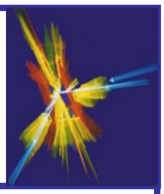
Electroweak unification

- Experimental facts
- $SU(2)_L \otimes U(1)_Y$ gauge theory
- Charged current interaction
- Neutral current interaction
- Gauge self-interactions



EXPERIMENTAL FACTS

♠ Family Structure

$$\begin{bmatrix} \nu_l & q_u \\ l^- & q_d \end{bmatrix} \equiv \begin{cases} \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L \;, \quad (\nu_l)_R \;, \quad l_R^- \\ \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L \;, \quad (q_u)_R \;, \quad (q_d)_R \end{cases}$$

♠ Three Families

$$\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix} \;, \quad \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix} \;, \quad \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}$$

♠ Charged Currents $\quad W^\pm$

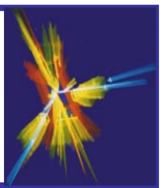
- Left-handed fermions only
- Flavour Changing: $\nu_l \Leftrightarrow l_l \quad , \quad q_u \Leftrightarrow q_d$

♠ Neutral Currents $\quad \gamma \,, Z$

- Flavour Conserving

♠ Universality (Family–Independent Couplings)

♠ $(\nu_l)_R$?



$SU(2)_L \otimes U(1)_Y$ GAUGE THEORY

Fields	$\psi_1(x)$	$\psi_2(x)$	$\psi_3(x)$
Quarks	$\begin{pmatrix} q_u \\ q_d \end{pmatrix}_L$	$(q_u)_R$	$(q_d)_R$
Leptons	$\begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L$	$(\nu_l)_R$	$(l^-)_R$

Free Lagrangian for Massless Fermions:

$$\mathcal{L}_0 = \sum_j i\overline{\psi}_j \gamma^\mu \partial_\mu \psi_j$$

$\vec{\tau}$ = Pauli matrices

$SU(2)_L \otimes U(1)_Y$ Flavour Symmetry:

$$\psi_1 \to \exp\{i\vec{\tau}\cdot\vec{\alpha}/2\}\ \exp\{iy_1\beta/2\}\,\psi_1$$

$$\overline{\psi}_1 \to \exp\{-i\vec{\tau}\cdot\vec{\alpha}/2\}\ \exp\{-iy_1\beta/2\}\,\overline{\psi}_1$$

$$\psi_2 \to \exp\{iy_2\beta/2\}\,\psi_2 \quad \overline{\psi}_2 \to \exp\{-iy_2\beta/2\}\,\overline{\psi}_2$$

$$\psi_3 \to \exp\{iy_3\beta/2\}\,\psi_3 \quad \overline{\psi}_3 \to \exp\{-iy_3\beta/2\}\,\overline{\psi}_3$$

Gauge Principle: $\vec{\alpha} = \vec{\alpha}(x), \beta = \beta(x)$

$$D_\mu \psi_1(x) \equiv [\partial_\mu - igW_\mu(x) - ig'(y_1/2)B_\mu(x)]\psi_1(x)$$

$$D_\mu \psi_1(x) \rightarrow U(x)\exp\{i(y_1/2)\beta(x)\}D_\mu \psi_1(x)$$
$$B_\mu(x) \rightarrow B_\mu(x) + (1/g')\partial_\mu \beta(x)$$
$$W_\mu(x) \rightarrow U(x)W_\mu(x)U^\dagger(x) + (1/g)U(x)\partial_\mu U^\dagger(x)$$

g (g') coupling connected to $SU(2)_L$ ($U(1)_Y$) interaction

$$W_\mu(x) \equiv \vec{\tau}/2 \cdot \vec{W}_\mu(x) \; ; \quad U(x) \equiv exp\{ i\vec{\tau}/2 \cdot \vec{\alpha}(x)\}$$

$$W^i_\mu \rightarrow W^i_\mu + \frac{1}{g}\partial_\mu \alpha^i - \varepsilon^{ijk}\alpha^j W^k_\mu + O(\alpha^j \alpha^k)$$

4 Massless Gauge Bosons

$$W^\pm_\mu \quad , \quad W^3_\mu \quad , \quad B^0_\mu$$

CHARGED CURRENTS

$$\sum_j i\,\bar{\psi}_j\,\gamma^\mu\,\mathbf{D}_\mu\,\psi_j$$

$$\longrightarrow\; g\;\bar{\psi}_1\,\gamma^\mu\,\mathbf{W}_\mu\,\psi_1 \;+\; g'\,B_\mu\,\sum_j y_j/2\,\bar{\psi}_j\,\gamma^\mu\,\psi_j$$

$$\mathbf{W}_\mu \;\equiv\; \frac{\vec{\tau}}{2}\cdot\vec{W}_\mu \;=\; \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}\,W_\mu^\dagger \\ \sqrt{2}\,W_\mu & -W_\mu^3 \end{pmatrix}$$

$$W_\mu \;\equiv\; W_\mu^1 + i\,W_\mu^2$$

$(1-\gamma_5)$ takes only left-handed (right-handed) component of the fermion (antifermion) field

$$\mathcal{L}_{CC} \;=\; \frac{g}{2\sqrt{2}}\,W_\mu^\dagger\;[\bar{q}_u\,\gamma^\mu\,(1-\gamma_5)\,q_d + \bar{\nu}_l\,\gamma^\mu\,(1-\gamma_5)\,l]$$

$$+\;\;\text{h.c.}$$

Quark / Lepton Universality

Left-Handed Interaction

NEUTRAL CURRENTS

$$\mathcal{L}_{NC} = \sum_j \bar{\psi}_j \gamma^\mu \left[g \, \frac{T_3}{2} \, W^3_\mu + g' \, y_j/2 \, B_\mu \right] \psi_j$$

Massless Fields ➡ Arbitrary Combination

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

Impose condition: existence of QED with photon exchange

A_μ has the **QED** Interaction **IF**

$$g \, \sin\theta_W = g' \, \cos\theta_W = e \quad ; \quad y = 2\,(\mathbf{Q} - \mathbf{T}_3)$$

$$(y_1/2 = Q_u - \tfrac{1}{2} = Q_d + \tfrac{1}{2} \; , \; y_2/2 = Q_u \; , \; y_3/2 = Q_d)$$

⬇ **Electroweak Unification**

$\theta_W \equiv$ Weinberg angle

$$\mathcal{L}_{NC} = e \, A_\mu \sum_j \bar{\psi}_j \gamma^\mu \mathbf{Q}_j \psi_j + \mathcal{L}^Z_{NC}$$

$$\mathbf{Q}_1 = \begin{pmatrix} Q_u & 0 \\ 0 & Q_d \end{pmatrix} \quad ; \quad Q_2 = Q_u \quad ; \quad Q_3 = Q_d$$

Nobel prize in physics 1979: Glashow, Salam & Weinberg

Neutral currents

$$\mathcal{L}_{NC}^{Z} = \frac{e}{\sin\theta_W \cos\theta_W} Z_\mu \sum_j \bar{\psi}_j \gamma^\mu \left[\frac{\tau_3}{2} - \sin^2\theta_W \mathbf{Q}_j\right] \psi_j$$

$$= \frac{e}{2\sin\theta_W \cos\theta_W} Z_\mu \sum_f \bar{f} \gamma^\mu \left[v_f - a_f \gamma_5\right] f$$

	$2\,v_f$	$2\,a_f$
q_u	$1 - \frac{8}{3}\sin^2\theta_W$	1
q_d	$-1 + \frac{4}{3}\sin^2\theta_W$	-1
ν_l	1	1
l^-	$-1 + 4\sin^2\theta_W$	-1

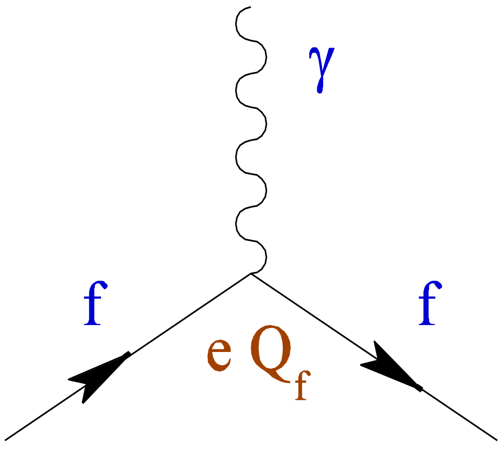
$v_f = T_f^3 - 2Q_j \sin^2\theta_W \quad a_f = T_f^3, \quad T_f^3$ is the weak isospin

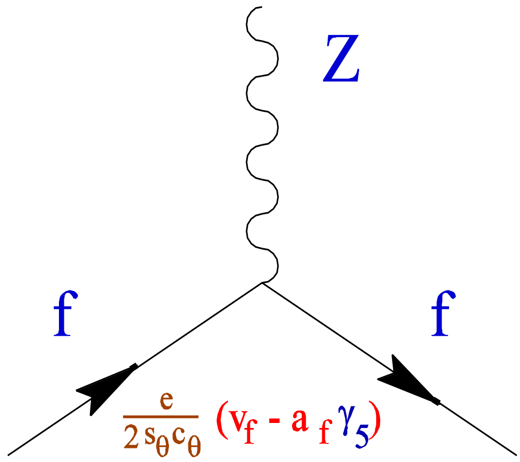
♠ **IF ν_R do exist :**

$y(\nu_R) = Q_\nu = 0$ ➡ No ν_R Interactions

Sterile Neutrinos

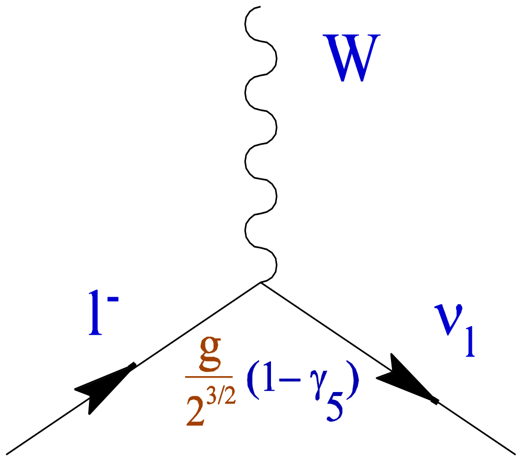
NEUTRAL CURRENTS

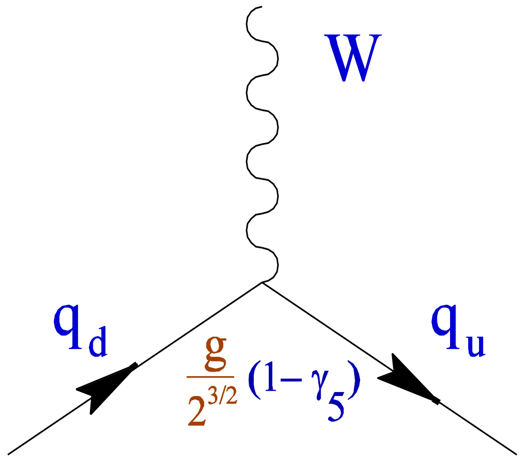




$$s_\theta = \sin\theta_W \qquad c_\theta = \cos\theta_W$$

CHARGED CURRENTS





Gauge self-interactions

$$\mathbf{W}_{\mu\nu} \equiv \frac{i}{g}\,[\mathbf{D}_\mu\,,\mathbf{D}_\nu] \equiv \frac{\vec{\tau}}{2}\cdot\vec{W}_{\mu\nu} \longrightarrow \mathbf{U}\;\mathbf{W}_{\mu\nu}\,\mathbf{U}^\dagger$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\,\epsilon^{ijk}\,W^j_\mu\,W^k_\nu$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \longrightarrow B_{\mu\nu}$$

$$\mathcal{L}_K = -\frac{1}{4}B_{\mu\nu}\,B^{\mu\nu} - \frac{1}{2}\mathrm{Tr}\,(\mathbf{W}_{\mu\nu}\,\mathbf{W}^{\mu\nu})$$

$$= -\frac{1}{4}B_{\mu\nu}\,B^{\mu\nu} - \frac{1}{4}\vec{W}_{\mu\nu}\,\vec{W}_{\mu\nu}$$

$$= \mathcal{L}_{\mathrm{kin}} + \mathcal{L}_3 + \mathcal{L}_4$$

$$\mathcal{L}_3 = -i\,e\,\cot\theta_W\,\{(\partial^\mu W^\nu - \partial^\nu W^\mu)\,W^\dagger_\mu\,Z_\nu - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}\right)\,W_\mu\,Z_\nu + W_\mu\,W^\dagger_\nu\,(\partial^\mu Z^\nu - \partial^\nu Z^\mu)\}$$

$$-i\,e\,\{(\partial^\mu W^\nu - \partial^\nu W^\mu)\,W^\dagger_\mu\,A_\nu - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}\right)\,W_\mu\,A_\nu + W_\mu\,W^\dagger_\nu\,(\partial^\mu A^\nu - \partial^\nu A^\mu)\}$$

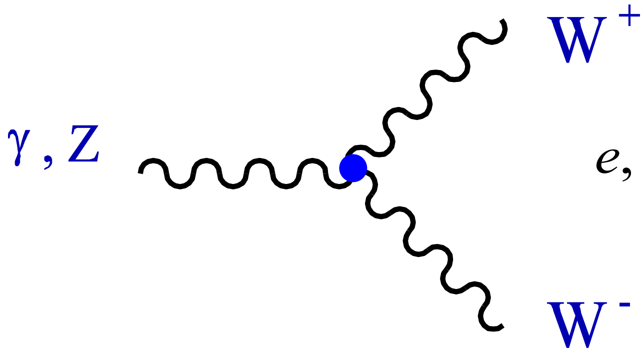
$$\mathcal{L}_4 = -\frac{e^2}{2\sin^2\theta_W}\left\{\left(W^\dagger_\mu\,W^\mu\right)^2 - W^\dagger_\mu\,W^{\mu\dagger}\,W_\nu\,W^\nu\right\}$$

$$-\,e^2\cot^2\theta_W\,\{W^\dagger_\mu\,W^\mu\,Z_\nu\,Z^\nu - W^\dagger_\mu\,Z^\mu\,W_\nu\,Z^\nu\}$$

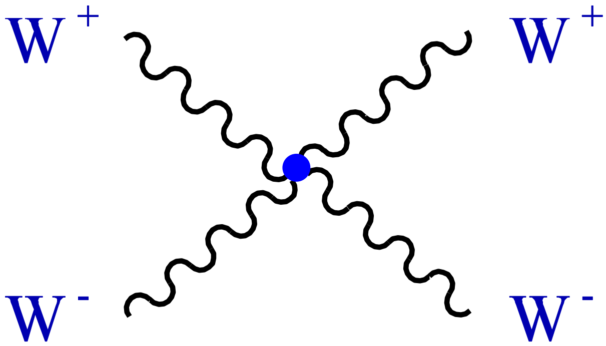
$$-\,e^2\cot\theta_W\,\{2\,W^\dagger_\mu\,W^\mu\,Z_\nu\,A^\nu - W^\dagger_\mu\,Z^\mu\,W_\nu\,A^\nu - W^\dagger_\mu\,A^\mu\,W_\nu\,Z^\nu\}$$

$$-\,e^2\,\{W^\dagger_\mu\,W^\mu\,A_\nu\,A^\nu - W^\dagger_\mu\,A^\mu\,W_\nu\,A^\nu\}$$

