\begin{array}{c|c} l'_j & d'_j \end{array} \right) & Q = -\frac{1}{3} \end{array} \quad (j = 1, \dots, N_G)$$

Masses are the only difference

$$\mathcal{L}_Y = \sum_{jk} \left\{ (\bar{u}'_j, \bar{d}'_j)_L \left[c_{jk}^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c_{jk}^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] \right. \\ \left. + (\bar{\nu}'_j, \bar{l}'_j)_L c_{jk}^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l'_{kR} \right\} + \text{h.c.}$$

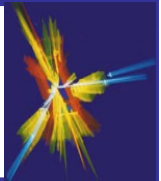
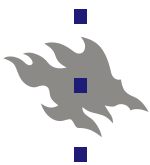


SSB

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \{ \bar{d}'_L M'_d d'_R + \bar{u}'_L M'_u u'_R + \bar{l}'_L M'_l l'_R + \text{h.c.} \}$$

Arbitrary Non-Diagonal Complex Mass Matrices

$$[M'_d, M'_u, M'_l]_{ij} = - \left[c_{ij}^{(d)}, c_{ij}^{(u)}, c_{ij}^{(l)} \right] \frac{v}{\sqrt{2}}$$



DIAGONALIZATION OF MASS MATRICES

$$M'_d = H_d U_d = S_d^\dagger \mathcal{M}_d S_d U_d$$

$$M'_u = H_u U_u = S_u^\dagger \mathcal{M}_u S_u U_u$$

$$M'_l = H_l U_l = S_l^\dagger \mathcal{M}_l S_l U_l$$

$$H_f = H_f^\dagger$$

$$U_f U_f^\dagger = 1$$

$$S_f S_f^\dagger = 1$$

$$\mathcal{L}_Y = - \left(1 + \frac{H}{v}\right) \left\{ \bar{d} \mathcal{M}_d d + \bar{u} \mathcal{M}_u u + \bar{l} \mathcal{M}_l l \right\}$$

$$\mathcal{M}_d = \text{diag} (m_d, m_s, m_b) \quad , \quad \mathcal{M}_u = \text{diag} (m_u, m_c, m_t)$$

$$\mathcal{M}_l = \text{diag} (m_e, m_\mu, m_\tau)$$

Mass Eigenstates $\neq$ Weak Eigenstates

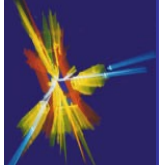
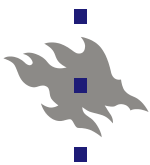
$$\begin{aligned} d_L &\equiv S_d d'_L & , & & u_L &\equiv S_u u'_L & , & & l_L &\equiv S_l l'_L \\ d_R &\equiv S_d U_d d'_R & , & & u_R &\equiv S_u U_u u'_R & , & & l_R &\equiv S_l U_l l'_R \end{aligned}$$

$$\bar{f}'_L f'_L = \bar{f}_L f_L \quad , \quad \bar{f}'_R f'_R = \bar{f}_R f_R \quad \longrightarrow \quad \mathcal{L}'_{NC} = \mathcal{L}_{NC}$$

$$\bar{u}'_L d'_L = \bar{u}_L \mathbf{V} d_L \quad \longrightarrow \quad \mathcal{L}'_{CC} \neq \mathcal{L}_{CC}$$

$$\mathbf{V} \equiv S_u S_d^\dagger$$

QUARK MIXING

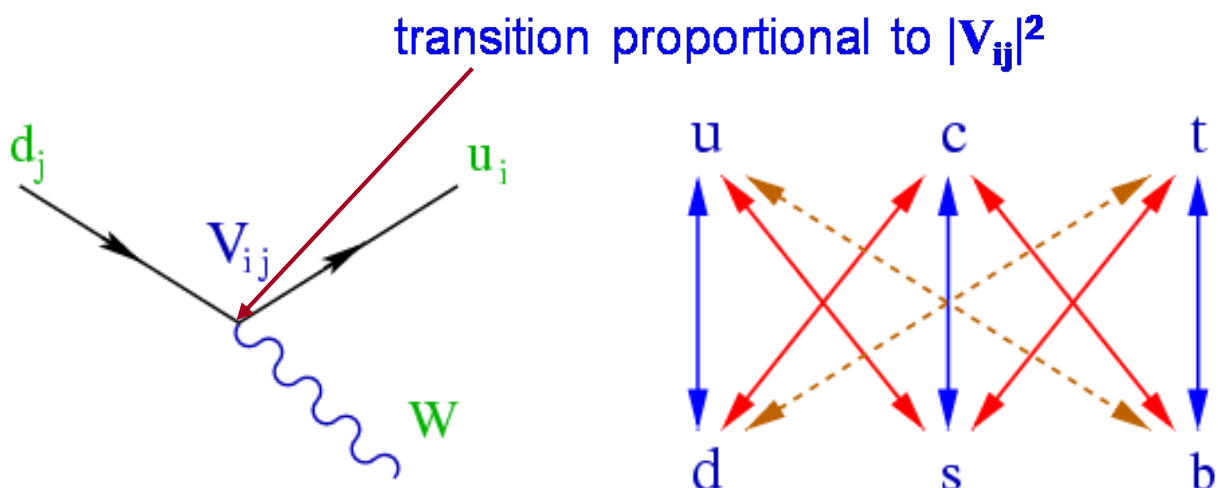


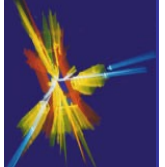
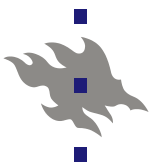
$$\mathcal{L}_{NC}^Z = \frac{e}{2 \sin \theta_W \cos \theta_W} Z_\mu \sum_f \bar{f} \gamma^\mu [v_f - a_f \gamma_5] f$$

Flavour Conserving Neutral Currents

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\sum_{ij} \bar{u}_i \gamma^\mu (1 - \gamma_5) V_{ij} d_j + \sum_l \bar{\nu}_l \gamma^\mu (1 - \gamma_5) l \right] + \text{h.c.}$$

Flavour Changing Charged Currents





QUARK MIXING MATRIX

♠ Unitary $N_G \times N_G$ Matrix: N_G^2 parameters

♠ $2 N_G - 1$ arbitrary phases:

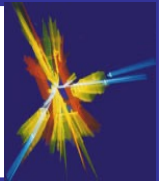
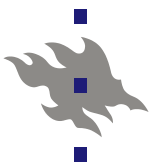
$$\left. \begin{array}{l} u_i \rightarrow e^{i\phi_i} u_i \\ d_j \rightarrow e^{i\theta_j} d_j \end{array} \right\} \rightarrow V_{ij} \rightarrow e^{i(\theta_j - \phi_i)} V_{ij}$$



V_{ij} Physical Parameters

$\frac{1}{2} N_G (N_G - 1)$ moduli
or angles

$\frac{1}{2} (N_G - 1) (N_G - 2)$ phases



QUARK MIXING MATRIX

Unitarity: $V \cdot V^\dagger = V^\dagger \cdot V = 1$

$N_f = 2$:

1 angle, 0 phases

θ_c = Cabibbo angle

$$V = \begin{bmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{bmatrix} \rightarrow \text{No } \cancel{CP}$$

CKM = Cabibbo-Kobayashi-Maskawa

$N_f = 3$: (CKM)

3 angles, 1 phase

Source: PDG review on the CKM matrix

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

$c_{ij} \equiv \cos \theta_{ij}$ $s_{ij} \equiv \sin \theta_{ij}$, where $i, j = 1, 2$ or 3

$$\approx \begin{bmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4) \quad \lambda \approx \sin \theta_c$$

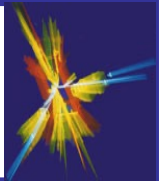
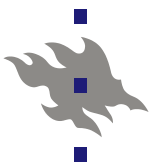
$$\lambda = 0.22500 \pm 0.00067 \quad A = 0.826^{+0.018}_{-0.015},$$

$$\bar{\rho} = 0.159 \pm 0.010 \quad \bar{\eta} = 0.348 \pm 0.010$$

with $\bar{\rho} = \rho(1 - \lambda^2/2 + \dots)$ & $\bar{\eta} = \eta(1 - \lambda^2/2 + \dots)$