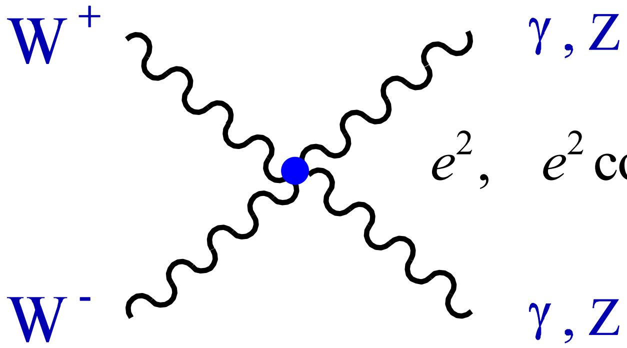
GAUGE SELF-INTERACTIONS



$e, \quad e\cot\theta_W$



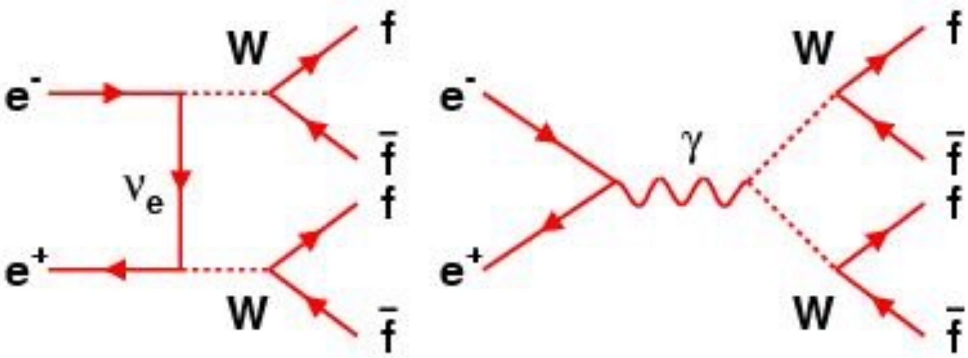
$e^2/\sin^2\theta_W$



$e^2, \quad e^2\cot\theta_W, \quad e^2\cot^2\theta_W$

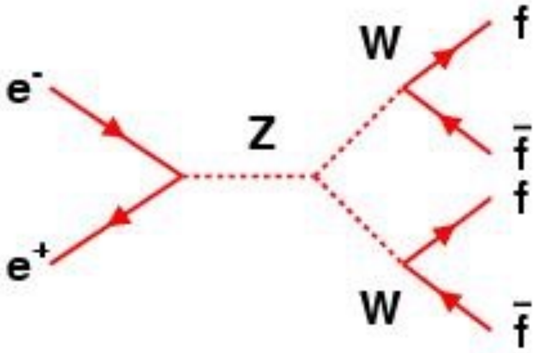
$e^+e^- \rightarrow W^+W^-$

W pair production in e^+e^- annihilation:



3 diagrams "CC03"

Notice 3-boson vertices



W decays to 68% $q\bar{q}$, 32% $\ell\nu$, so WW events are:

- 46% $q\bar{q}q\bar{q}$ – typically 4 jets

  efficiency/purity $\sim$ 90%/80%

- 44% $q\bar{q}\ell\nu$ – 2 jets, one charged lepton, missing momentum

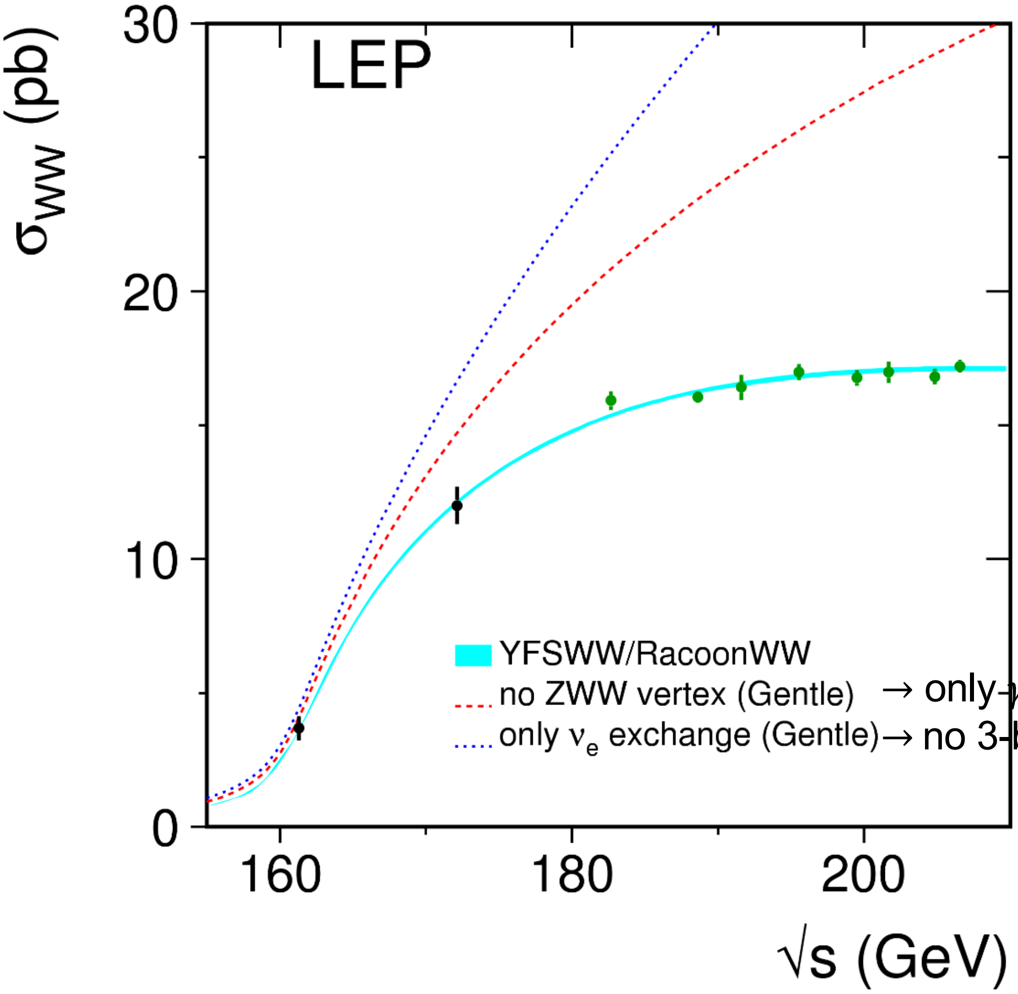
  efficiency/purity $\sim$ 80%/90%

- 10% $\ell\nu\ell\nu$ – two charged leptons, missing momentum

  efficiency/purity $\sim$ 60–80%/90%

12 000 WW produced/experiment ($\sim$17 pb $\times$ 700 pb^{-1}).

W production cross section



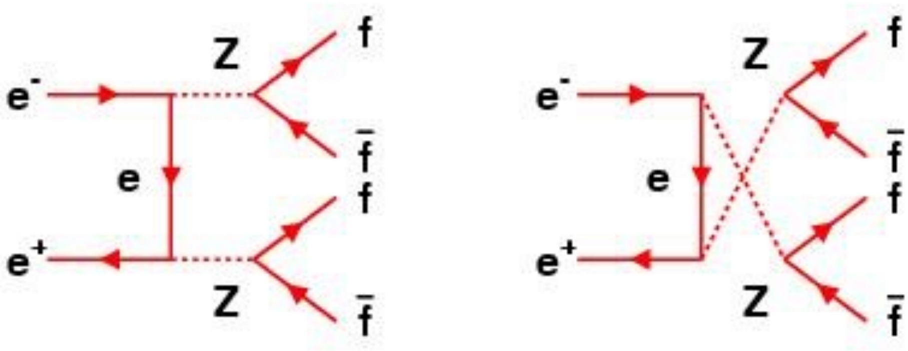
→ only-γWW vertex
→ no 3-boson vertices

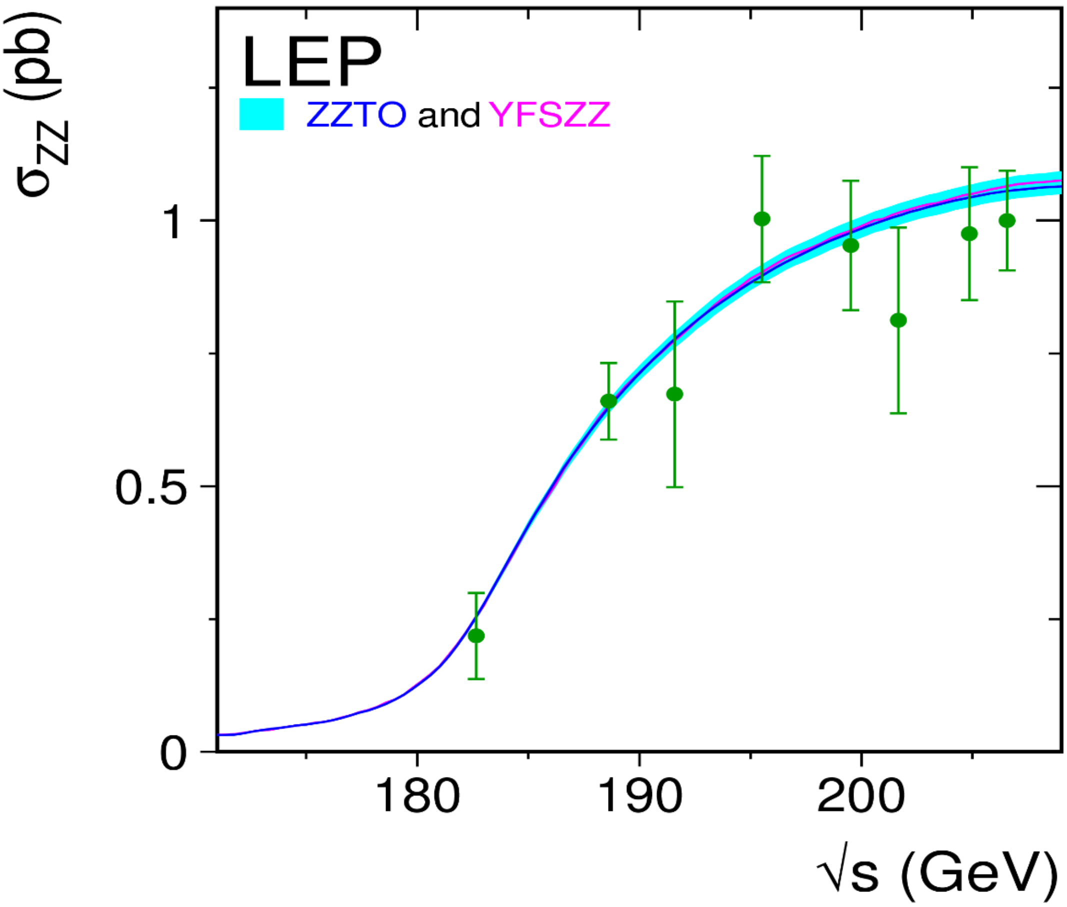
- Sensitivity to mass at threshold (only 10 pb^{-1}).
- Sensitivity to gauge couplings - beautiful demonstration of non-abelian nature of electroweak theory.
- $BR(W \to e\nu, \mu\nu, \tau\nu)$ measured. Test of lepton universality at 2.0% level. $Br(W \to q\bar{q})$ = (67.41 ± 0.27) % (PDG)

Z pair production at LEP2

$e^+e^- \rightarrow ZZ$

2 diagrams "NC02"





NB! No $e^+e^- \rightarrow Z/\gamma \rightarrow ZZ$ vertex in the Standard Model

PROBLEM WITH MASS SCALES

Gauge Symmetry

→ $m_\gamma = 0$ Good

$m_W = m_Z = 0$ Bad!

Experimentally:
m_W = 80.37 GeV (PDG)
m_Z = 91.19 GeV (PDG)

Moreover

$$\mathcal{L}_{m_f} \equiv -m_f \, \bar{f} \, f = -m_f \left(\bar{f}_L \, f_R + \bar{f}_R \, f_L \right)$$

Also Forbidden by Gauge Symmetry

→ $m_f = 0$ $\forall \, f$

All Particles Massless

Higgs mechanism

- Spontaneous symmetry breaking
- Higgs mechanism
- The Higgs boson
- Fermion masses
- Fermion mixing

Spontaneous symmetry breaking

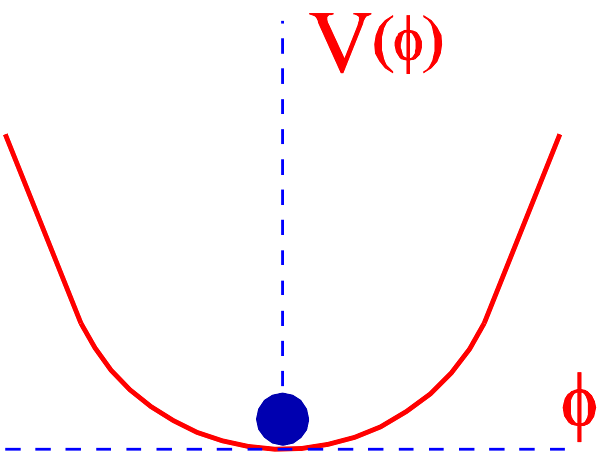
We have been able to generate electromagnetism and the strong interactions through local gauge invariance. We would like to do the same for the weak interactions, but we immediately face a problem. Unlike the photon and the gluons, the W and Z have non-zero masses. A mass term in the Lagrangian, $\frac{1}{2} m^2 A_\mu A^\mu$, would not be gauge invariant.
Suppose we forget about gauge invariance and just put it in anyway. Then theory becomes non-renormalizable, i.e. the counterterms required to cancel all divergences become infinite and theory looses all predictive power.

There is a more subtle way to generate a mass for the W and Z, called ***spontaneous symmetry breaking***.
Let us first consider a real scalar field ϕ with an arbitrary ”potential”.

$$L = \tfrac{1}{2} (\partial_\mu \phi)^2 - V(\phi)$$

We require the L to be invariant under $\phi \to \phi'$.
Then, if we expand V, it will have only even values of ϕ:

$$L = \tfrac{1}{2} (\partial_\mu \phi)^2 - \tfrac{1}{2} \mu^2 \phi^2 - \tfrac{1}{4}\lambda \phi^4 \ldots$$

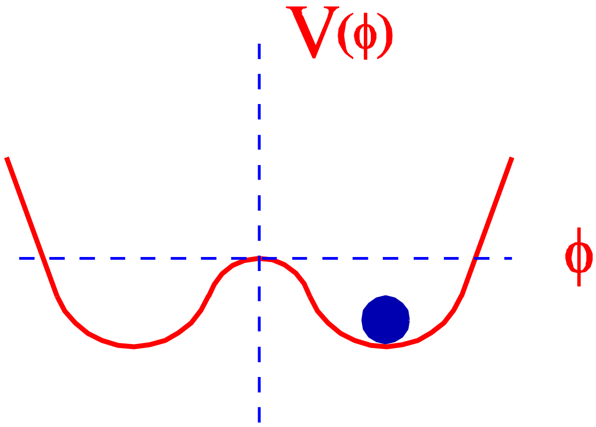


Spontaneous symmetry breaking

But what happens if μ^2 is allowed to be negative

$$L = \tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4 \ldots$$

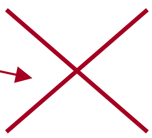
The sign of the mass term has changed giving an imaginary mass. This makes no sense. The potential looks like in the figure. The minimum is now at $\phi = \pm\sqrt{-\mu^2/\lambda}$.