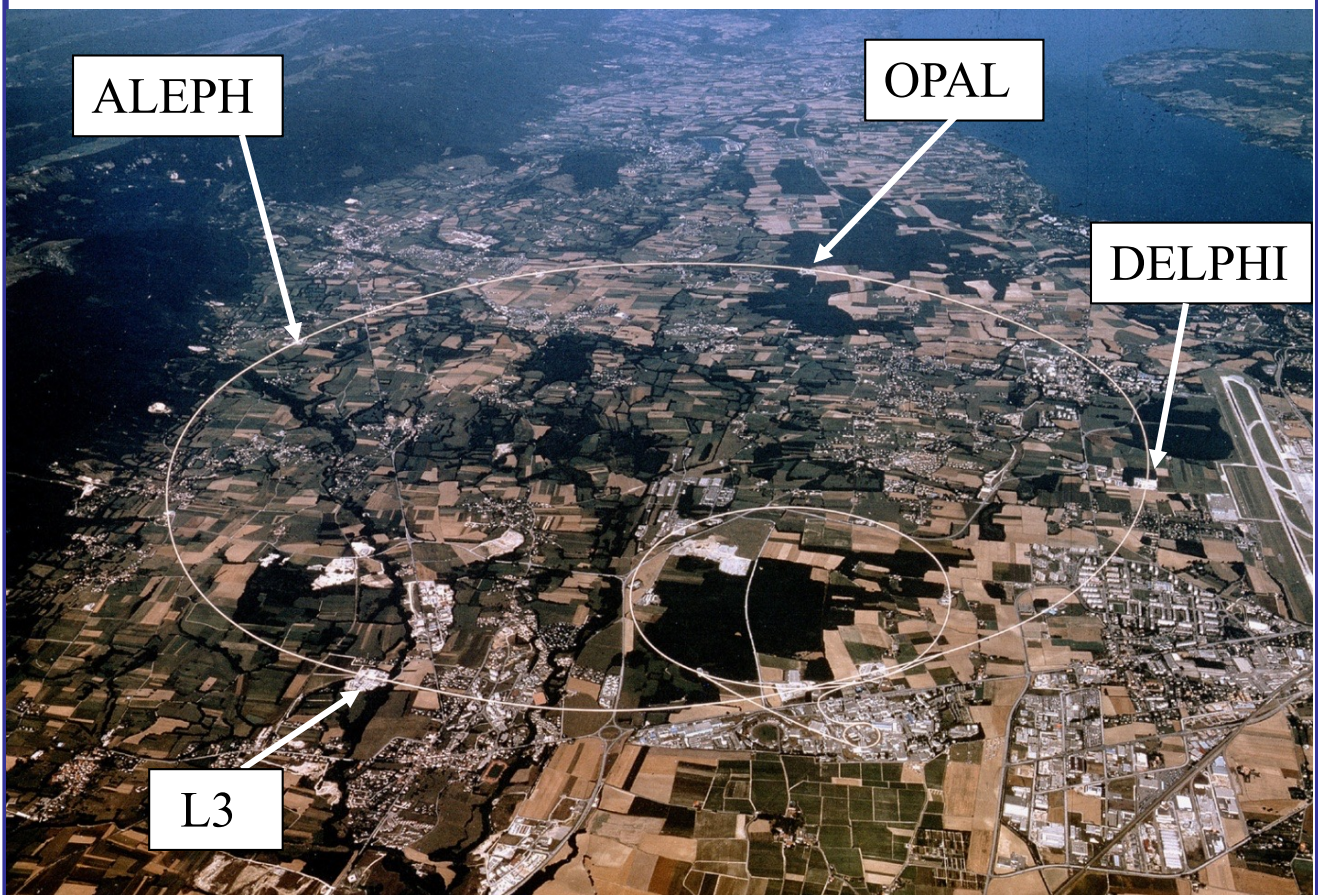
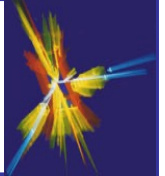
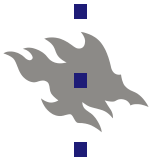
As observed for K^0 , D^0 and B^0 mesons:

$$\delta_{13} \neq 0 \quad (\eta \neq 0) \rightarrow \cancel{CP}$$



Electroweak precision measurement

- ◆ **LEP and its experiments**
- ◆ **Electroweak radiative corrections**
- ◆ **Z precision measurements**
- ◆ **Global electroweak fit**



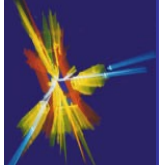
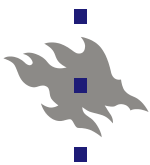
An electron–positron collider
at CERN, European laboratory
for particle physics, in the same
tunnel that is now used by LHC.

Four experiments focused on
studying the Standard Model
(electroweak measurements $\sim W$ & Z ,
QCD measurement & Higgs searches)

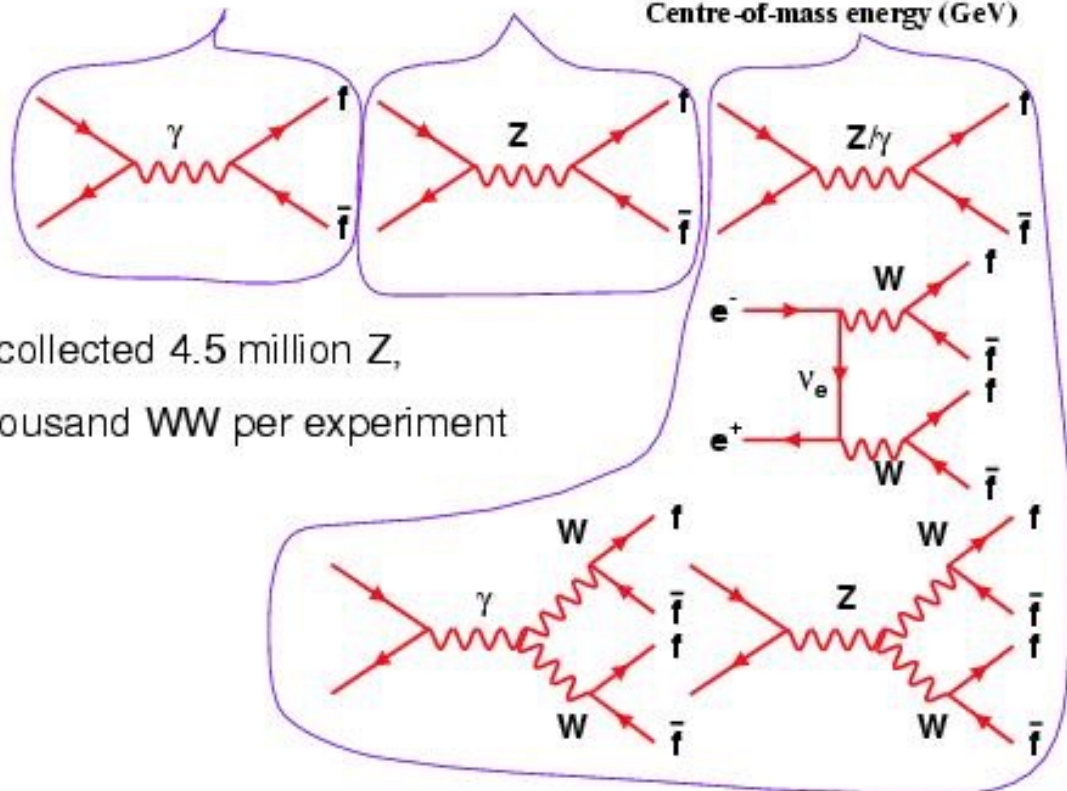
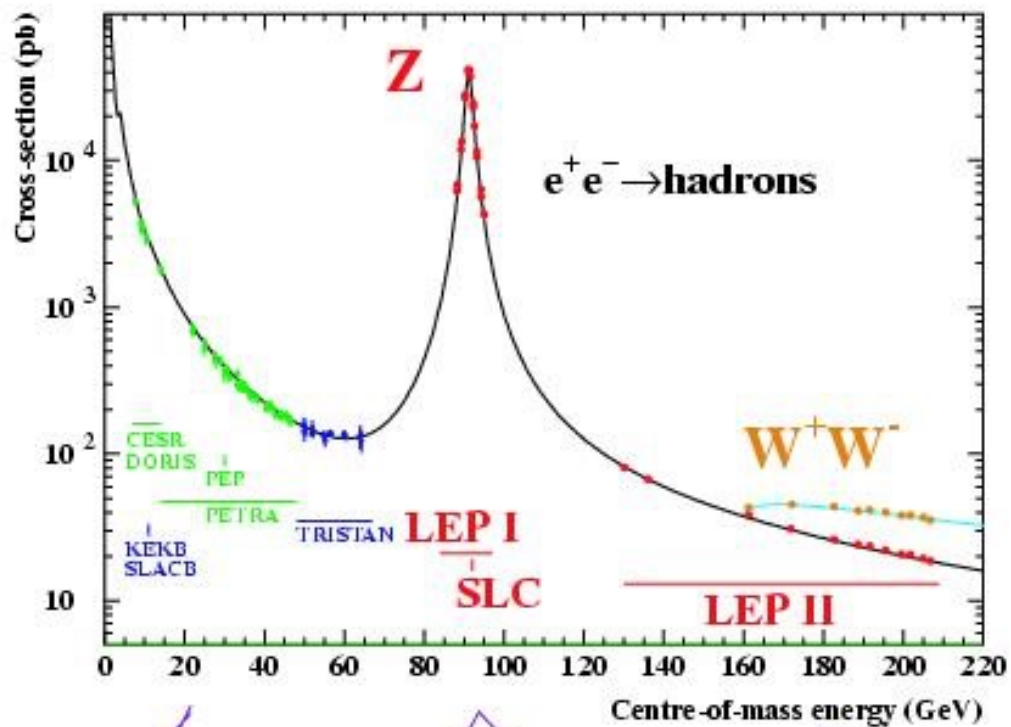
LEPI – $\sqrt{s} \approx m_Z$ (1989-95)

LEPII – $\sqrt{s} = 140\text{-}209$ GeV (1995-2000)

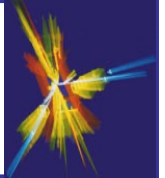
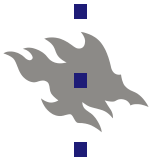




Electron positron annihilation



LEP collected 4.5 million Z,
12 thousand WW per experiment



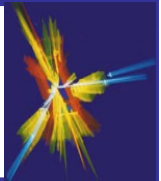
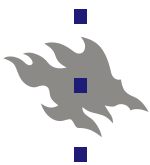
Z resonance lineshape

To measure the Z mass, total width and cross-section, partial widths (branching ratios) and couplings:

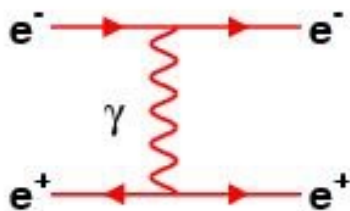
- LEP machine gives e^+e^- collisions at a few energies on and near the Z peak and precise measurement of E_{beam}
- Detectors **ALEPH, DELPHI, L3, OPAL** distinguish Z final states and measure the luminosity from QED t-channel process $e^+e^- \rightarrow e^+e^-$ (Bhabha scattering)

$$\sigma(\sqrt{s}) = (N_{\text{observed}} - N_{\text{background}})/\epsilon\mathcal{L}$$

- Monte Carlo simulation of the signal efficiency and background.
- Precise theoretical prediction of the lineshape
- Match precision from 4.5 million Z events per experiment - relative statistical error about 5×10^{-4} .
- Several thousand people involved: machine/experiment/theory
- $\sigma(m_Z) \approx 340$ MeV from UA2+CDF in 1989. Hoped to reduce to ≈ 10 MeV (limited by beam energy precision)
- Count the number of generations. 2.5 generations were known in 1989, top quark and ν_τ not yet established. Number of light neutrinos limited by big bang nucleosynthesis to $\lesssim 4$. Expected precision of about ± 0.2 on the number.



Luminosity Measurement

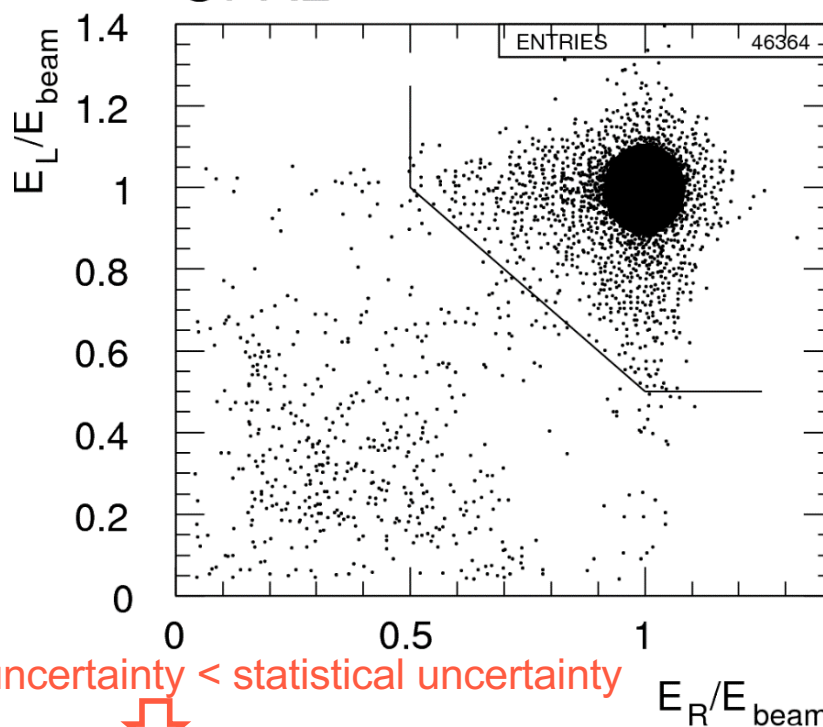


“Bhabha scattering”

The t-channel contribution to $e^+e^- \rightarrow e^+e^-$ dominates at small angles. Detectors typically 25 to 60 mrad from beam.

Very clear electron signal in forward detectors (calorimeters).

OPAL



systematic uncertainty < statistical uncertainty

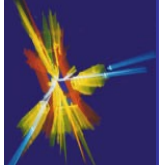
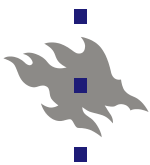


Accepted cross section at least $2 \times \sigma_{\text{had}}$. $1/\theta^3$ variation.

Experimental difficulty: define geometric edge of acceptance to give cross-section precision $\lesssim 0.05\%$.

Common theory error of $\sim 0.05\%$ (cf $\sim 1\%$ in 1989).

(BHLUMI program: S. Jadach, B.F.L. Ward et al.)

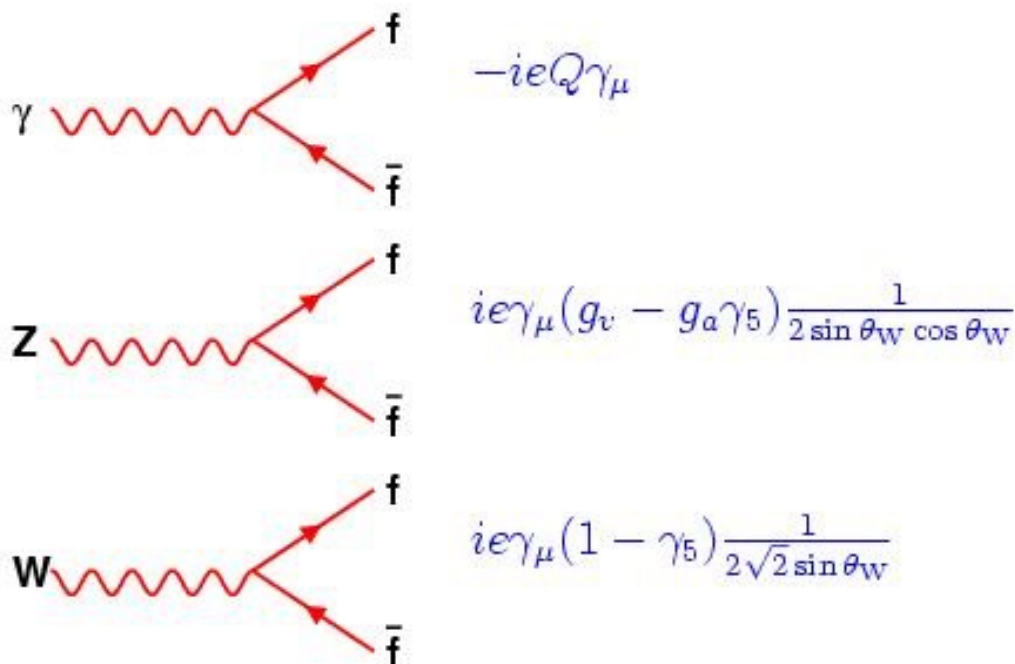


Standard Model relationships

Masses of heavy gauge bosons and their couplings to fermions depend on SAME mixing angle

$$\cos \theta_W = m_W / m_Z$$

$SU(2) \times U(1)$ coupling constants, g, g' , proportional to electric charge e : $g = e \sin \theta_W$, $g' = e \cos \theta_W$

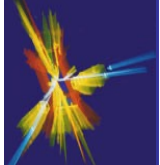


where Q , g_a and g_v depend on fermion type, with

$$g_a = T^3 = \pm \frac{1}{2} \equiv a_f$$

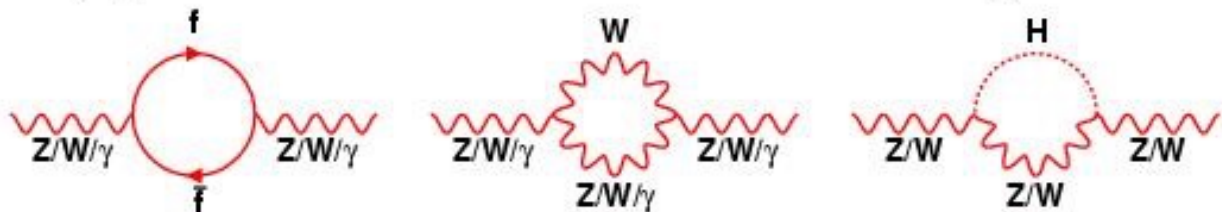
$$g_v = (T^3 - 2Q \sin^2 \theta_W) = \pm \frac{1}{2}(1 - 4|Q| \sin^2 \theta_W) \equiv v_f$$

g_v/g_a gives $\sin^2 \theta_W$ if you know $|Q|$.

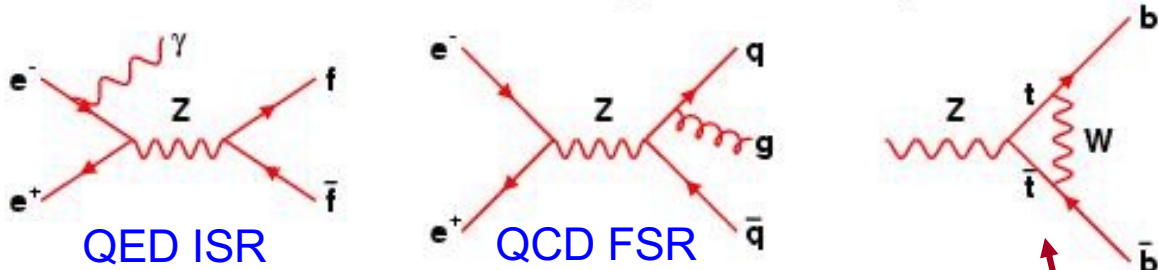


Radiative corrections

Propagator corrections are the same for each fermion type.



QED, QCD and vertex corrections give fermion dependent terms.