Feynman diagrams represent a perturbation serie. The serie would not converge if we expand about $\phi = 0$, since this is a local maximum. We need to expand about the global minimum, say $+\sqrt{-\mu^2/\lambda}$. $\phi(x^\mu) = +\sqrt{-\mu^2/\lambda} + \eta(x^\mu)$, and then $\eta = 0$ corresponds to the minimum. Then

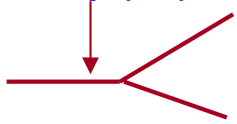
$$L = \tfrac{1}{2}[\partial_\mu(\sqrt{-\mu^2/\lambda} + \eta(x^\mu))]^2 - \tfrac{1}{2}\mu^2(\sqrt{-\mu^2/\lambda} + \eta(x^\mu))^2 - \tfrac{1}{4}\lambda(\sqrt{-\mu^2/\lambda} + \eta(x^\mu))^4$$

$$L = \tfrac{1}{2}(\partial_\mu\eta(x^\mu))^2 + \tfrac{1}{4}\mu^4/\lambda + \tfrac{1}{2}\mu^2\eta^2(x^\mu) - \sqrt{-\mu^2\lambda}\,\eta^3(x^\mu) - \tfrac{1}{4}\lambda\eta^4(x^\mu)$$



= constant

$m = \sqrt{-2\mu^2}$



Spontaneous symmetry breaking

The η and ϕ fields are the same fields, but we now see that the η has a mass of $\sqrt{-\mu^2}$, and does not have reflection symmetry $\eta \Leftrightarrow -\eta$. This symmetry has been "spontaneously broken".

A more interesting case occurs, when we consider a complex field, $\phi = (1/\sqrt{2})(\varphi_1 + i\varphi_2)$

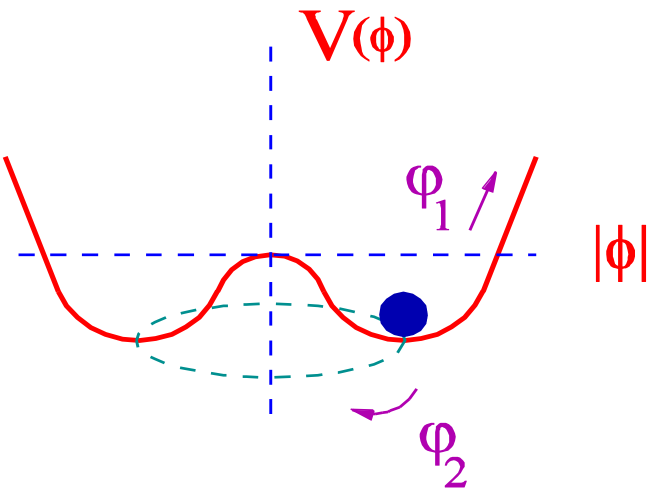
$$L = (\partial_\mu \phi)^*(\partial^\mu \phi) - \mu^2 \phi^*\phi - \lambda(\phi^*\phi)^2$$

$$L = \tfrac{1}{2}(\partial_\mu \varphi_1)^2 + \tfrac{1}{2}(\partial_\mu \varphi_2)^2 - \tfrac{1}{2}\mu^2(\varphi_1^{\,2} + \varphi_2^{\,2}) - \tfrac{1}{4}\lambda(\varphi_1^{\,2} + \varphi_2^{\,2})^2$$

The minimum is now a circle: $\varphi_1^{\,2} + \varphi_2^{\,2} = -\mu^2/\lambda$

We choose to expand about $\varphi_1 = \sqrt{-\mu^2/\lambda}$, $\varphi_2 = 0$.

$\phi(x^\mu) = (1/\sqrt{2})$
$[\sqrt{-\mu^2/\lambda} + \eta(x^\mu) + i\xi(x^\mu)]$



We get: $L = \tfrac{1}{2}(\partial_\mu \eta(x^\mu))^2 + \tfrac{1}{2}(\partial_\mu \xi(x^\mu))^2 + \tfrac{1}{4}\mu^4/\lambda$
$+ \mu^2\eta^2(x^\mu) + \sqrt{-\mu^2\lambda}\,[\eta^3(x^\mu) + \eta(x^\mu)\xi^2(x^\mu)]$
$- \tfrac{1}{4}\lambda[\eta^4(x^\mu) + \xi^4(x^\mu) + 2\eta^2(x^\mu)\xi^2(x^\mu)]$

The η field gets a mass $\sqrt{-\mu^2}$, as before, but the ξ field is massless. This always happens whenever a continuous symmetry is spontaneously broken. The massless particle is called a ***Goldstone boson***.

Spontaneous symmetry breaking

Let us consider a complex scalar field which is invariant under a local gauge transformation

$$\phi \to \exp(i\alpha(x^\mu))\,\phi$$

$$\partial_\mu \to D_\mu \equiv \partial_\mu - iqA_\mu \qquad A_\mu \to A_\mu + (1/q)\,\partial_\mu \alpha(x^\mu)$$

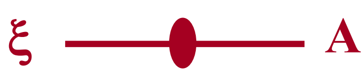
$$L = (\partial^\mu + iqA^\mu)\phi^*(\partial_\mu - iqA_\mu)\phi - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

As before $\phi = (1/\sqrt{2})(\varphi_1 + i\varphi_2)$, we choose
$\varphi_1(x^\mu) = \sqrt{-\mu^2/\lambda} + \eta(x^\mu)$; $\varphi_2(x^\mu) = \xi(x^\mu)$ and we get

$$L = \tfrac{1}{2}(\partial_\mu \eta)^2 + \tfrac{1}{2}(\partial_\mu \xi)^2 + \tfrac{1}{4}\mu^4/\lambda + \mu^2\eta^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$-\tfrac{1}{2}q^2\,(\sqrt{-\mu^2/\lambda})^2 A_\mu A^\mu + q(\sqrt{-\mu^2/\lambda})(\partial_\mu \xi)A^\mu$$

(+ interaction terms in η, ξ and A.)

Note: (a) The A field has aquired a mass $q(\sqrt{-\mu^2/\lambda})$
(b) There is still a Goldstone boson, ξ.
(c) There is a strange term $q(\sqrt{-\mu^2/\lambda})(\partial_\mu \xi)A^\mu$



This indicates that we haven't chosen the correct fields.

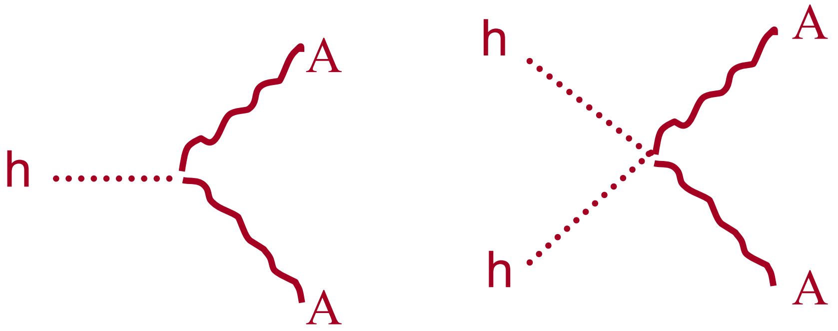
We can use our freedom to choose a gauge so that

$$\phi(x^\mu) \to (1/\sqrt{2})[\sqrt{-\mu^2/\lambda} + h(x^\mu)]\, \exp\{i\theta(x^\mu)/(\sqrt{-\mu^2/\lambda})\}$$

($h(x^\mu)$ and $\theta(x^\mu)$ are real)

$$A_\mu \to A_\mu + (1/q(\sqrt{-\mu^2/\lambda}))\, \partial_\mu \theta(x^\mu)$$

$$L = \tfrac{1}{2}(\partial_\mu h(x^\mu))^2 + \mu^2 h^2(x^\mu) + \tfrac{1}{4}\mu^4/\lambda - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$- \tfrac{1}{2}q^2(\sqrt{-\mu^2/\lambda})\, A_\mu A^\mu + (\sqrt{-\mu^2/\lambda})q^2 h\, A_\mu A^\mu + \tfrac{1}{2}q^2 h^2\, A_\mu A^\mu$$
$$- \sqrt{-\mu^2/\lambda}\, h^3 - \tfrac{1}{4}\lambda h^4$$



etc..

We have 1 massive scalar h and 1 massive vector A. Note that we started with two scalars (η and ξ) or (h and θ) and one massless vector A. This is 4 spin degrees of freedom, (2x1 + 1x2). We end with one scalar and one massive vector. This is still 4 spin degrees of freedom (1x1 + 1x3). The gauge field has eaten up the Goldstone boson. This is the ***Higgs mechanism***, and h is called the Higgs boson.

ELECTROWEAK SSB

New Scalar Doublet

$$\phi(x) \equiv \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix} \quad ; \quad y_\phi = 1$$

$$\mathcal{L}(\phi) = (\mathbf{D}_\mu \phi)^\dagger \, \mathbf{D}^\mu \phi - \mu^2 \, \phi^\dagger \phi - h \, \left(\phi^\dagger \phi\right)^2$$

$$\mathbf{D}^\mu \phi = \left[\partial^\mu - i\, g\, \mathbf{W}^\mu - i\, g'\, y_\phi/2\, B^\mu\right] \phi \; ; \; \mathbf{W}^\mu = \frac{\vec{\tau}}{2} \cdot \vec{W}^\mu$$

$\mathbf{SU(2)_L \otimes U(1)_Y}$ Symmetry

Degenerate Vacuum States: $\mu^2 < 0 \; , \; h > 0$

$$|\langle 0|\, \phi_0\, |0\rangle| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}}$$

$v \equiv$ vacuum expectation value

Spontaneous Symmetry Breaking:

$$\phi(x) = \exp\left\{i\, \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x)\right\} \; \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

HIGGS MECHANISM

$$\phi(x) = \exp\left\{i\frac{\vec{\tau}}{2}\cdot\vec{\theta}(x)\right\} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

$SU(2)_L$ Invariance ➡ $\vec{\theta}(x)$ Unphysical

Unitary Gauge: $\vec{\theta}(x) = 0$

$$(\mathbf{D}_\mu \phi)^\dagger \mathbf{D}^\mu \phi \longrightarrow \frac{1}{2}\partial_\mu H \,\partial^\mu H$$

$$+ \frac{g^2}{4}(v+H)^2 \left\{ W_\mu^\dagger W^\mu + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu \right\}$$

➡ $$\boxed{M_Z \cos\theta_W = M_W = \frac{1}{2} v\, g}$$

$M_W^2 = g^2 v^2/4$ $\qquad$ $M_Z^2 = (g^2 + g'^2)v^2/4$

Massive Gauge Bosons

Bosonic Degrees of Freedom

Massless $W^{\pm}, Z$

3×2 helicities $= 6$

$+$

3 Goldstones $\vec{\theta}$

↓ SSB

Massive $W^{\pm}, Z$

3×3 helicities $= 9$

SAME PHYSICS

THE HIGGS BOSON

$$\mathcal{L}_S = \frac{h\,v^4}{4} + \mathcal{L}_H + \mathcal{L}_{HG^2}$$

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H\,\partial^\mu H - \frac{1}{2}M_H^2\,H^2 - \frac{M_H^2}{2\,v}\,H^3 - \frac{M_H^2}{8\,v^2}\,H^4$$

$$\mathcal{L}_{HG^2} = M_W^2\,W_\mu^\dagger W^\mu \left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\} + \frac{1}{2}M_Z^2\,Z_\mu Z^\mu \left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\}$$

1 Scalar Particle H^0 should exist !!!

$$M_H = \sqrt{-2\,\mu^2} = \sqrt{2\,h}\,v$$

Free Parameter

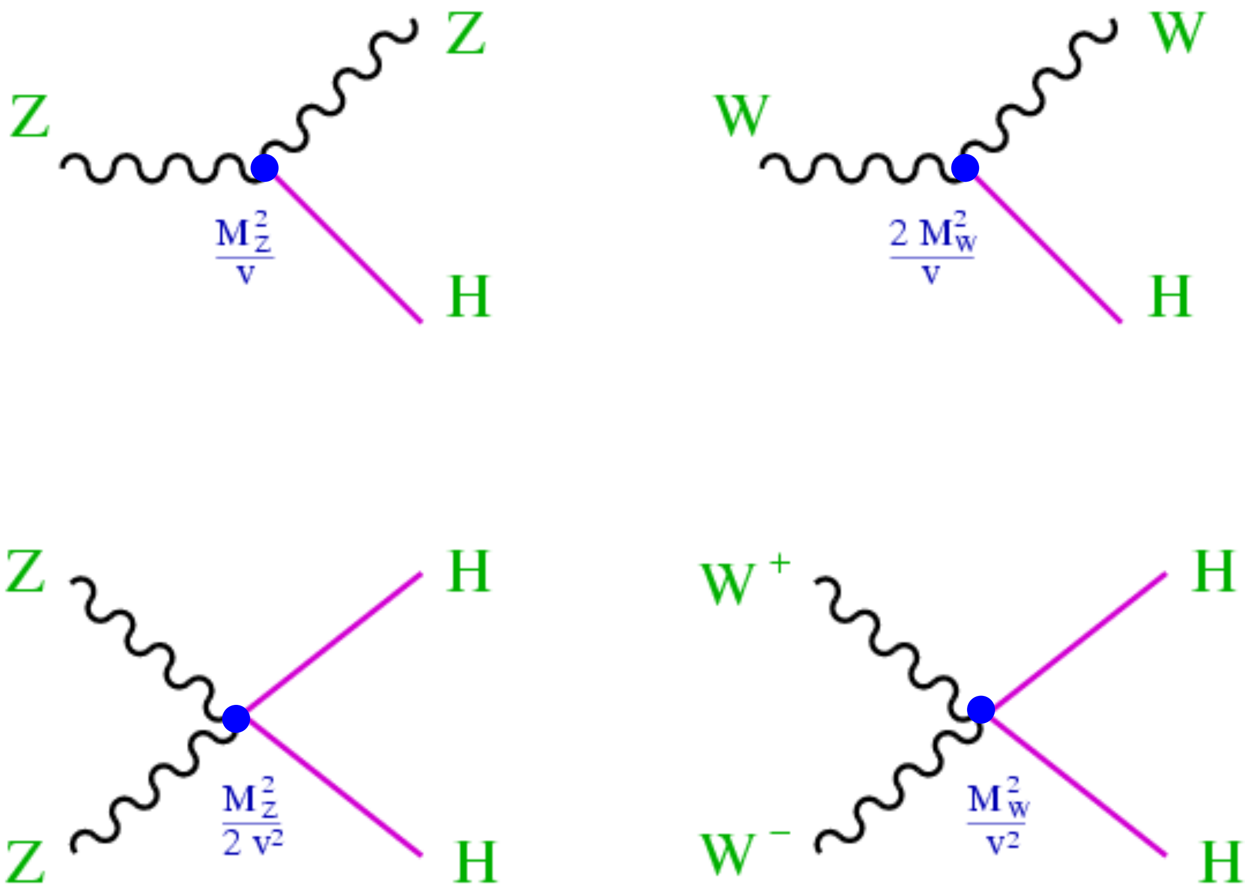
LHC:

discovery of a Higgs-like particle 4.7.2012 !!!

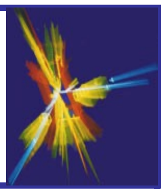
$m_{\mathrm{H}} = 125.20 \pm 0.11$ GeV (2025 CMS & ATLAS average by PDG)

Up to now all measurements of the newly discovered particle consistent with the Standard Model Higgs boson !