QED ISR

QCD FSR

ISR/FSR = initial/final state radiation

Electroweak corrections absorbed into effective couplings:

$$g_V \equiv g_V^{\text{eff}} = \sqrt{(1 + \Delta\rho)}(T^3 - 2Q \sin^2 \theta_{\text{eff}})$$

$$g_A \equiv g_A^{\text{eff}} = \sqrt{(1 + \Delta\rho)}T^3$$

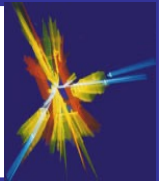
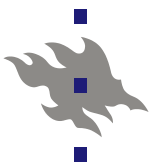
$$\sin^2 \theta_{\text{eff}} = (1 + \Delta\kappa) \sin^2 \theta_W$$

$$\Delta\rho = \frac{3G_F M_W^2}{8\sqrt{2}\pi^2} \left(\frac{m_t^2}{m_W^2} - \tan^2 \theta_W \left[\ln \frac{m_H^2}{m_W^2} - \frac{5}{6} \right] \right) + \dots$$

$$\Delta\kappa = \frac{3G_F M_W^2}{8\sqrt{2}\pi^2} \left(\cot^2 \theta_W \frac{m_t^2}{m_W^2} - \frac{11}{9} \left[\ln \frac{m_H^2}{m_W^2} - \frac{5}{6} \right] \right) + \dots$$

Extra m_t^2/m_W^2 contributions for b quark

m_H = Higgs boson mass, m_t = top quark mass, m_W = W boson mass



Radiative corrections

The value of G_F is also modified:

Fermi constant $G_F = \frac{\pi\alpha}{\sqrt{2}m_W^2 \sin^2 \theta_W} \frac{1}{1 - \Delta r}$ $\propto m_t^2 \text{ \& } \ln m_H^2$

where

$$\Delta r = \Delta\alpha + \Delta r_w = \Delta\alpha - \Delta\kappa + \dots$$

$\Delta\alpha$ term incorporates the running of the electromagnetic coupling due to fermion loops in the photon propagator. The difficult part of the calculation is to account for all the hadronic states. Use experimental measurement of $e^+e^- \rightarrow \text{hadrons}$ at low $\sqrt{s}$.

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha}$$

$$\alpha(m_e^2) = 1/137.035999180(10) \quad \alpha(m_Z^2) = 1/127.95$$

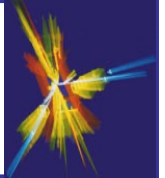
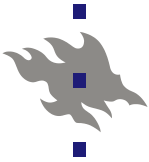
Quadratic dependence on m_t

Logarithmic dependence on m_H

Can fit both m_t and m_H

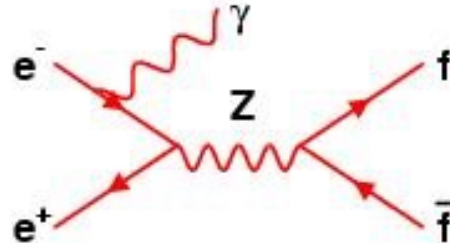
Use programs such as ZFITTER (D Bardin et al.) and TOPAZ0 (G Montagna et al.) for calculations to higher order.

Leading order expressions above are for large m_H .



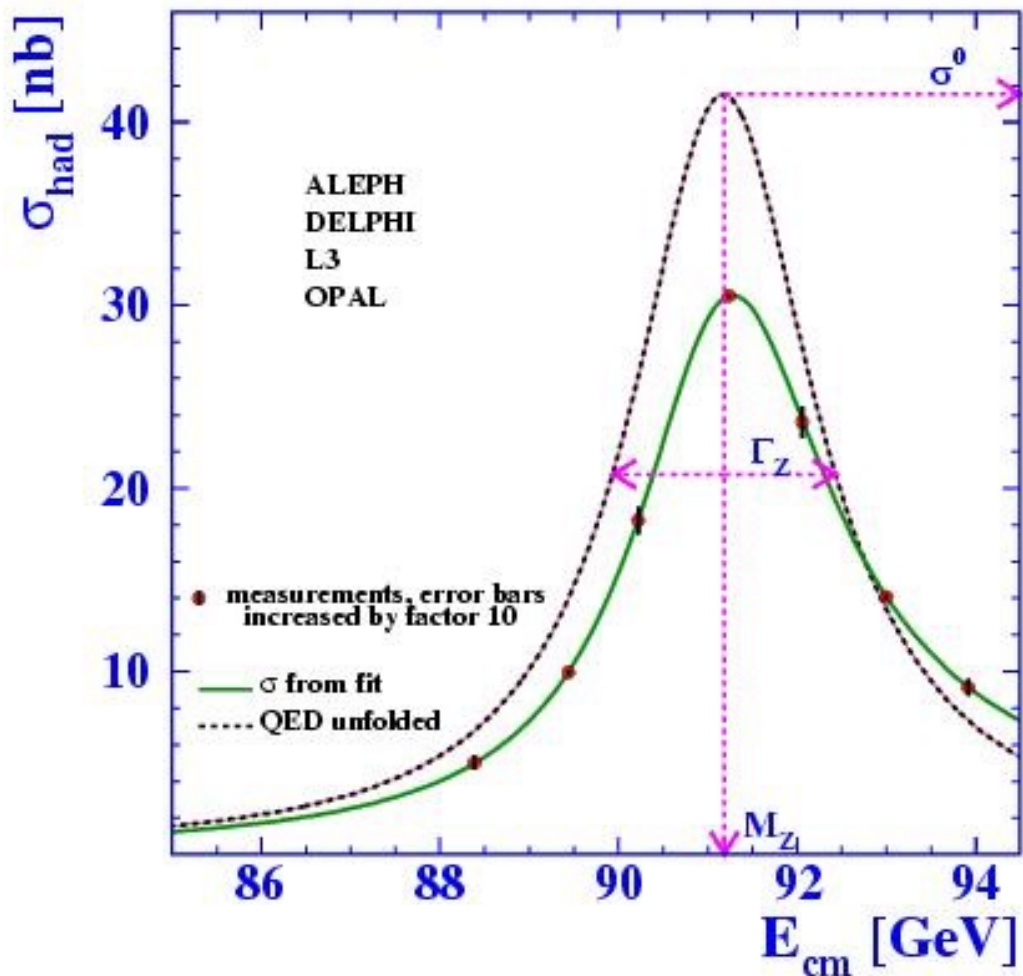
QED corrections

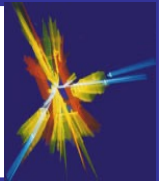
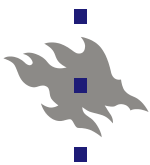
Dominant QED correction from initial state radiation.



Accounted for by radiator function H . We want $\sigma_{\text{ew}}(s)$

$$\sigma(s) = \int_{4m_f^2/s}^1 dz H_{\text{QED}}^{\text{tot}}(z, s) \sigma_{\text{ew}}(zs).$$

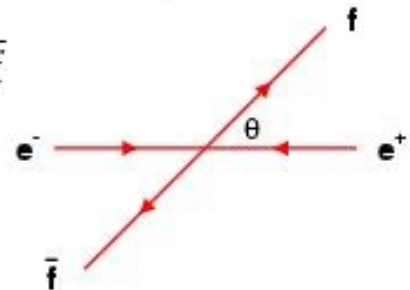




Differential cross-section

Improved Born Approximation for $e^+e^- \rightarrow f\bar{f}$

(ignoring fermion masses, QED/QCD ISR/FSR ...)



$$\frac{d\sigma_{ew}}{d\cos\theta} = \frac{\pi N_c^f}{2s} 16 |\chi(s)|^2 \times$$

$$\left[(g_{Ve}^2 + g_{Ae}^2)(g_{Vf}^2 + g_{Af}^2)(1 + \cos^2\theta) + 8g_{Ve}g_{Ae}g_{Vf}g_{Af}\cos\theta \right]$$

$$+ [\gamma \text{ exchange}] + [\gamma Z \text{ interference}]$$

Where

$$\chi(s) = \frac{G_F m_Z^2}{8\pi\sqrt{2}} \frac{s}{s - m_Z^2 + is\Gamma_Z/m_Z}$$

Even term in $\cos\theta$ gives total cross-section

$$\sigma_{\text{H}} \propto (g_{Ve}^2 + g_{Ae}^2)(g_{Vf}^2 + g_{Af}^2)$$

Odd term in $\cos\theta$ leads to **forward-backward asymmetry**:

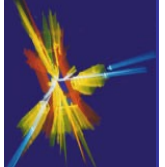
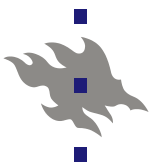
$$A_{\text{FB}} = \frac{\sigma_{\text{F}} - \sigma_{\text{B}}}{\sigma_{\text{F}} + \sigma_{\text{B}}}$$

where $\sigma_{\text{F}} = \int_0^1 (d\sigma/d\cos\theta) d\cos\theta$
(B) $\int_{-1}^0 (d\sigma/d\cos\theta) d\cos\theta$

$$A_{\text{FB}}^{0,f} = \frac{3}{4} \frac{2g_{Ve}g_{Ae}}{g_{Ve}^2 + g_{Ae}^2} \frac{2g_{Vf}g_{Af}}{g_{Vf}^2 + g_{Af}^2} \equiv \frac{3}{4} \mathcal{A}_e \mathcal{A}_f \quad @ Z^0 \text{ pole}$$

(Interference term means A_{FB} varies with s)

Cross-section plus A_{FB} allow g_{Vf} and g_{Af} to be derived.