Higgs Couplings $\propto$ Masses



$$v = \left(\sqrt{2}\,G_F\right)^{-1/2} = 246 \ \text{GeV}$$



FERMION MASSES

Scalar–Fermion Couplings allowed by Gauge Symmetry

$$\mathcal{L}_Y = (\bar{q}_u, \bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right]$$

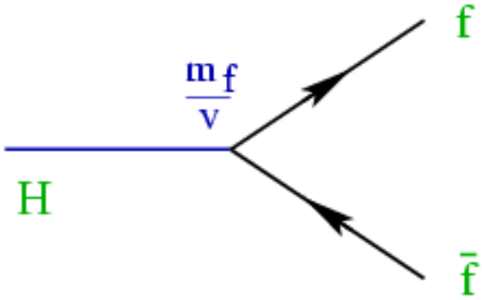
$$+ \ (\bar{\nu}_l, \bar{l})_L \ c^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l_R \quad + \quad \text{h.c.}$$

$\downarrow$ SSB

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \left\{ m_{q_d} \, \bar{q}_d \, q_d + m_{q_u} \, \bar{q}_u \, q_u + m_l \, \bar{l} \, l \right\}$$

Fermion Masses are New Free Parameters

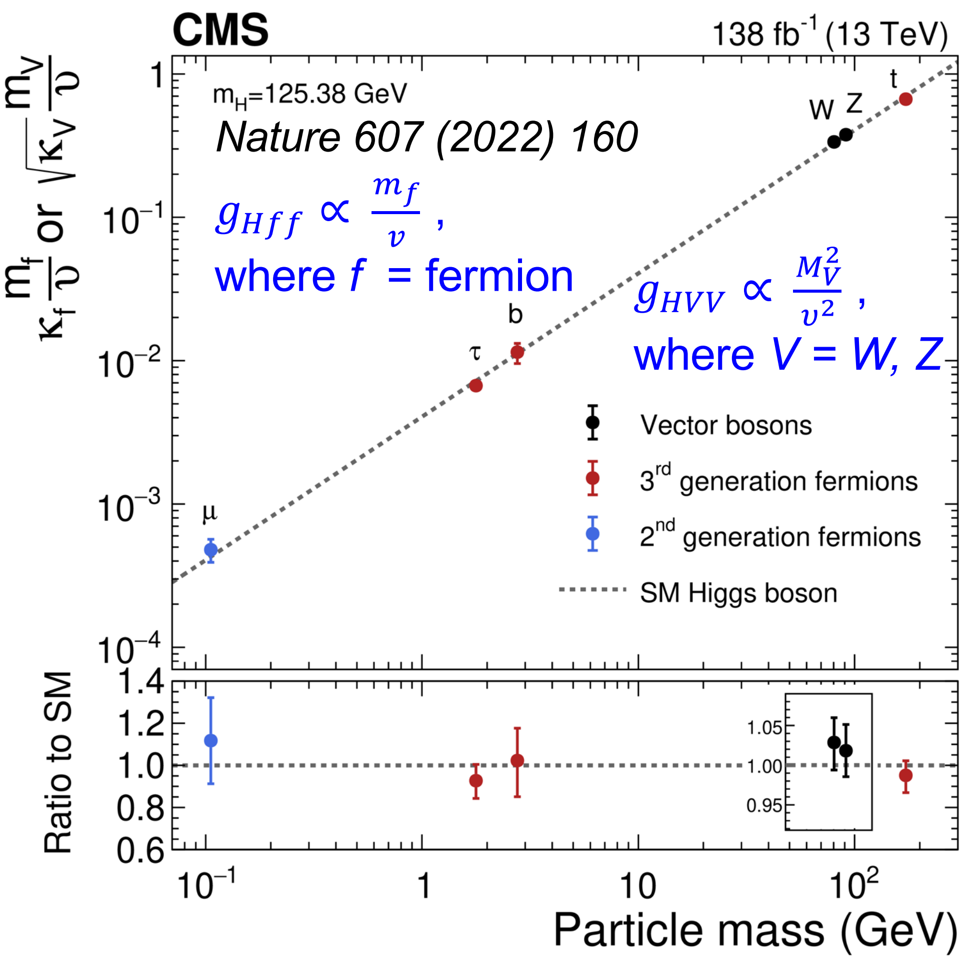
$$[m_{q_d}, m_{q_u}, m_l] = -\left[c^{(d)}, c^{(u)}, c^{(l)}\right] \frac{v}{\sqrt{2}}$$



Couplings Fixed

$$g_{Hf\bar{f}} = \frac{m_f}{v}$$

Higgs coupling & decay



BR(%)	Higgs 125 GeV	Z boson
$q\bar{q}$		70
$b\bar{b}$	58	15
$c\bar{c}$	3	12
gg	8	0
$\ell^+\ell^-$	0.02 (μ)	10
$\tau^+\tau^-$	6.4	3
γγ	0.2	
W*W*	22	
Z*Z*	2.7	

at 1st order: $$\Gamma(H \to f\bar{f}) = \frac{N_C^f G_F m_f^2}{4\sqrt{2}\pi} m_H \beta_f,$$ where β_f is the fermion velocity after the H decay.

FERMION GENERATIONS

$N_G = 3$ **Identical Copies** **WHY ?**

$$Q = 0 \quad\quad \begin{pmatrix} \nu'_j & | & u'_j \\ l'_j & | & d'_j \end{pmatrix} \quad\quad Q = +\tfrac{2}{3} \quad\quad (j = 1, \cdots, N_G)$$
$$Q = -1 \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad Q = -\tfrac{1}{3}$$

Masses are the only difference

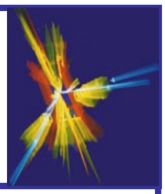
$$\mathcal{L}_Y = \sum_{jk} \left\{ (\bar{u}'_j, \bar{d}'_j)_L \left[c^{(d)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c^{(u)}_{jk} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] + (\bar{\nu}'_j, \bar{l}'_j)_L \, c^{(l)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l'_{kR} \right\} + \text{h.c.}$$

↓ SSB

$$\mathcal{L}_Y = -\left(1 + \tfrac{H}{v}\right) \left\{ \bar{\mathrm{d}}'_L \, \mathrm{M}'_d \, \mathrm{d}'_R + \bar{\mathrm{u}}'_L \, \mathrm{M}'_u \, \mathrm{u}'_R + \bar{\mathrm{l}}'_L \, \mathrm{M}'_l \, \mathrm{l}'_R + \text{h.c.} \right\}$$

Arbitrary Non-Diagonal Complex Mass Matrices

$$[\mathrm{M}'_d, \mathrm{M}'_u, \mathrm{M}'_l]_{ij} = -\left[c^{(d)}_{ij}, c^{(u)}_{ij}, c^{(l)}_{ij} \right] \frac{v}{\sqrt{2}}$$



DIAGONALIZATION OF MASS MATRICES

$$\mathbf{M'}_d = \mathbf{H}_d\,\mathbf{U}_d = \mathbf{S}_d^\dagger\,\mathcal{M}_d\,\mathbf{S}_d\,\mathbf{U}_d \qquad \mathbf{H}_f = \mathbf{H}_f^\dagger$$

$$\mathbf{M'}_u = \mathbf{H}_u\,\mathbf{U}_u = \mathbf{S}_u^\dagger\,\mathcal{M}_u\,\mathbf{S}_u\,\mathbf{U}_u \qquad \mathbf{U}_f\,\mathbf{U}_f^\dagger = 1$$

$$\mathbf{M'}_l = \mathbf{H}_l\,\mathbf{U}_l = \mathbf{S}_l^\dagger\,\mathcal{M}_l\,\mathbf{S}_l\,\mathbf{U}_l \qquad \mathbf{S}_f\,\mathbf{S}_f^\dagger = 1$$

$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right)\left\{\bar{\mathbf{d}}\,\mathcal{M}_d\,\mathbf{d} + \bar{\mathbf{u}}\,\mathcal{M}_u\,\mathbf{u} + \bar{\mathbf{l}}\,\mathcal{M}_l\,\mathbf{l}\right\}$$

$$\mathcal{M}_d = \mathrm{diag}\,(m_d, m_s, m_b) \quad , \quad \mathcal{M}_u = \mathrm{diag}\,(m_u, m_c, m_t)$$

$$\mathcal{M}_l = \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$

Mass Eigenstates $\neq$ Weak Eigenstates

$$\mathbf{d}_L \equiv \mathbf{S}_d\,\mathbf{d}'_L \quad , \quad \mathbf{u}_L \equiv \mathbf{S}_u\,\mathbf{u}'_L \quad , \quad \mathbf{l}_L \equiv \mathbf{S}_l\,\mathbf{l}'_L$$

$$\mathbf{d}_R \equiv \mathbf{S}_d\,\mathbf{U}_d\,\mathbf{d}'_R \quad , \quad \mathbf{u}_R \equiv \mathbf{S}_u\,\mathbf{U}_u\,\mathbf{u}'_R \quad , \quad \mathbf{l}_R \equiv \mathbf{S}_l\,\mathbf{U}_l\,\mathbf{l}'_R$$

$$\bar{\mathbf{f}}'_L\,\mathbf{f}'_L = \bar{\mathbf{f}}_L\,\mathbf{f}_L \;,\; \bar{\mathbf{f}}'_R\,\mathbf{f}'_R = \bar{\mathbf{f}}_R\,\mathbf{f}_R \quad \longrightarrow \quad \mathcal{L}'_{NC} = \mathcal{L}_{NC}$$

$$\bar{\mathbf{u}}'_L\,\mathbf{d}'_L = \bar{\mathbf{u}}_L\,\mathbf{V}\,\mathbf{d}_L \quad \longrightarrow \quad \mathcal{L}'_{CC} \neq \mathcal{L}_{CC}$$

$$\mathbf{V} \equiv \mathbf{S}_u\,\mathbf{S}_d^\dagger$$

QUARK MIXING

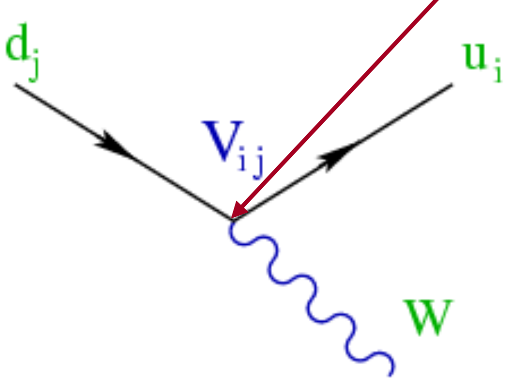
$$\mathcal{L}_{NC}^{Z} = \frac{e}{2\sin\theta_W\cos\theta_W} Z_\mu \sum_f \bar{f}\gamma^\mu \left[v_f - a_f\gamma_5\right] f$$

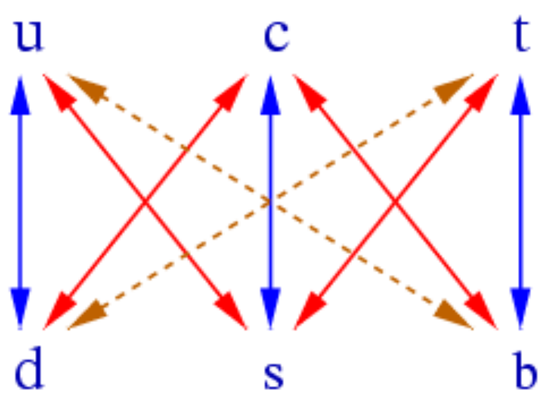
Flavour Conserving Neutral Currents

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\sum_{ij} \bar{u}_i \gamma^\mu (1-\gamma_5) \mathbf{V}_{ij} d_j + \sum_l \bar{\nu}_l \gamma^\mu (1-\gamma_5) l \right] + \text{h.c.}$$

Flavour Changing Charged Currents

transition proportional to $|\mathbf{V_{ij}}|^2$





QUARK MIXING MATRIX

- Unitary $N_G \times N_G$ Matrix: N_G^2 parameters

- $2\,N_G - 1$ arbitrary phases:

$$\left.\begin{array}{l} u_i \rightarrow e^{i\phi_i}\, u_i \\ d_j \rightarrow e^{i\theta_j}\, d_j \end{array}\right\} \longrightarrow \mathrm{V}_{ij} \rightarrow e^{i(\theta_j - \phi_i)}\, \mathrm{V}_{ij}$$

$$\Downarrow$$

V_{ij} Physical Parameters

$\frac{1}{2}\,N_G\,(N_G - 1)$	moduli or angles
$\frac{1}{2}\,(N_G - 1)\,(N_G - 2)$	phases

QUARK MIXING MATRIX

Unitarity: $V \cdot V^\dagger = V^\dagger \cdot V = 1$

$N_f = 2$: 1 angle, 0 phases

θ_c= Cabbibo angle

$$V = \begin{bmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{bmatrix} \rightarrow \text{No } \not{CP}$$

CKM = Cabibbo-Kobayashi-Maskawa

$N_f = 3$: (CKM) 3 angles, 1 phase

Source: PDG review on the CKM matrix

$$\begin{bmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta_{13}} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & c_{23}\,c_{13} \end{bmatrix}$$

$c_{ij} \equiv cos\theta_{ij}$ $s_{ij} \equiv sin\theta_{ij}$, where $i,j = 1,2$ or 3

$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3\,(\rho - i\,\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3\,(1-\rho-i\,\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$

$\lambda \approx sin\theta_c$

$\lambda \approx 0.225$ $A \approx 0.83, \bar{\rho} \approx 0.16$ $\bar{\eta} \approx 0.35$

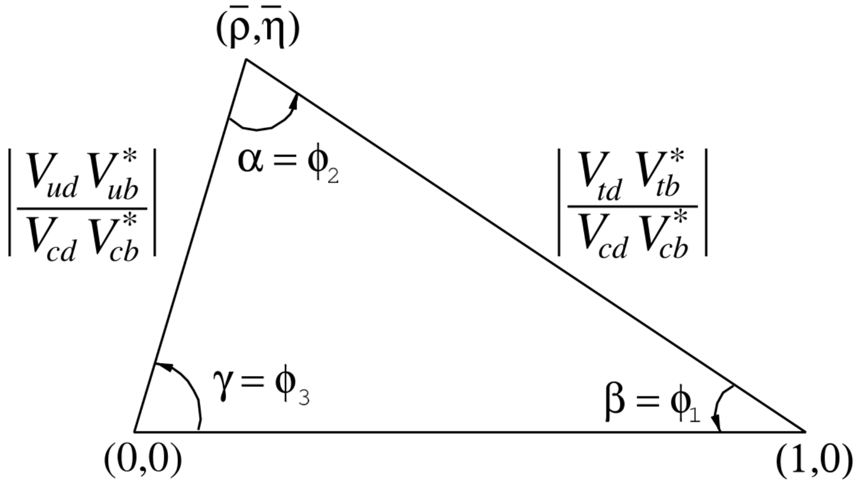
with $\bar{\rho} = \rho\,(1 - \lambda^2/2 + \ldots)$ & $\bar{\eta} = \eta\,(1 - \lambda^2/2 + \ldots)$
the CKM matrix becomes unitary to all orders of λ.

As observed for K^0, D^0 and B^0 mesons:

$$\delta_{13} \neq 0 \quad (\eta \neq 0) \rightarrow \not{CP}$$

Unitary of the CKM matrix requires 6 vanishing combinations to make a triangle in the complex plane e.g.

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



Tests of the unitary triangle at B-factories & LHC are sensitive to beyond Standard Model physics

$$\beta = (22.58 \pm 0.45)°, \alpha = \left(84.1^{+4.5}_{-3.8}\right)°, \gamma = (65.7 \pm 3.0)°$$

Summary of measurement of CKM matrix elements, B-meson oscillation (Δm_d & Δm_s), & CP violation in B hadron (α, β, γ) and K meson (ε_K) decays.