Cross-section and partial widths

Cross-section as a function of s : “Z lineshape”

$$\sigma_{ff}(s) = \sigma_{ff}^0 \frac{s\Gamma_Z^2}{(s - m_Z^2)^2 + s^2\Gamma_Z^2/m_Z^2}$$

where pole cross-section is

$$\sigma_{ff}^0 = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}.$$

with $\Gamma_{ff}/\Gamma_Z = \text{BR}(Z \rightarrow f\bar{f})$ and partial width is

$$\Gamma_{ff} = N_c^f \frac{G_F m_Z^3}{6\sqrt{2}\pi} (g_{Af}^2 + g_{Vf}^2)$$

+ QED/QCD corrections eg. QCD: $\Gamma_{q\bar{q}} \rightarrow \Gamma_{q\bar{q}}(1 + \alpha_s/\pi + \dots)$

Total width of Z

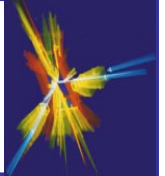
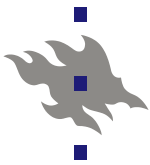
$$\Gamma_Z = \Gamma_{\text{had}} + 3\Gamma_{\ell\ell} + \Gamma_{\text{inv}} = \Sigma\Gamma_{q\bar{q}} + 3\Gamma_{\ell\ell} + N_\nu\Gamma_{\nu\nu}$$

Comparing total width to partial width gives $N_\nu = 2.9963 \pm 0.0074$

Cross-sections and widths correlated. Choose to fit:

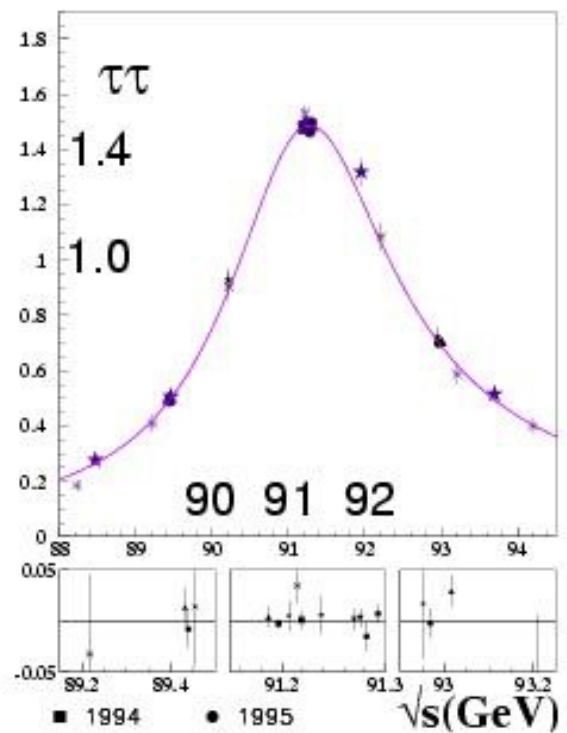
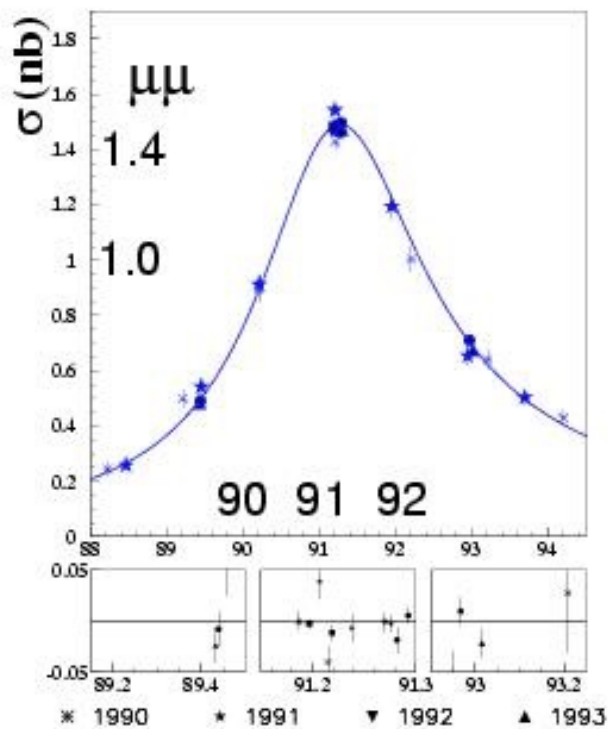
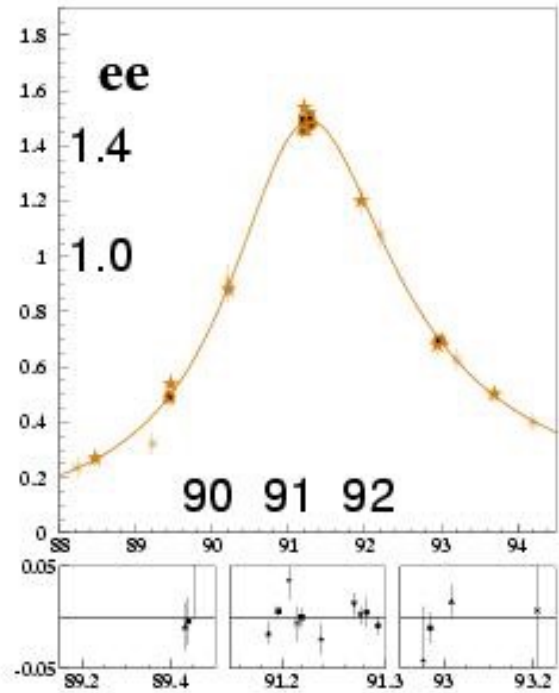
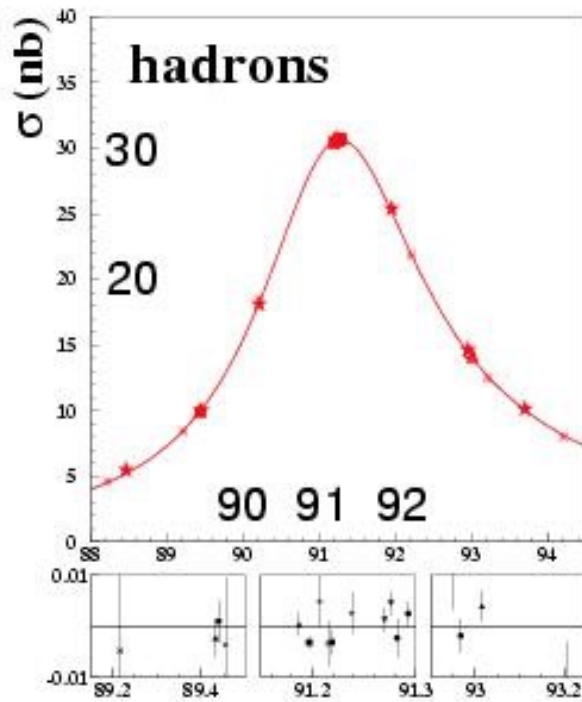
- $m_Z, \Gamma_Z, \sigma_h^0$
- Ratios: $R_e^0 \equiv \Gamma_{\text{had}}/\Gamma_{ee}, R_\mu^0 \equiv \Gamma_{\text{had}}/\Gamma_{\mu\mu}, R_\tau^0 \equiv \Gamma_{\text{had}}/\Gamma_{\tau\tau}$
or $R_\ell^0 \equiv \Gamma_{\text{had}}/\Gamma_{\ell\ell}$
- Asymmetries: $A_{\text{FB}}^{0,e}, A_{\text{FB}}^{0,\mu}$ and $A_{\text{FB}}^{0,\tau}$ or $A_{\text{FB}}^{0,\ell}$

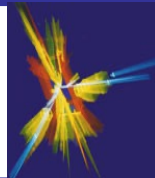
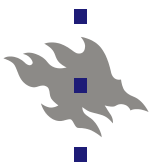
Extra information from tagging some quark flavours - see later.



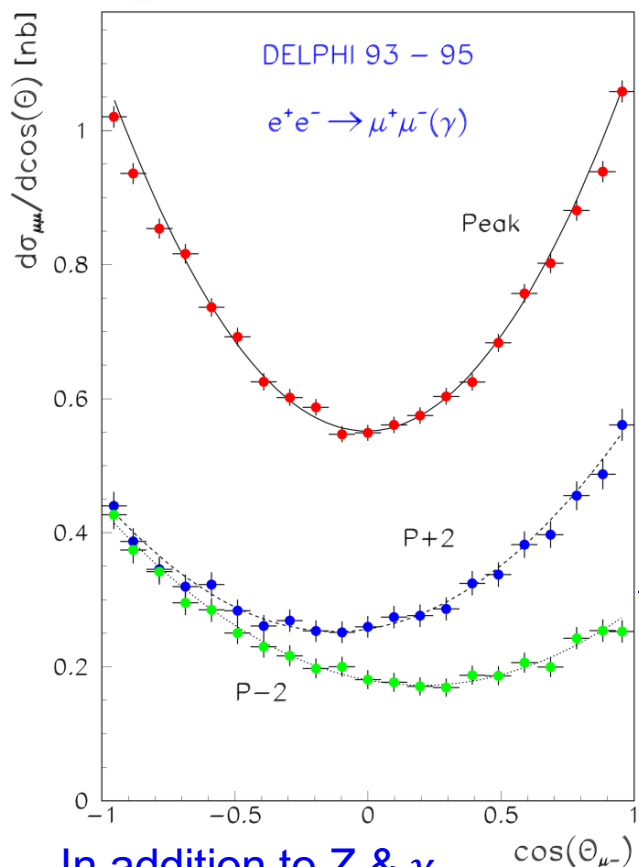
Cross-sections vs $\sqrt{s}$

ALEPH





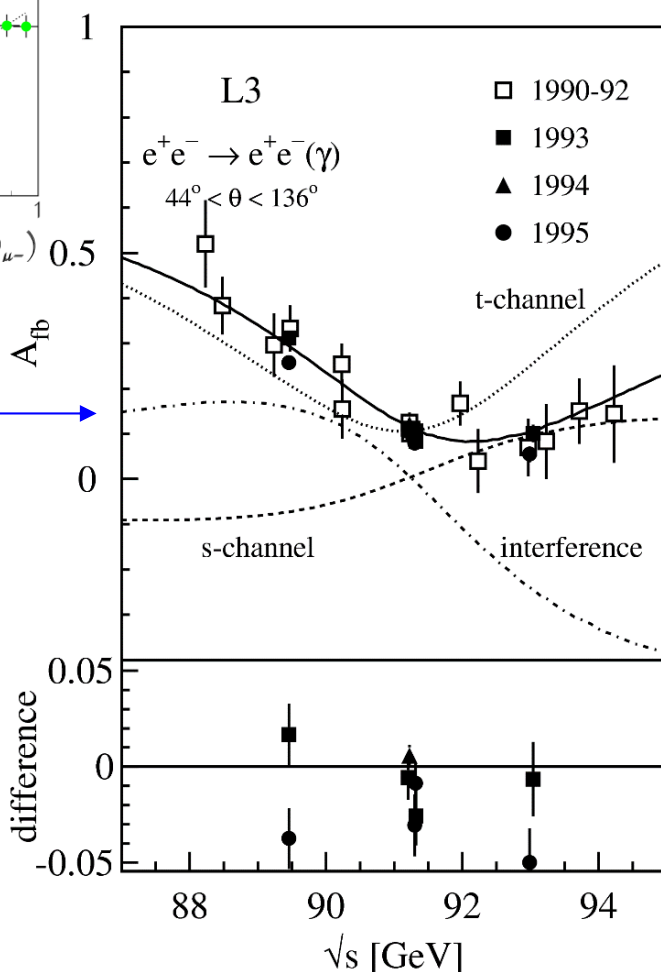
Lepton forward-backward asymmetries

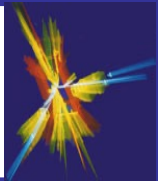
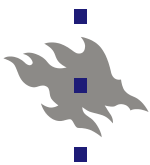


Forward-backward asymmetry for lepton pairs is straightforward to measure. Charge of lepton from tracking.

In addition to Z & γ exchange (“s-channel”), has for electrons also to take into account Bhabha scattering (“t-channel”) plus their interference.

Asymmetry varies with centre-of-mass energy.





Lepton Universality

Plot $A_{\text{FB}}^{0,\ell}$ vs. $R_\ell^0 = \Gamma_{\text{had}}/\Gamma_{\ell\ell}$. Contours contain 68% probability.

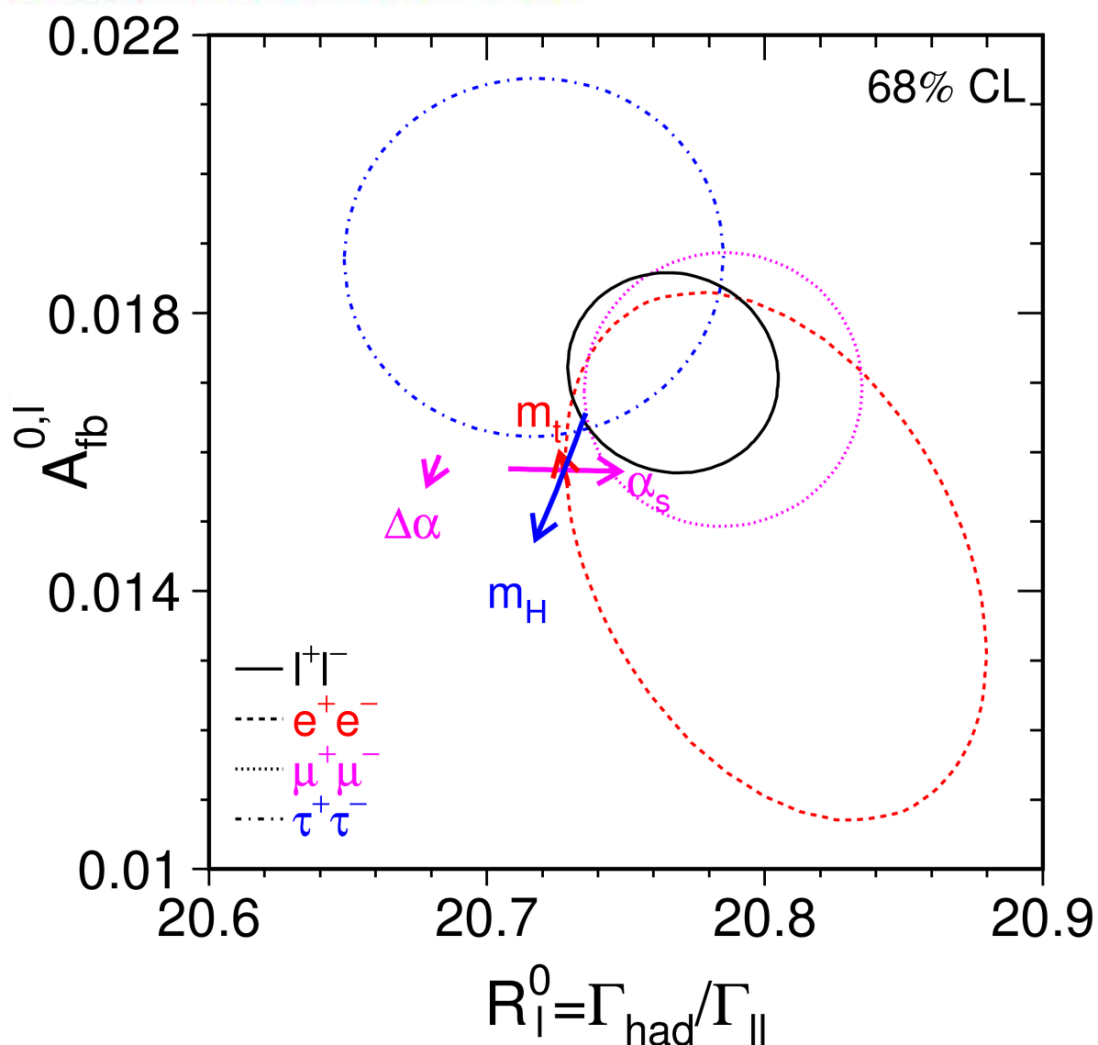
Lepton universality OK. Results agree with SM (arrows)

$$m_t = 178.0 \pm 4.3 \text{ GeV}$$

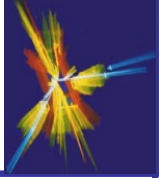
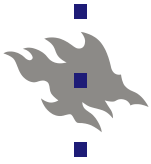
$$m_H = 300_{-186}^{+700} \text{ GeV (low } m_H \text{ preferred)}$$

$$\alpha_s(m_Z^2) = 0.118 \pm 0.002$$

$$\Delta\alpha_{\text{had}} = 0.02761 \pm 0.00036$$



Can be interpreted in terms of g_{V1} and g_{A1}



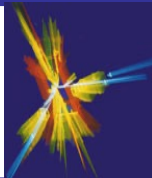
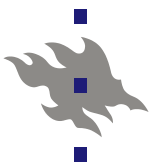
Inputs to global electroweak fits

Measurements of W, Z, quark & H properties + coupling constants from LEP/CERN, SLD/SLAC ($e^+e^- \rightarrow Z^0$)
Tevatron/Fermilab, LHC/CERN & lower $\sqrt{s}$ e^+e^- colliders.

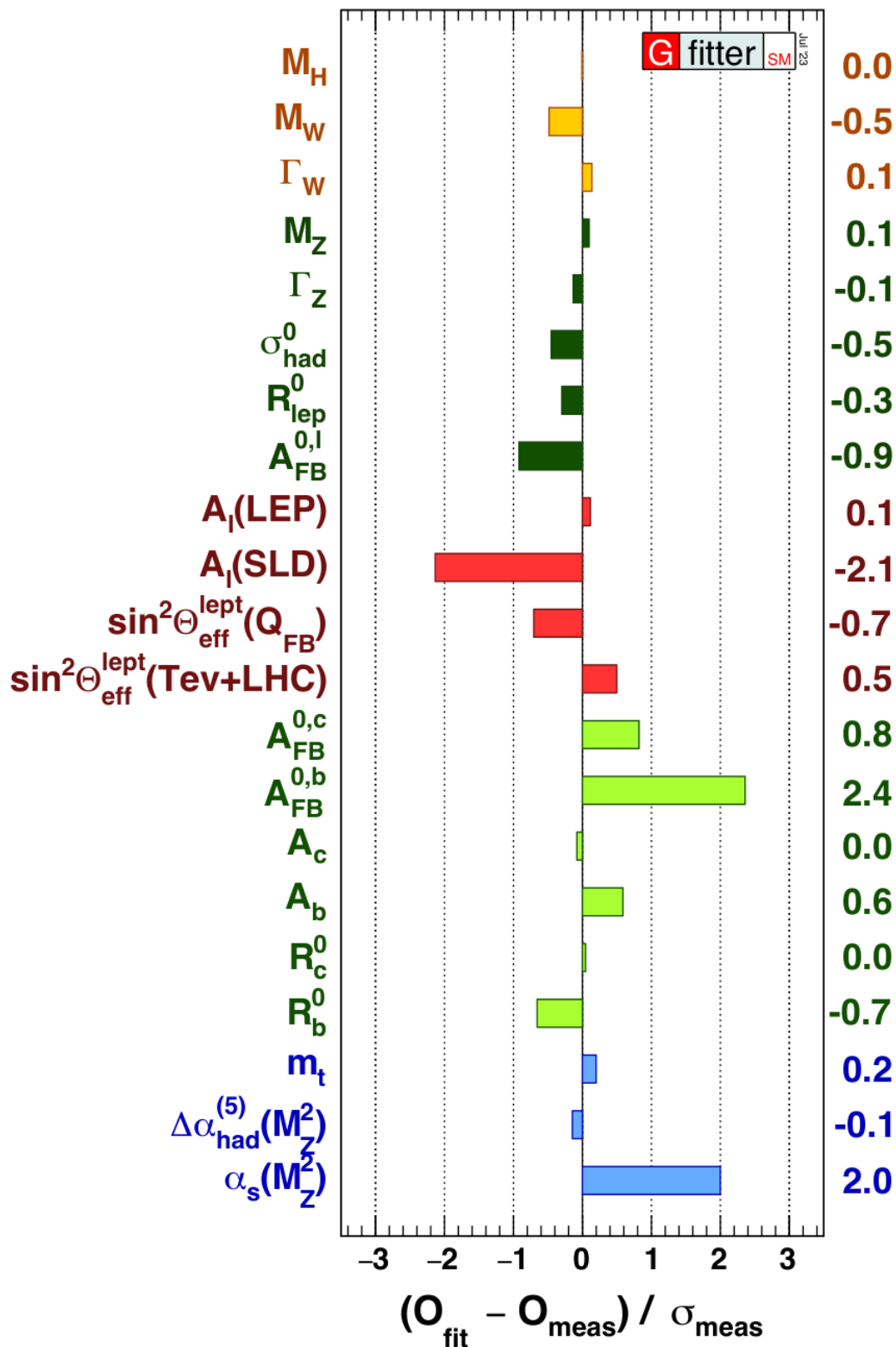
Results & plots: *Gfitter group, EPJC 78 (2018) 675, PoS ICHEP2022 (2023) 897, and PoS EPS-HEP2023 (2024) 304*