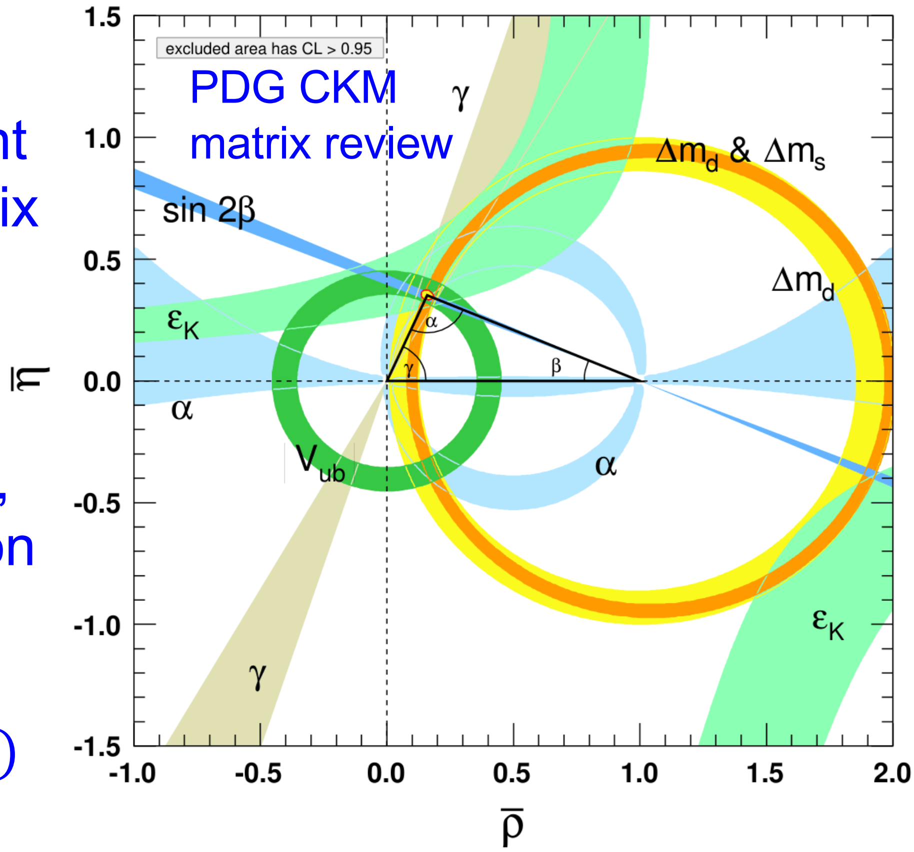
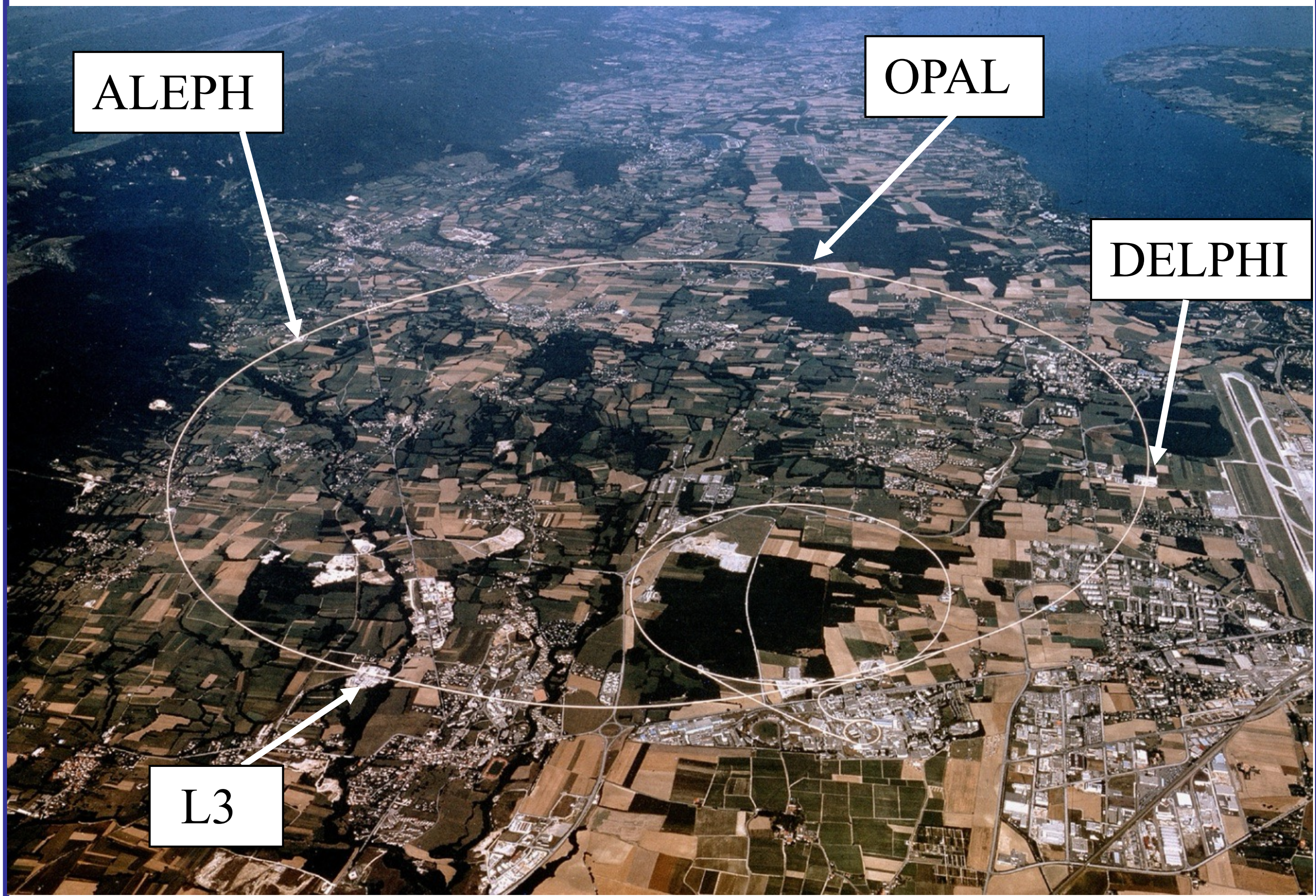


Figure 12.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

Electroweak precision measurement

- LEP and its experiments
- Electroweak radiative corrections
- Z precision measurements
- Global electroweak fit

The LEP collider



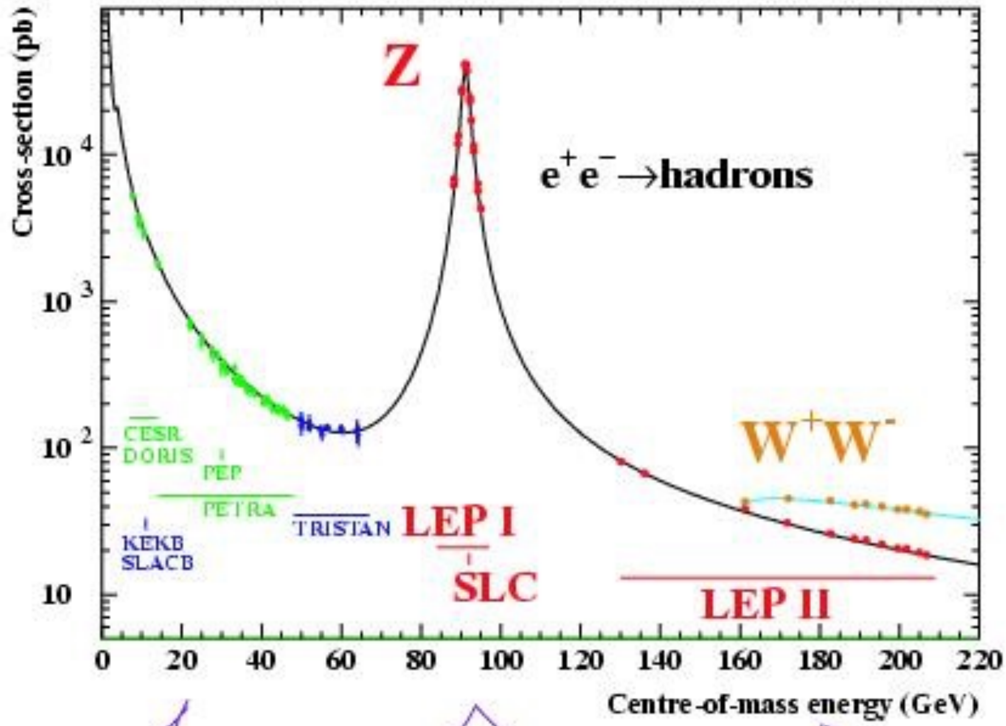
An electron–positron collider at CERN, European laboratory for particle physics, in the same tunnel that is now used by LHC.

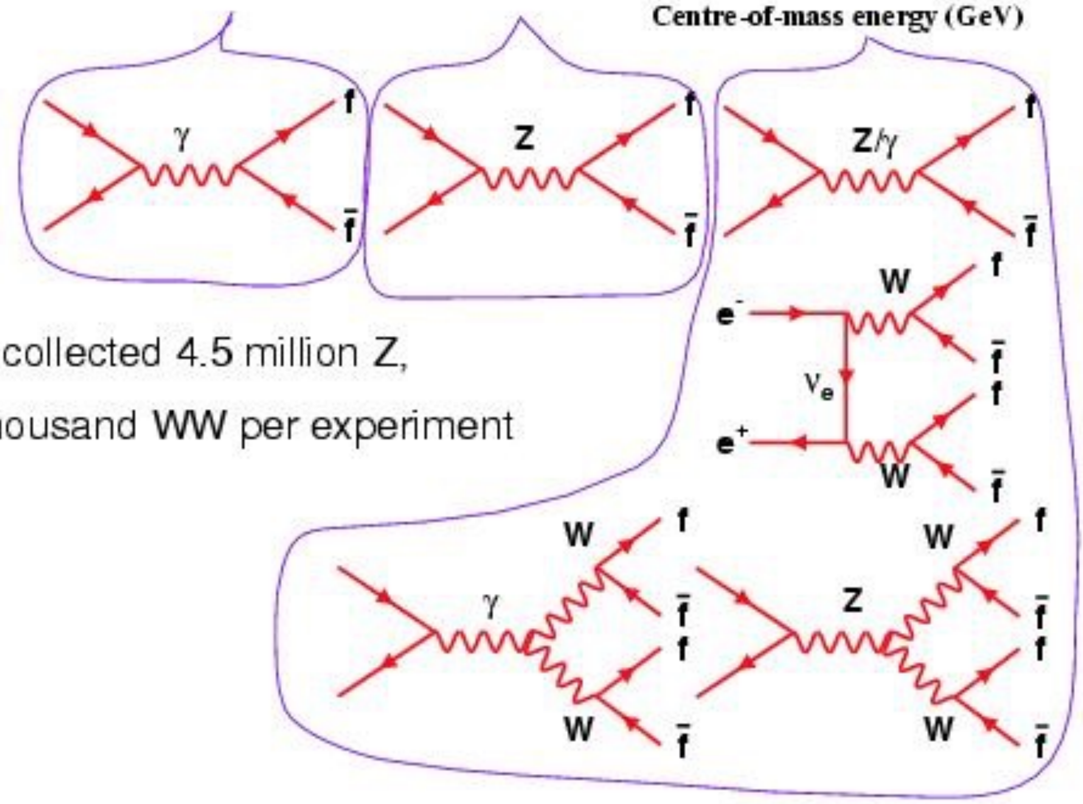
Four experiments focused on studying the Standard Model (electroweak measurements ~ W & Z, QCD measurement & Higgs searches)

LEPI – $\sqrt{s} \approx m_Z$ (1989-95)

LEPII – $\sqrt{s}$ = 140-209 GeV (1995-2000)

Electron positron annihilation





LEP collected 4.5 million Z,

12 thousand WW per experiment

Z resonance lineshape

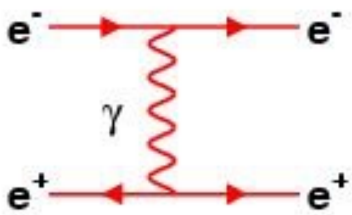
To measure the Z mass, total width and cross-section, partial widths (branching ratios) and couplings:

- LEP machine gives e^+e^- collisions at a few energies on and near the Z peak and precise measurement of E_{beam}
- Detectors ALEPH, DELPHI, L3, OPAL distinguish Z final states and measure the luminosity from QED t-channel process $e^+e^- \to e^+e^-$ (Bhabha scattering)

$$\sigma(\sqrt{s}) = (N_{\mathrm{observed}} - N_{\mathrm{background}})/\epsilon\mathcal{L}$$

- Monte Carlo simulation of the signal efficiency and background.
- Precise theoretical prediction of the lineshape
- Match precision from 4.5 million Z events per experiment - relative statistical error about 5×10^{-4}.
- Several thousand people involved: machine/experiment/theory
- $\sigma(m_Z) \approx 340$ MeV from UA2+CDF in 1989. Hoped to reduce to ≈ 10 MeV (limited by beam energy precision)
- Count the number of generations. 2.5 generations were known in 1989, top quark and ν_τ not yet established. Number of light neutrinos limited by big bang nucleosynthesis to $\lesssim 4$. Expected precision of about ± 0.2 on the number.

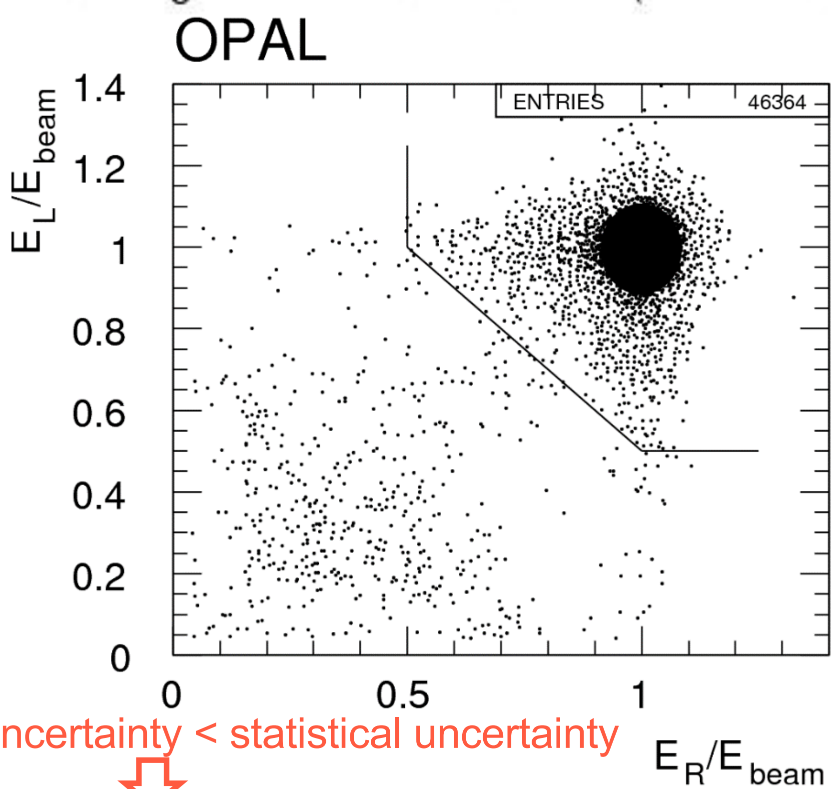
Luminosity Measurement



"Bhabha scattering"

The t-channel contribution to $e^+e^- \to e^+e^-$ dominates at small angles. Detectors typically 25 to 60 mrad from beam.

Very clear electron signal in forward detectors (calorimeters).



systematic uncertainty < statistical uncertainty

Accepted cross section at least $2 \times \sigma_{had}$. $1/\theta^3$ variation.

Experimental difficulty: define geometric edge of acceptance to give cross-section precision $\lesssim 0.05\%$.

Common theory error of $\sim 0.05\%$ (cf $\sim 1\%$ in 1989).

(BHLUMI program: S. Jadach, B.F.L. Ward et al.)