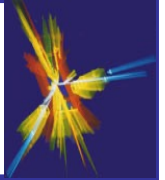
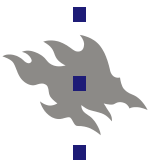
Parameter	Input value	Free in fit	Fit Result	w/o exp. input in line	w/o exp. input in line, no theo. unc
M_H [GeV]	125.1 ± 0.2	yes	$125.1^{+0.2}_{-0.2}$	$100.8^{+25.4}_{-21.1}$	$101.0^{+24.0}_{-20.1}$
M_W [GeV]	80.362 ± 0.014	–	80.356 ± 0.006	80.354 ± 0.007	80.353 ± 0.005
Γ_W [GeV]	2.085 ± 0.042	–	2.091 ± 0.001	2.091 ± 0.001	2.091 ± 0.001
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1878 ± 0.0021	91.1969 ± 0.0105	91.1969 ± 0.0100
Γ_Z [GeV]	2.4955 ± 0.0023	–	2.4952 ± 0.0014	2.4949 ± 0.0016	2.4949 ± 0.0016
σ_{had}^0 [nb]	41.500 ± 0.037	–	41.484 ± 0.015	41.481 ± 0.016	41.481 ± 0.015
R_l^0	20.767 ± 0.025	–	20.759 ± 0.017	20.750 ± 0.026	20.750 ± 0.026
$A_{\text{FB}}^{0,l}$	0.0171 ± 0.0010	–	0.01619 ± 0.0001	0.01618 ± 0.0001	0.01617 ± 0.0001
$A_\ell^{(*)}$	0.1499 ± 0.0018	–	0.1469 ± 0.0005	0.1469 ± 0.0005	0.1469 ± 0.0003
$\sin^2 \theta_{\text{eff}}^l(Q_{\text{FB}})$	0.2324 ± 0.0012	–	0.23154 ± 0.000060	0.23153 ± 0.00006	0.23154 ± 0.00004
$\sin^2 \theta_{\text{eff}}^l(\text{Tev} + \text{LHC})$	0.23141 ± 0.00026	–	0.23154 ± 0.000060	0.23154 ± 0.00006	0.23155 ± 0.00004
A_c	0.670 ± 0.027	–	0.6678 ± 0.00021	0.6678 ± 0.00021	0.6678 ± 0.00014
A_b	0.923 ± 0.020	–	0.93475 ± 0.000040	0.93475 ± 0.00004	0.93474 ± 0.00002
$A_{\text{FB}}^{0,c}$	0.0707 ± 0.0035	–	0.0736 ± 0.0003	0.0736 ± 0.0003	0.0736 ± 0.0002
$A_{\text{FB}}^{0,b}$	0.0992 ± 0.0016	–	0.1030 ± 0.0003	0.1031 ± 0.0003	0.1030 ± 0.0002
R_c^0	0.1721 ± 0.0030	–	$0.17223^{+0.00007}_{-0.00006}$	0.17224 ± 0.00007	0.17223 ± 0.00006
R_b^0	0.21629 ± 0.00066	–	$0.21586^{+0.00004}_{-0.00005}$	0.21585 ± 0.00005	0.21586 ± 0.00004
$\bar{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	–	–
$\bar{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$	–	–
m_t [GeV] ^(∇)	172.47 ± 0.68	yes	172.65 ± 0.65	$174.84^{+2.37}_{-2.39}$	$174.85^{+2.30}_{-2.32}$
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)^{(\dagger\Delta)}$	2761 ± 9	yes	2759 ± 10	2730 ± 39	2731 ± 37
$\alpha_s(M_Z^2)$	–	yes	0.1196 ± 0.0029	0.1196 ± 0.0029	0.1195 ± 0.0028

(*) Average of LEP ($A_\ell = 0.1465 \pm 0.0033$) and SLD ($A_\ell = 0.1513 \pm 0.0021$) measurements, used as two measurements in the fit. The fit w/o the LEP (SLD) measurement gives $A_\ell = 0.1469 \pm 0.0005$ ($A_\ell = 0.1467 \pm 0.0005$). (∇) Combination of experimental (0.46 GeV) and theory uncertainty (0.5 GeV). ($\dagger$) In units of 10^{-5} . (Δ) Rescaled due to α_s dependency.



Results of global electroweak fits





Electroweak precision measurements:

$$m_W = \left(\frac{\pi \alpha_{EM}}{\sqrt{2} G_F} \right)^{1/2} \frac{1}{\sin \theta_W \sqrt{1 - \Delta r}} \leftarrow \propto m_{\text{top}}^2 \& \ln(m_H^2)$$

since G_F , α_{em} , $\sin \theta_W$ are known with high precision,
precise measurements of m_{top} & m_W constrain radiative
corrections & Higgs mass (weakly due logarithmic dependence)

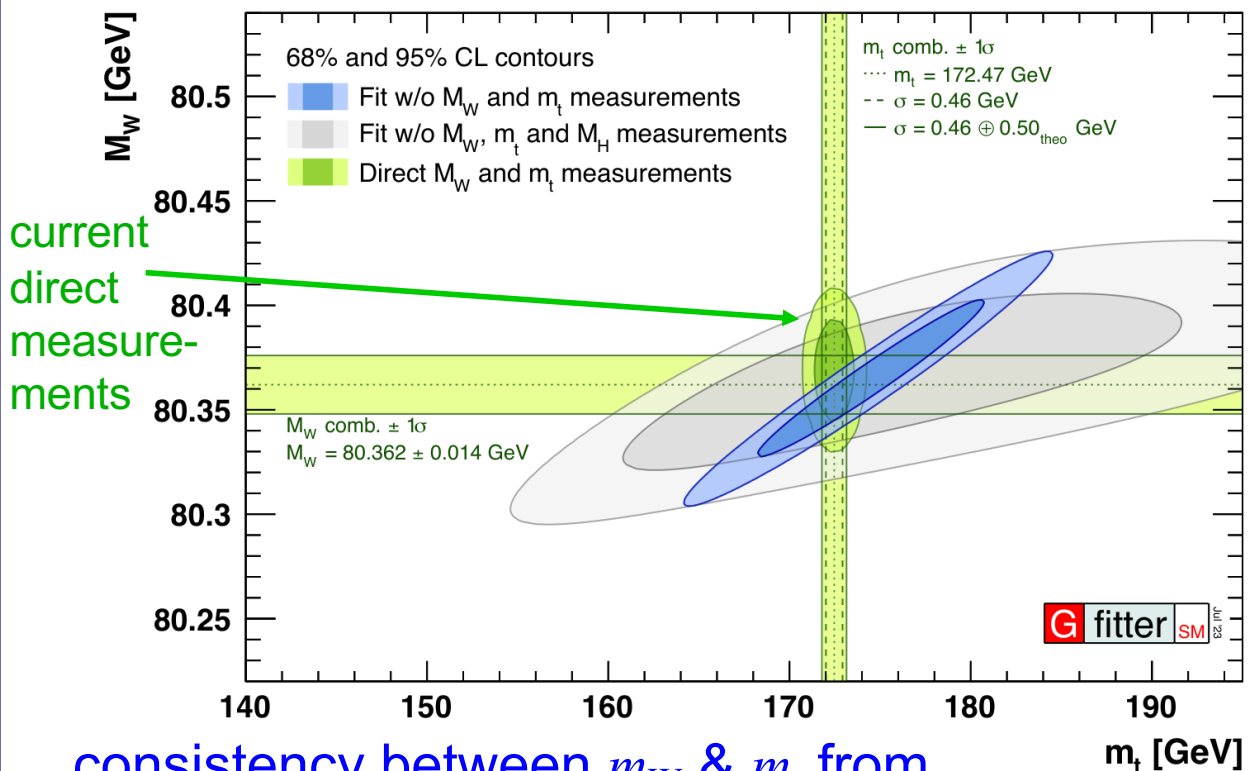
$$m_W \text{ (LEP II//LHC)} = 80.369 \pm 0.016 \text{ GeV}$$

$$m_{\text{top}} \text{ (LHC)} = 172.47 \pm 0.46 \text{ GeV}$$

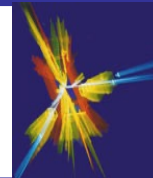
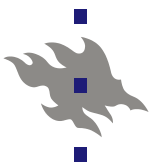
$2 \cdot 10^{-4}$