Standard Model relationships

Masses of heavy gauge bosons and their couplings to fermions depend on SAME mixing angle

$$\cos\theta_W = m_W/m_Z$$

$SU(2) \times U(1)$ coupling constants, g, g', proportional to electric charge e: $g = e\sin\theta_W$, $g' = e\cos\theta_W$

$$-ieQ\gamma_\mu$$

$$ie\gamma_\mu(g_v - g_a\gamma_5)\frac{1}{2\sin\theta_W\cos\theta_W}$$

$$ie\gamma_\mu(1-\gamma_5)\frac{1}{2\sqrt{2}\sin\theta_W}$$

where Q, g_a and g_v depend on fermion type, with

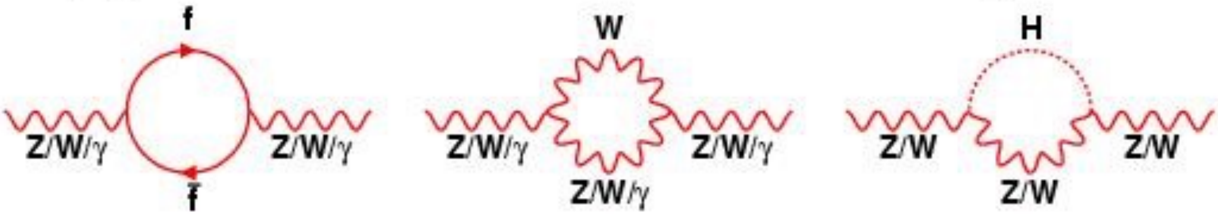
$$g_a = \quad T^3 \quad = \pm\frac{1}{2} \quad \equiv a_f$$

$$g_v = \quad (T^3 - 2Q\sin^2\theta_W) \quad = \pm\frac{1}{2}(1 - 4|Q|\sin^2\theta_W) \quad \equiv v_f$$

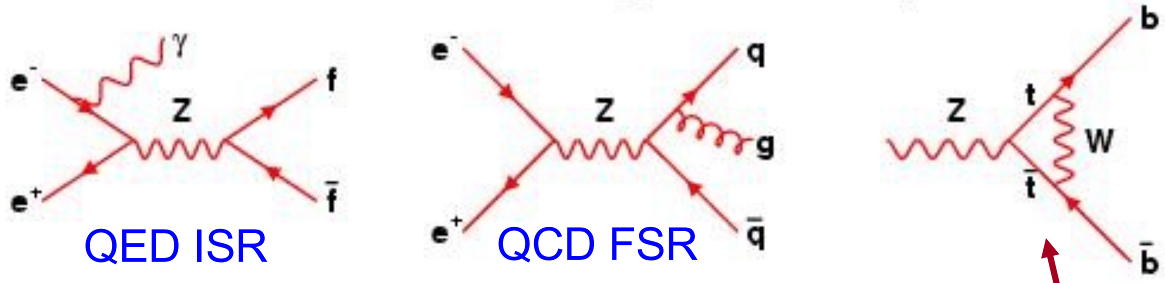
g_v/g_a gives $\sin^2\theta_W$ if you know $|Q|$.

Radiative corrections

Propagator corrections are the same for each fermion type.



QED, QCD and vertex corrections give fermion dependent terms.



QED ISR QCD FSR

ISR/FSR = initial/final state radiation

Electroweak corrections absorbed into effective couplings:

$$g_V \equiv g_V^{\text{eff}} = \sqrt{(1+\Delta\rho)}(T^3 - 2Q\sin^2\theta_{\text{eff}})$$
$$g_A \equiv g_A^{\text{eff}} = \sqrt{(1+\Delta\rho)}T^3$$
$$\sin^2\theta_{\text{eff}} = (1+\Delta\kappa)\sin^2\theta_W$$

$$\Delta\rho = \frac{3G_F M_W^2}{8\sqrt{2}\pi^2}\left(\frac{m_t^2}{m_W^2} - \tan^2\theta_W\left[\ln\frac{m_H^2}{m_W^2} - \frac{5}{6}\right]\right) + \cdots$$
$$\Delta\kappa = \frac{3G_F M_W^2}{8\sqrt{2}\pi^2}\left(\cot^2\theta_W\frac{m_t^2}{m_W^2} - \frac{11}{9}\left[\ln\frac{m_H^2}{m_W^2} - \frac{5}{6}\right]\right) + \cdots$$

Extra m_t^2/m_W^2 contributions for b quark

m_H = Higgs boson mass, m_t = top quark mass, m_W = W boson mass

Radiative corrections

The value of G_F is also modified:

$\propto m_t^2$ & $\ln m_H^2$

Fermi constant $$G_F = \frac{\pi\alpha}{\sqrt{2}\, m_W^2 \sin^2\theta_W} \frac{1}{1-\Delta r}$$

where

$$\Delta r = \Delta\alpha + \Delta r_w = \Delta\alpha - \Delta\kappa + \cdots$$

$\Delta\alpha$ term incorporates the running of the electromagnetic coupling due to fermion loops in the photon propagator. The difficult part of the calculation is to account for all the hadronic states. Use experimental measurement of $e^+e^- \to \text{hadrons}$ at low $\sqrt{s}$.

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha}$$

$\alpha(m_e^2) = 1/137.035999177(21)$ $\alpha(m_z^2) = 1/127.93$

Quadratic dependence on m_t

Logarithmic dependence on m_H

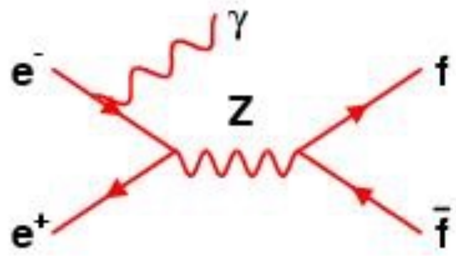
Can fit both m_t and m_H

Use programs such as ZFITTER (D Bardin et al.) and TOPAZ0 (G Montagna et al.) for calculations to higher order.

Leading order expressions above are for large m_H.

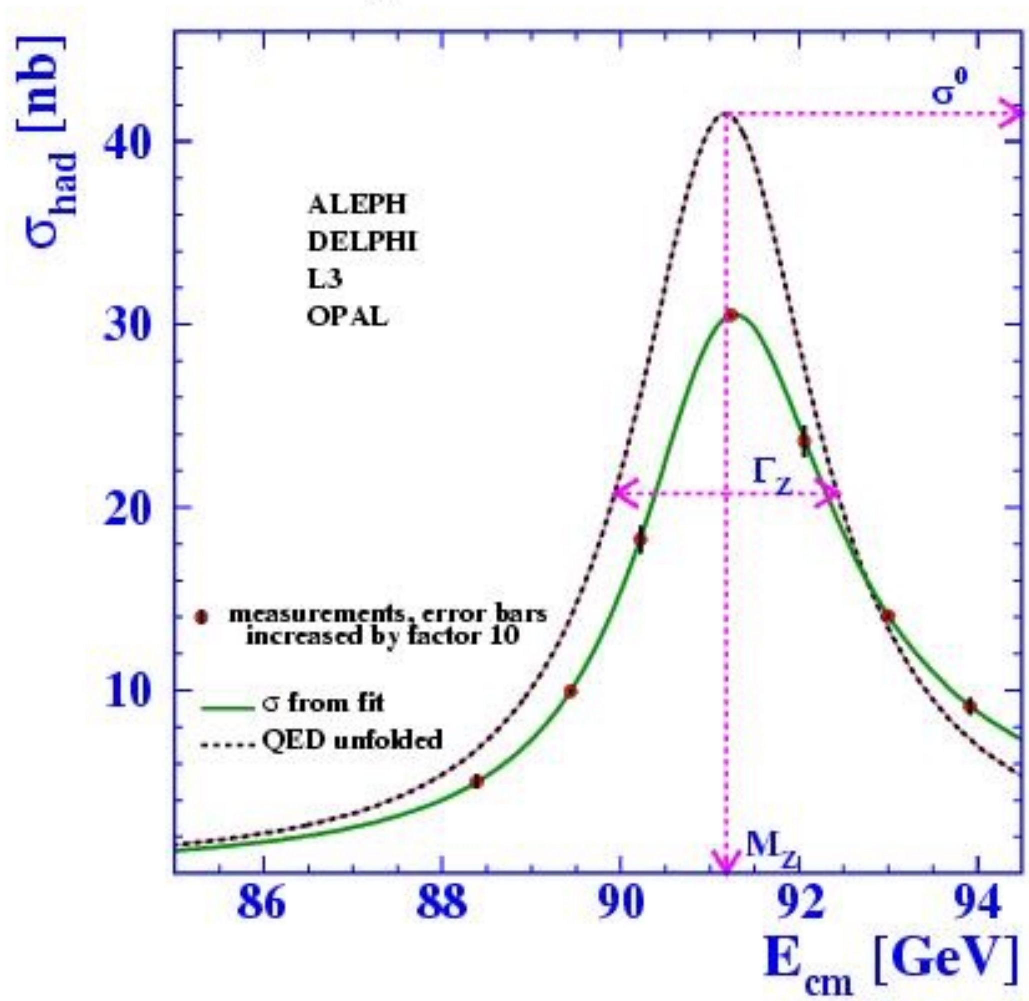
QED corrections

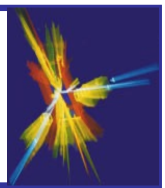
Dominant QED correction from initial state radiation.



Accounted for by radiator function H. We want $\sigma_{\mathrm{ew}}(s)$

$$\sigma(s) = \int_{4m_f^2/s}^{1} dz H_{\mathrm{QED}}^{\mathrm{tot}}(z,s)\sigma_{\mathrm{ew}}(zs).$$

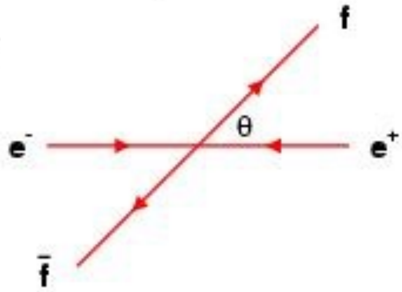




Differential cross-section

Improved Born Approximation for $e^+e^- \longrightarrow f\bar{f}$

(ignoring fermion masses, QED/QCD ISR/FSR ...)



$$\frac{d\sigma_{ew}}{d\cos\theta} = \frac{\pi N_c^f}{2s} 16|\chi(s)|^2 \times$$

$$\left[(g_{Ve}^2 + g_{Ae}^2)(g_{Vf}^2 + g_{Af}^2)(1+\cos^2\theta) + 8 g_{Ve} g_{Ae} g_{Vf} g_{Af} \cos\theta\right]$$

$$+ \ [\gamma \text{ exchange}] \ + \ [\gamma Z \text{ interference}]$$

Where

$$\chi(s) = \frac{G_F m_Z^2}{8\pi\sqrt{2}} \frac{s}{s - m_Z^2 + is\Gamma_Z/m_Z}$$

Even term in $\cos\theta$ gives total cross-section

$$\sigma_{f\bar{f}} \propto (g_{Ve}^2 + g_{Ae}^2)(g_{Vf}^2 + g_{Af}^2)$$

Odd term in $\cos\theta$ leads to forward-backward asymmetry:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

where $\sigma_{F\,(B)} = \int_{0\,(-1)}^{1\,(0)} (d\sigma/d\cos\theta)\, d\cos\theta$

$$A_{FB}^{0,f} = \frac{3}{4} \frac{2 g_{Ve} g_{Ae}}{g_{Ve}^2 + g_{Ae}^2} \frac{2 g_{Vf} g_{Af}}{g_{Vf}^2 + g_{Af}^2} \equiv \frac{3}{4} \mathcal{A}_e \mathcal{A}_f$$

@ Z^0 pole

(Interference term means A_{FB} varies with s)

Cross-section plus A_{FB} allow g_{Vf} and g_{Af} to be derived.

Cross-section and partial widths

Cross-section as a function of s: "Z lineshape"

$$\sigma_{f\bar{f}}(s) = \sigma^0_{f\bar{f}} \frac{s\Gamma_Z^2}{(s - m_Z)^2 + s^2\Gamma_Z^2/m_Z^2}$$

where pole cross-section is

$$\sigma^0_{f\bar{f}} = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}.$$

with $\Gamma_{f\bar{f}}/\Gamma_Z = \mathrm{BR}(Z \to f\bar{f})$ and partial width is

$$\Gamma_{f\bar{f}} = N_c^f \frac{G_F m_Z^3}{6\sqrt{2}\pi} \left(g^2_{Af} + g^2_{Vf}\right)$$

+ QED/QCD corrections eg. QCD: $\Gamma_{q\bar{q}} \to \Gamma_{q\bar{q}}(1 + \alpha_s/\pi + \cdots)$

Total width of Z

$$\Gamma_Z = \Gamma_{had} + 3\Gamma_{\ell\ell} + \Gamma_{inv} = \Sigma\Gamma_{q\bar{q}} + 3\Gamma_{\ell\ell} + N_\nu\Gamma_{\nu\nu}$$

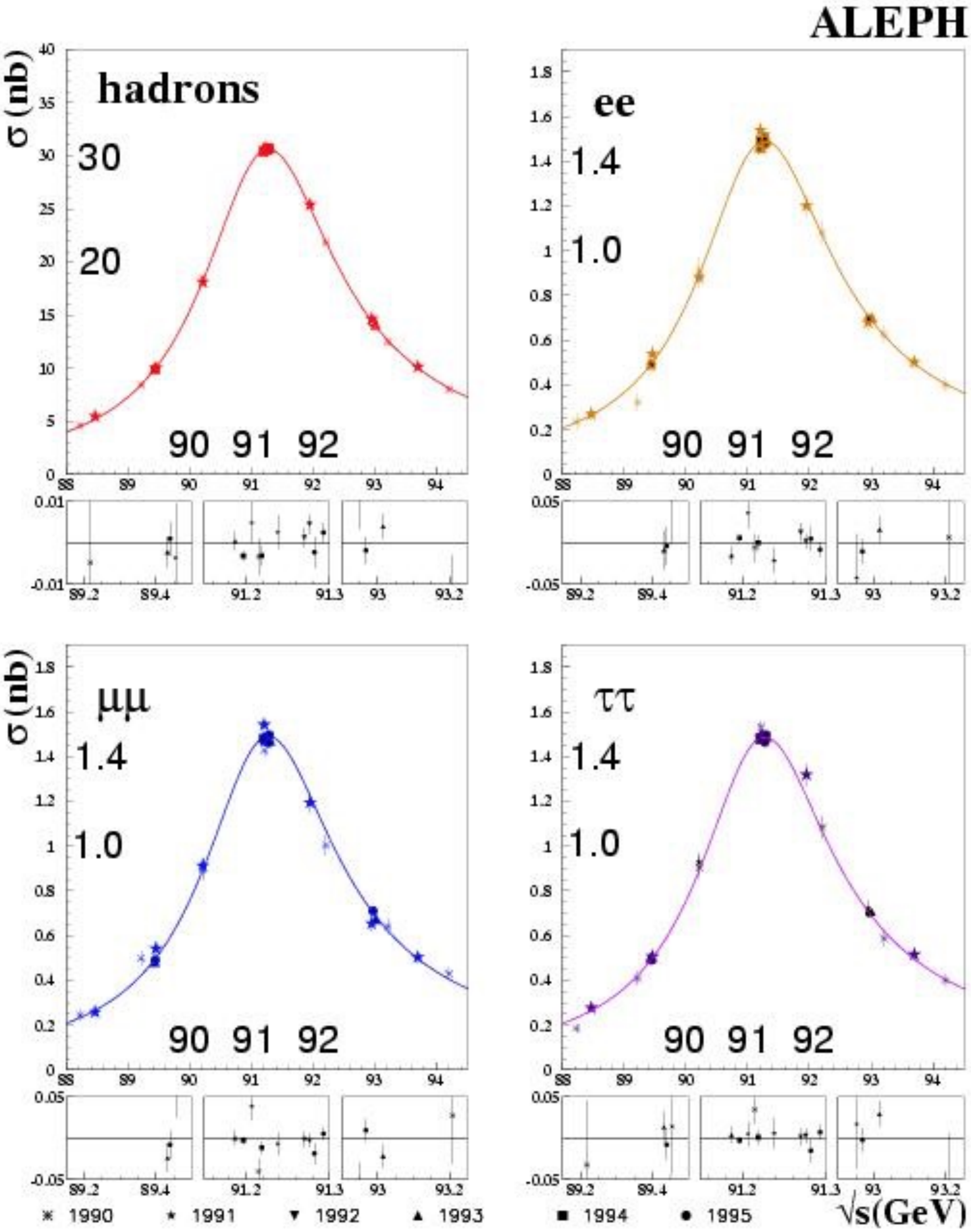
Comparing total width to partial width gives $N_\nu = 2.9963 \pm 0.0074$