$3 \cdot 10^{-3}$

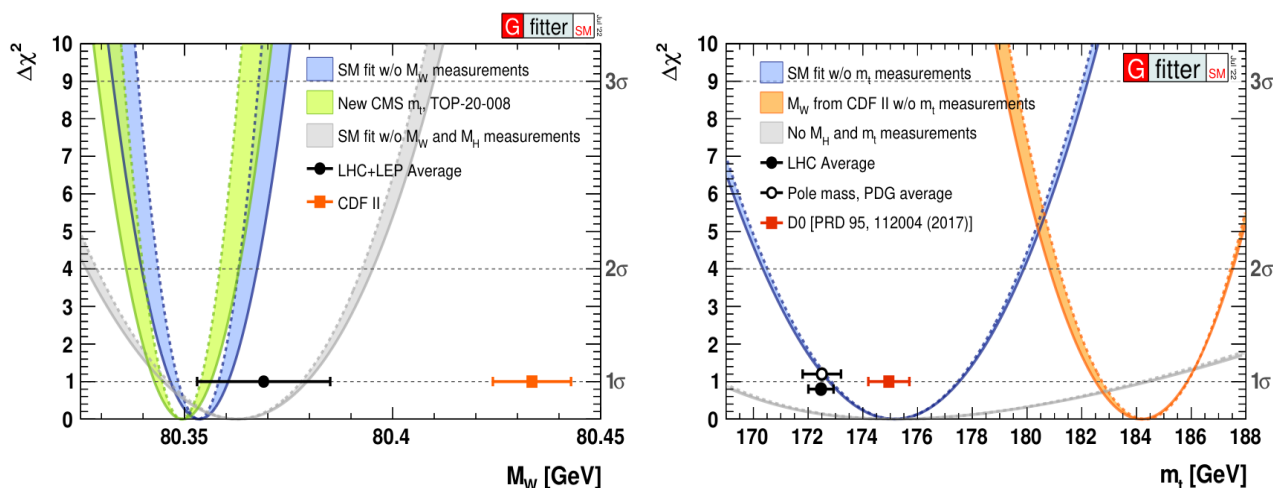
ultimate Standard Model test: compare direct
Higgs mass with radiative correction prediction.



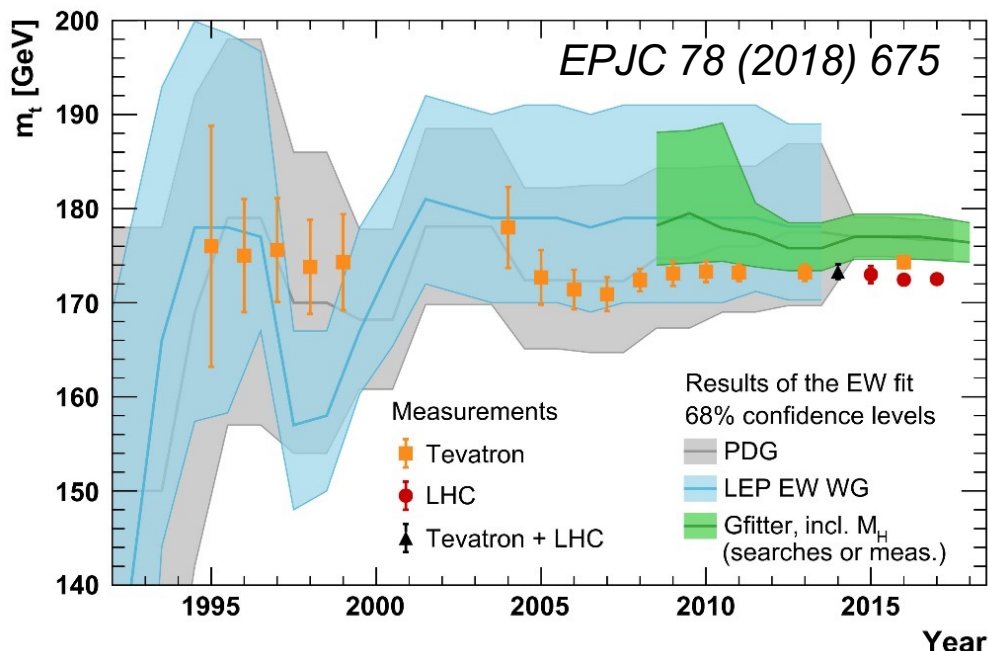
consistency between m_W & m_t from
radiative corrections and direct measurements.



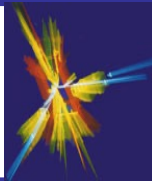
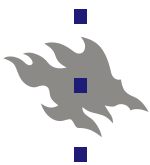
Top and W mass



Consistency (with the exception of Tevatron measurements) between m_W & m_t from radiative corrections and various direct measurements.



A measure of Standard Model success: m_t from radiative corrections vs m_t direct measurement.

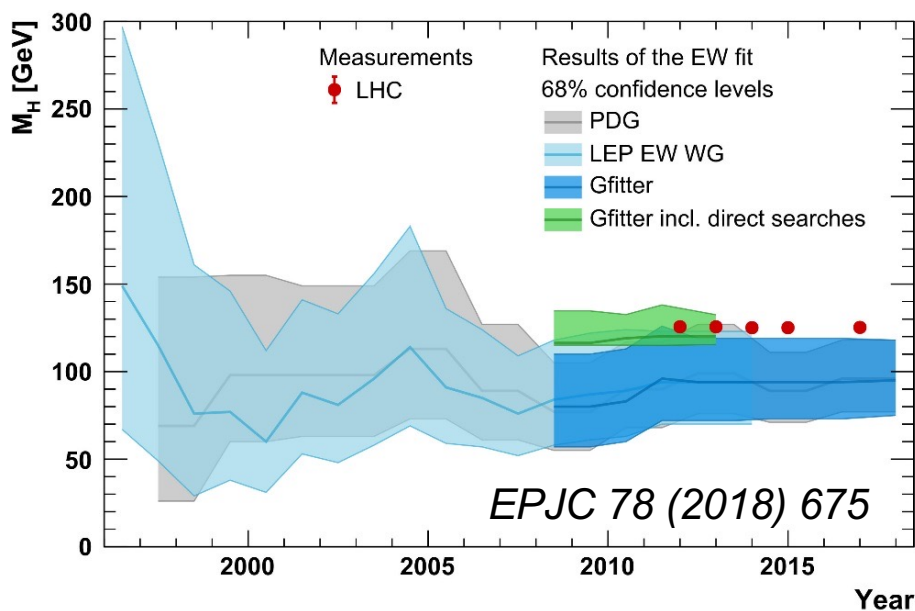
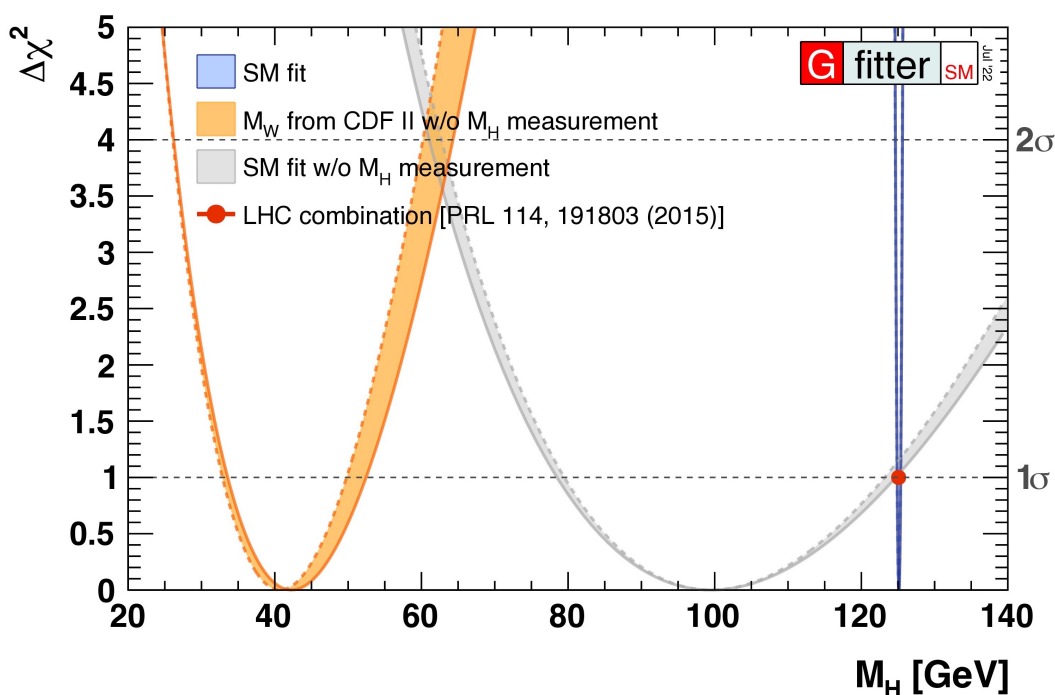


Standard Model Higgs

Electroweak fits $\Rightarrow m_H < 152$ GeV (95% CL).

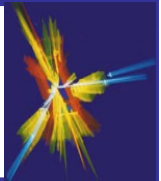
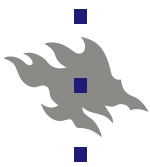
Preferred minimum $m_H = 101^{+25}_{-21}$ GeV

Direct measurement (ATLAS+CMS) $m_H = 125.1 \pm 0.2$ GeV



Overall SM fit:
 $\chi^2 = 13.8$ for
 $n_{\text{dof}} = 15 \Rightarrow$
P-value = 0.55
(August 2023)

✓ SM still very
reasonable !!



Standard Model Parameters

♠ **QCD:** $\alpha_s(M_Z)$ 1

♠ **EW Gauge / Scalar Sector:** 4

$$g, g', \mu^2, h \Leftrightarrow \alpha, \theta_W, M_W, M_H \Leftrightarrow \alpha, G_F, M_Z, M_H$$

♠ **Yukawa Sector:** 13

$$m_e, m_\mu, m_\tau$$

$$m_d, m_s, m_b$$

$$m_u, m_c, m_t$$

$$\theta_1, \theta_2, \theta_3, \delta$$



18 Free Parameters

(+ Neutrino Masses / Mixings ?)

TOO MANY !