Cross-sections and widths correlated. Choose to fit:

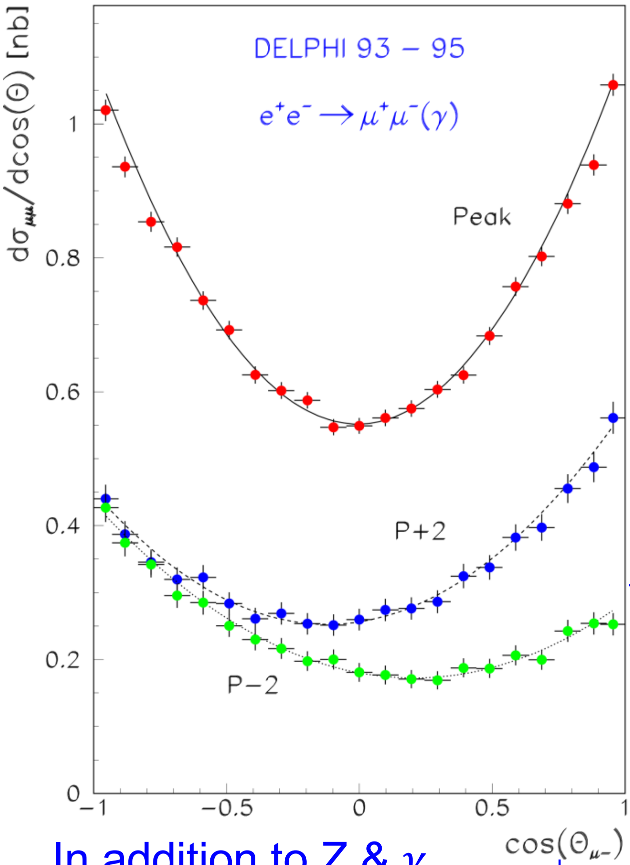
- m_Z, Γ_Z, σ^0_h
- Ratios: $R^0_e \equiv \Gamma_{had}/\Gamma_{ee}$, $R^0_\mu \equiv \Gamma_{had}/\Gamma_{\mu\mu}$, $R^0_\tau \equiv \Gamma_{had}/\Gamma_{\tau\tau}$ or $R^0_\ell \equiv \Gamma_{had}/\Gamma_{\ell\ell}$
- Asymmetries: $A^{0,e}_{FB}$, $A^{0,\mu}_{FB}$ and $A^{0,\tau}_{FB}$ or $A^{0,\ell}_{FB}$

Extra information from tagging some quark flavours - see later.

Cross-sections vs $\sqrt{s}$



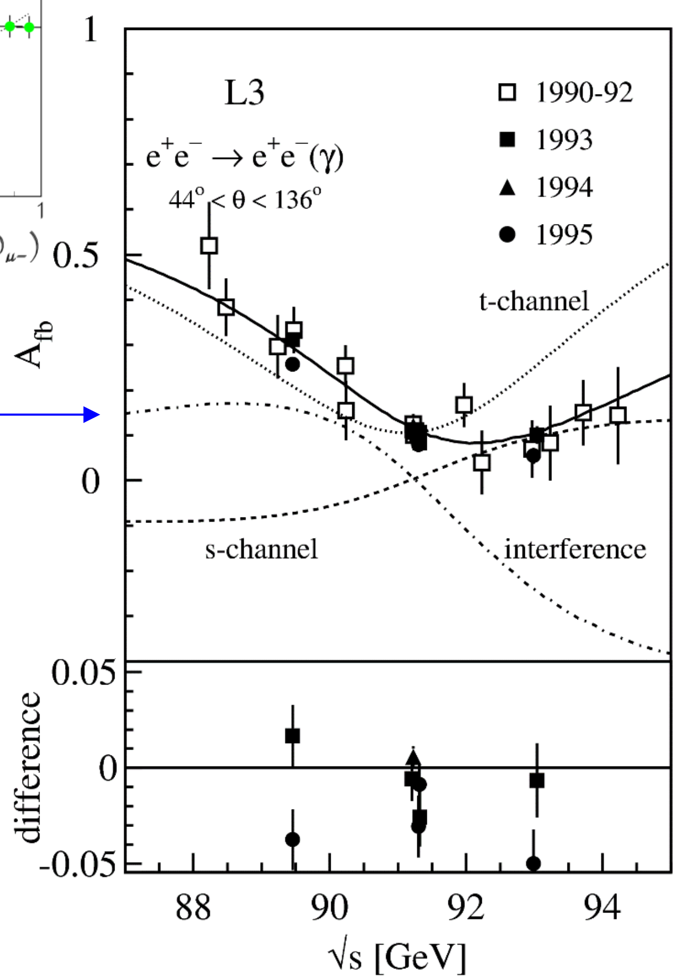
Lepton forward-backward asymmetries



Forward-backward asymmetry for lepton pairs is straightforward to measure. Charge of lepton from tracking.

In addition to Z & γ exchange ("s-channel"), has for electrons also to take into account Bhabha scattering ("t-channel") plus their interference.

Asymmetry varies with centre-of-mass energy.



Lepton Universality

Plot $A_{\rm FB}^{0,\ell}$ vs. $R_\ell^0 = \Gamma_{\rm had}/\Gamma_{\ell\ell}$. Contours contain 68% probability.

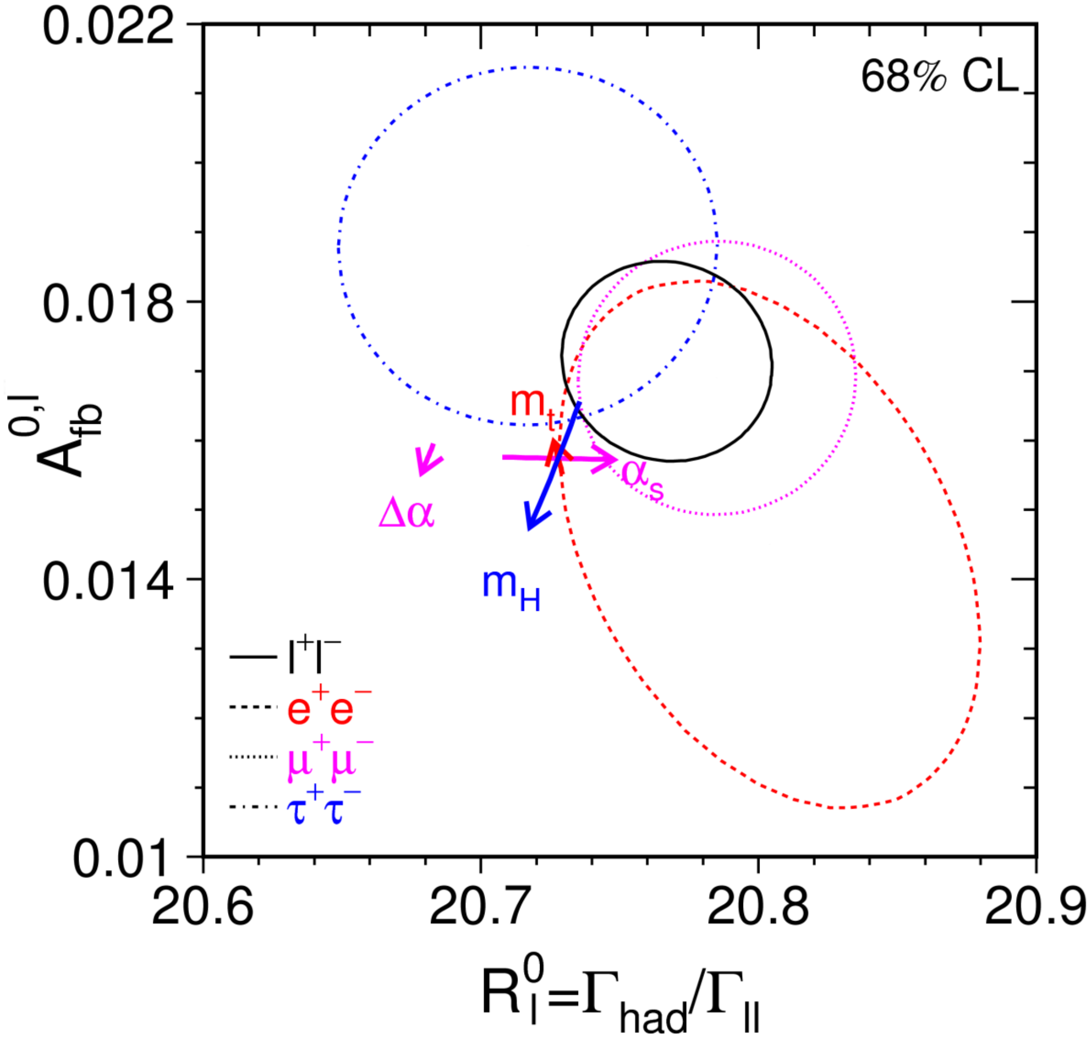
Lepton universality OK. Results agree with SM (arrows)

$m_{\rm t} = 178.0 \pm 4.3$ GeV

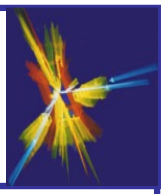
$m_{\rm H} = 300^{+700}_{-186}$ GeV (low $m_{\rm H}$ preferred)

$\alpha_s(m_{\rm Z}^2) = 0.118 \pm 0.002$

$\Delta\alpha_{\rm had} = 0.02761 \pm 0.00036$



Can be interpretated in terms of g_{V1} and g_{A1}



Inputs to global electroweak fits

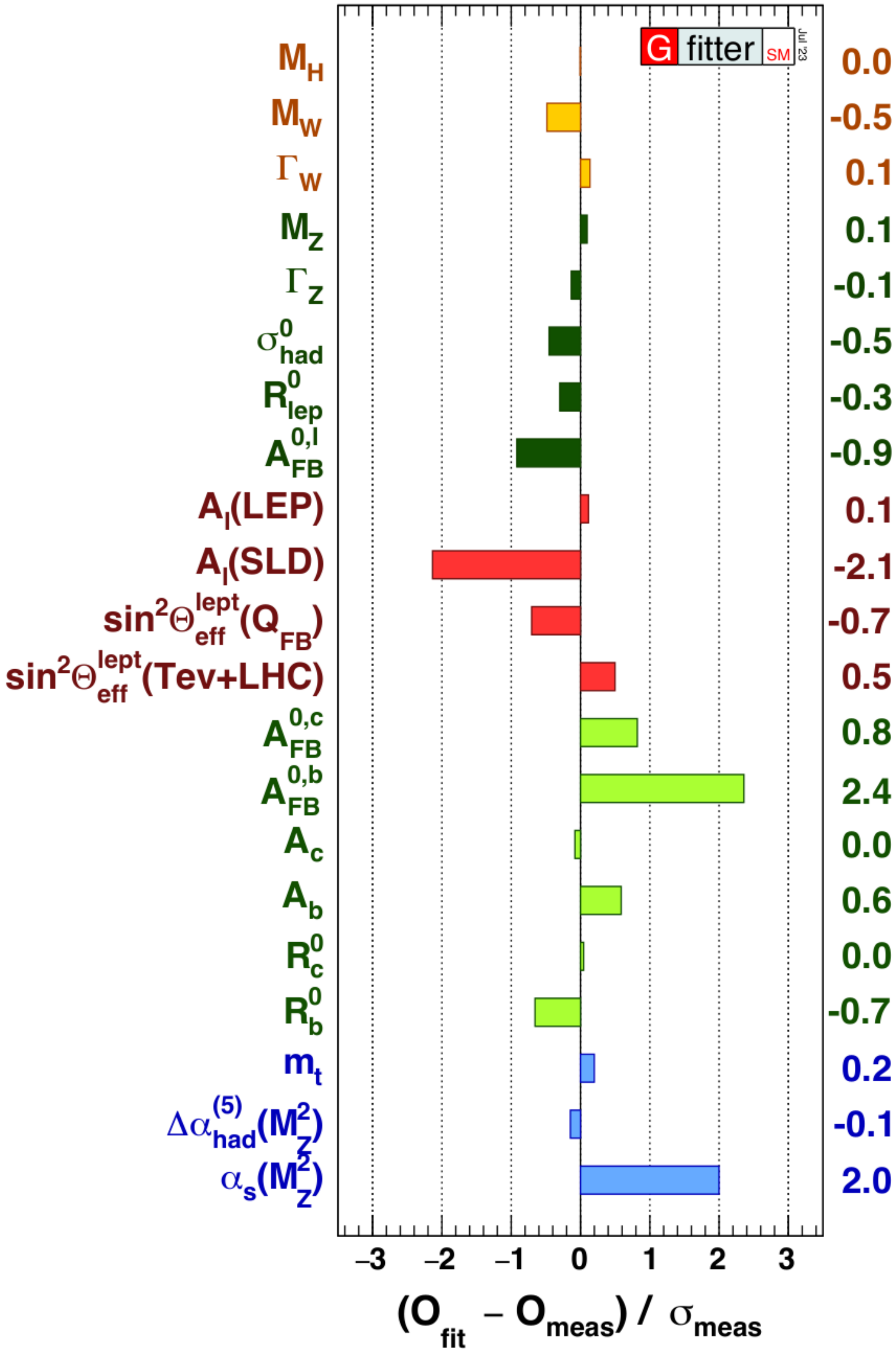
Measurements of W, Z, quark & H properties + coupling constants from LEP/CERN, SLD/SLAC ($e^+e^- \rightarrow Z^0$) Tevatron/Fermilab, LHC/CERN & lower $\sqrt{s}$ e^+e^- colliders.

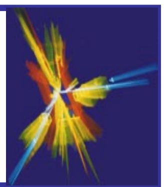
Results & plots: *Gfitter group, EPJC 78 (2018) 675, PoS ICHEP2022 (2023) 897, and PoS EPS-HEP2023 (2024) 304*

Parameter	Input value	Free in fit	Fit Result	w/o exp. input in line	w/o exp. input in line, no theo. unc
M_H [GeV]	125.1 ± 0.2	yes	$125.1^{+0.2}_{-0.2}$	$100.8^{+25.4}_{-21.1}$	$101.0^{+24.0}_{-20.1}$
M_W [GeV]	80.362 ± 0.014	–	80.356 ± 0.006	80.354 ± 0.007	80.353 ± 0.005
Γ_W [GeV]	2.085 ± 0.042	–	2.091 ± 0.001	2.091 ± 0.001	2.091 ± 0.001
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1878 ± 0.0021	91.1969 ± 0.0105	91.1969 ± 0.0100
Γ_Z [GeV]	2.4955 ± 0.0023	–	2.4952 ± 0.0014	2.4949 ± 0.0016	2.4949 ± 0.0016
$\sigma^0_{\rm had}$ [nb]	41.500 ± 0.037	–	41.484 ± 0.015	41.481 ± 0.016	41.481 ± 0.015
R^0_l	20.767 ± 0.025	–	20.759 ± 0.017	20.750 ± 0.026	20.750 ± 0.026
$A^{0,l}_{\rm FB}$	0.0171 ± 0.0010	–	0.01619 ± 0.0001	0.01618 ± 0.0001	0.01617 ± 0.0001
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	–	0.1469 ± 0.0005	0.1469 ± 0.0005	0.1469 ± 0.0003
$\sin^2\theta^l_{eff}(Q_{\rm FB})$	0.2324 ± 0.0012	–	0.23154 ± 0.00006	0.23153 ± 0.00006	0.23154 ± 0.00004
$\sin^2\theta^l_{eff}(\rm Tev + LHC)$	0.23141 ± 0.00026	–	0.23154 ± 0.00006	0.23154 ± 0.00006	0.23155 ± 0.00004
A_c	0.670 ± 0.027	–	0.6678 ± 0.00021	0.6678 ± 0.00021	0.6678 ± 0.00014
A_b	0.923 ± 0.020	–	0.93475 ± 0.00004	0.93475 ± 0.00004	0.93474 ± 0.00002
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	–	0.0736 ± 0.0003	0.0736 ± 0.0003	0.0736 ± 0.0002
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	–	0.1030 ± 0.0003	0.1031 ± 0.0003	0.1030 ± 0.0002
R^0_c	0.1721 ± 0.0030	–	$0.17223^{+0.00007}_{-0.00006}$	0.17224 ± 0.00007	0.17223 ± 0.00006
R^0_b	0.21629 ± 0.00066	–	$0.21586^{+0.00004}_{-0.00005}$	0.21585 ± 0.00005	0.21586 ± 0.00004
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	–	–
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$	–	–
m_t [GeV]$^{(\triangledown)}$	172.47 ± 0.68	yes	172.65 ± 0.65	$174.84^{+2.37}_{-2.39}$	$174.85^{+2.30}_{-2.32}$
$\Delta\alpha^{(5)}_{had}(M^2_Z)$ $^{(\dagger\triangle)}$	2761 ± 9	yes	2759 ± 10	2730 ± 39	2731 ± 37
$\alpha_s(M^2_Z)$	–	yes	0.1196 ± 0.0029	0.1196 ± 0.0029	0.1195 ± 0.0028

$^{(\star)}$Average of LEP ($A_\ell = 0.1465 \pm 0.0033$) and SLD ($A_\ell = 0.1513 \pm 0.0021$) measurements, used as two measurements in the fit. The fit w/o the LEP (SLD) measurement gives $A_\ell = 0.1469 \pm 0.0005$ ($A_\ell = 0.1467 \pm 0.0005$). $^{(\triangledown)}$Combination of experimental (0.46 GeV) and theory uncertainty (0.5 GeV).$^{(\dagger)}$In units of 10^{-5}. $^{(\triangle)}$Rescaled due to α_s dependency.

Results of global electroweak fits





Electroweak precision measurements:

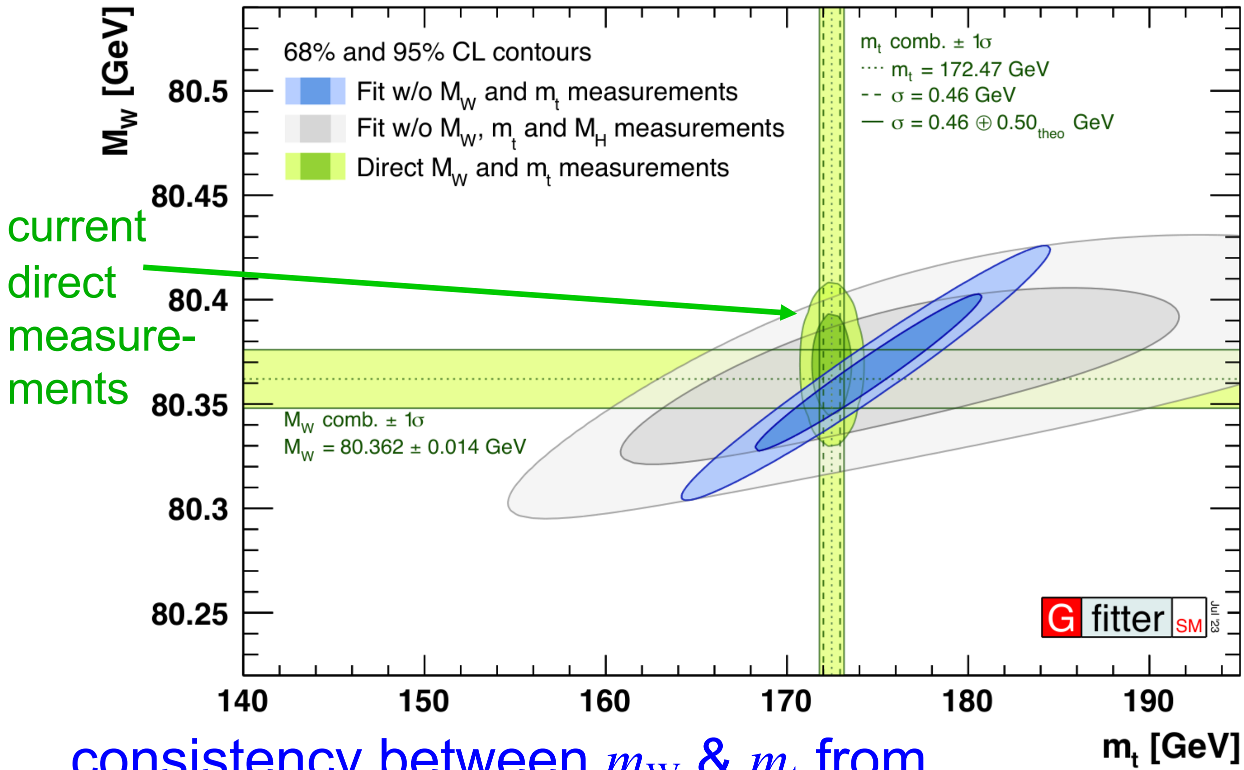
$$m_W = \left(\frac{\pi \alpha_{EM}}{\sqrt{2}\, G_F} \right)^{1/2} \frac{1}{\sin\theta_W \sqrt{1-\Delta r}} \longleftarrow \propto m_{top}^2 \;\&\; \ln(m_H^2)$$

since G_F, α_{em}, $\sin\theta_W$ are known with high precision,
precise measurements of m_{top} & m_W constrain radiative corrections & Higgs mass (weakly due logarithmic dependence)

m_W (LEPII//LHC) = 80.369 ± 0.013 GeV — $2 \cdot 10^{-4}$

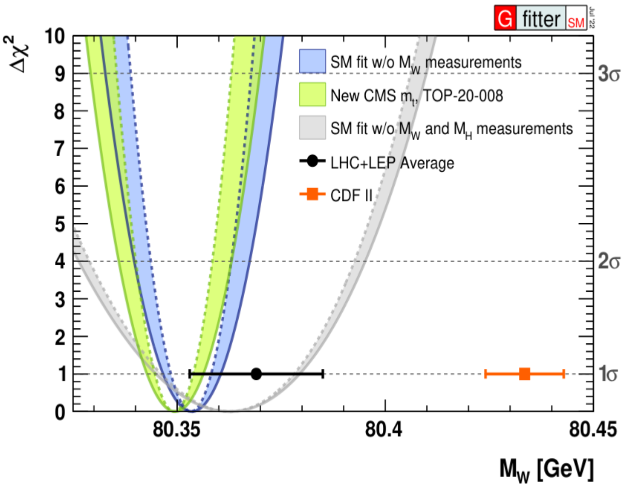
m_{top} (LHC) = 172.52 ± 0.33 GeV — $2 \cdot 10^{-3}$

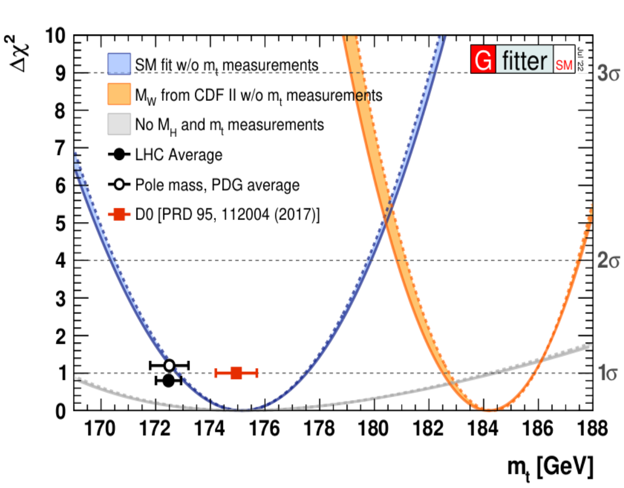
ultimate Standard Model test: compare direct Higgs mass with radiative correction prediction.



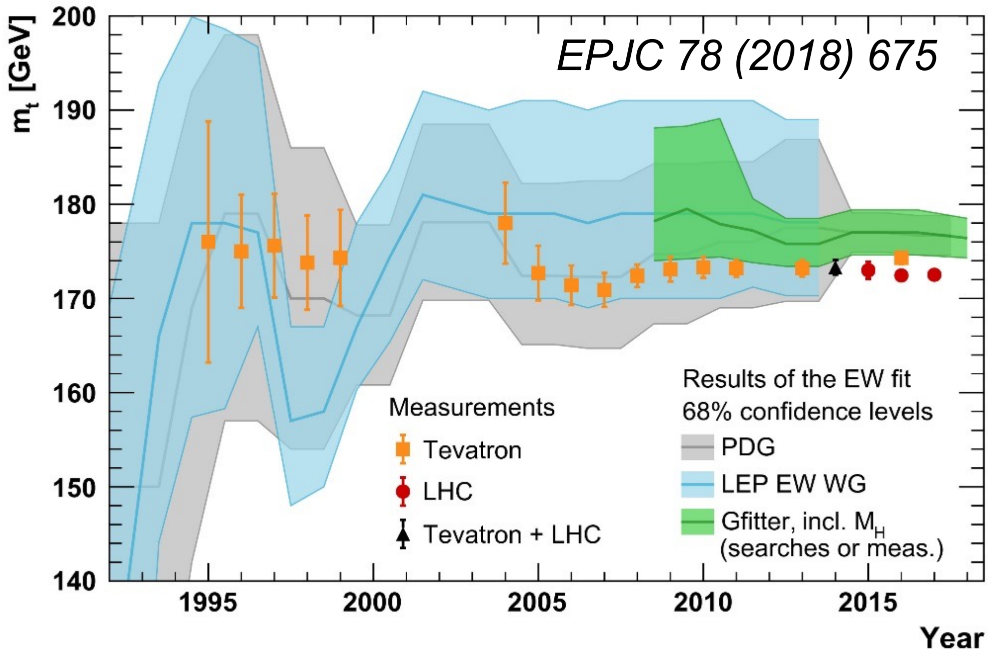
consistency between m_W & m_t from radiative corrections and direct measurements.

Top and W mass





Consistency (with the exception of Tevatron measurements) between m_W & m_t from radiative corrections and various direct measurements.



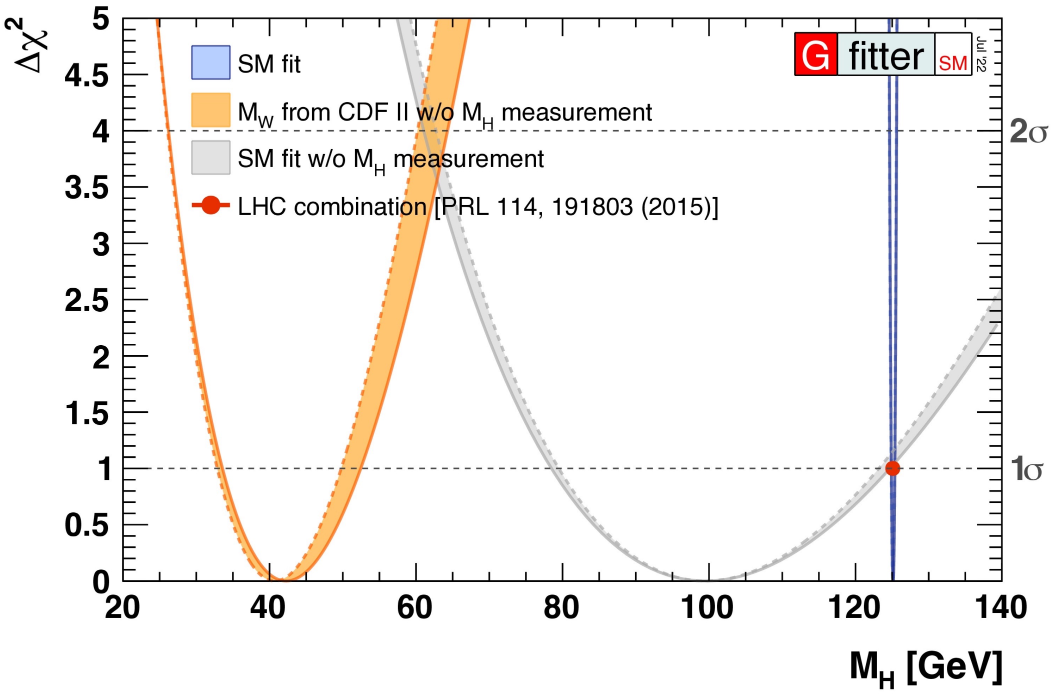
A measure of Standard Model success: m_t from radiative corrections vs m_t direct measurement.

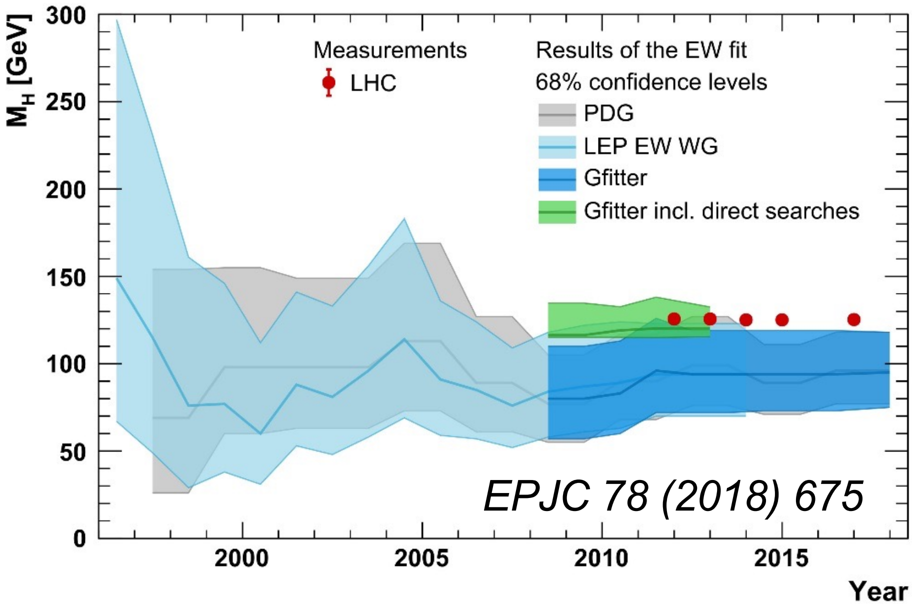
Standard Model Higgs

Electroweak fits $\Rightarrow m_{\mathrm{H}} < 152$ GeV (95% CL).

Prefered minimum $m_{\mathrm{H}} = 101^{+25}_{-21}$ GeV

Direct measurement (ATLAS+CMS) $m_{\mathrm{H}} = 125.20 \pm 0.11$ GeV





Overall SM fit:
$\chi^2 = 13.8$ for
$n_{dof} = 15 \Rightarrow$
P-value = 0.55
(August 2023)

√ SM still very reasonable !!

Standard Model Parameters

♠ QCD: $\alpha_s(M_Z)$ **1**

♠ EW Gauge / Scalar Sector: **4**

$$g, g', \mu^2, h \;\Leftrightarrow\; \alpha, \theta_W, M_W, M_H \;\Leftrightarrow\; \alpha, G_F, M_Z, M_H$$

♠ Yukawa Sector: **13**

$$m_e \;,\; m_\mu \;,\; m_\tau$$

$$m_d \;,\; m_s \;,\; m_b$$

$$m_u \;,\; m_c \;,\; m_t$$

$$\theta_1 \;,\; \theta_2 \;,\; \theta_3 \;,\; \delta$$

→ **18 Free Parameters**

(+ Neutrino Masses / Mixings ?)

TOO MANY !