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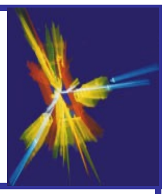
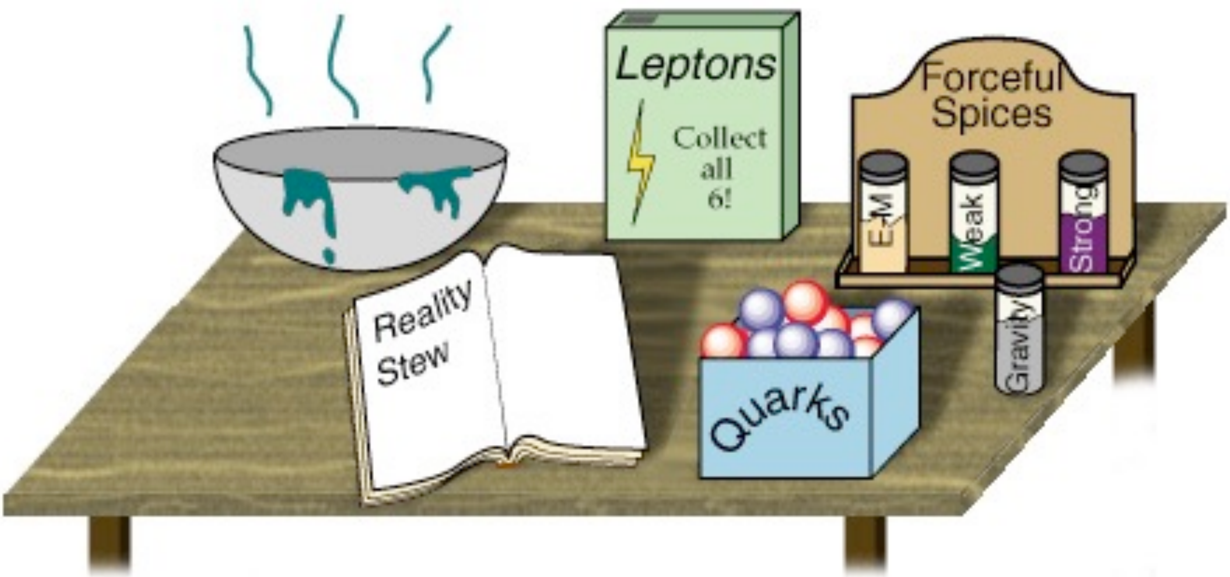


Standard Model

Constituents & Interactions

Gauge Invariance



Quantum Chromodynamics

Electroweak Unification

Electroweak Symmetry Breaking

follows A. Pich: The Standard Model of Electroweak Interactions, arXiv:1201.0537

Theoretical Framework

QM ($\hbar$) + special relativity (c) $\Rightarrow$ Quantum Field Theory

STANDARD MODEL:

- Electricity + Magnetism: γ

Quantum Electrodynamics (QED)

- QED + Weak Interaction: $\gamma, Z^0, W^\pm$

Electroweak Theory $SU(2)_L \otimes U(1)_Y$

- Strong Interaction: **8 gluons**

Quantum Chromodynamics (QCD) $SU(3)_C$

Standard Model = $SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$

NB! $Q = T_3 + Y/2$ (Y = weak hypercharge, T_3 = weak isospin)

OPEN QUESTIONS (at least some):

- Grand Unification **electroweak & strong combined ?**
- Supersymmetry **bosons & fermions linked together?**
- Gravitation **quantum theory of gravity? graviton?**

Leptons

- Do not have strong interactions
- Spin ½
- Seen as free particles
- Pointlike (at least for dist ≥ few · 10^{-19} m)

Family Structure:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L , \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

Why are there 3?

Transformation of fields under SM gauge symmetries:

(2,-1,1)

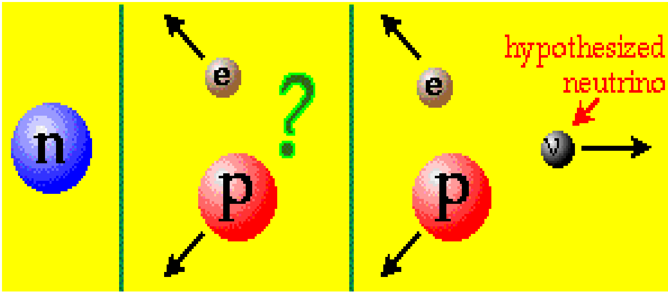
$(SU(2)_L, U(1)_Y, SU(3)_C)$

Lepton	Mass (MeV)	τ (s)	q	L	J^P	L_e	L_μ	L_τ	Decay	BR
e (electron)	0.510	stable	-1	1	$½^+$	1	0	0		
ν_e (neutrino)	~0	stable	0	1	$½^+$	1	0	0		
μ (muon)	105.6	$2.2 \cdot 10^{-6}$	-1	1	$½^+$	0	1	0	$\rightarrow e^- \bar{\nu}_e \nu_\mu$	100 %
ν_μ	~0	stable	0	1	$½^+$	0	1	0		
τ (tau)	1777	$2.9 \cdot 10^{-13}$	-1	1	$½^+$	0	0	1	$\rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ $\rightarrow e^- \bar{\nu}_e \nu_\tau$ →hadrons ν_τ	17.4% 17.8% 64.8%
ν_τ	~0	stable	0	1	$½^+$	0	0	1		

Neutrinos

- Particles interacting only weakly $\Rightarrow$ difficult to detect
- Abundantly produced in the sun & the atmosphere
- First signs in β decay ("non-energy conversation")

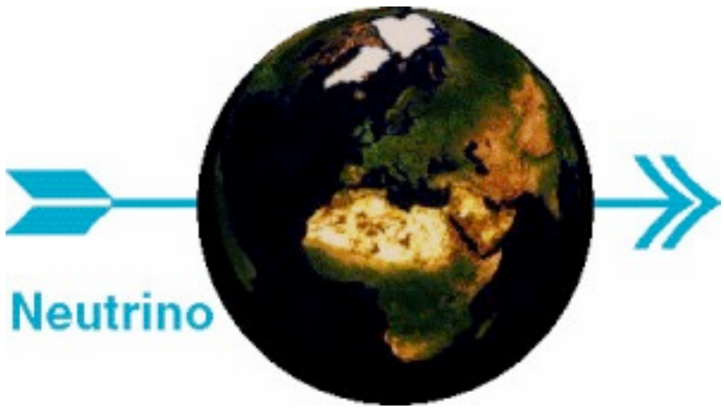
$${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + e^- + \bar{\nu}_e \quad (n \rightarrow p + e^- + \bar{\nu}_e)$$



$$q_{\bar{\nu}_e} = q_{\nu_e} = 0$$
$$m_{\bar{\nu}_e} \approx m_{\nu_e} \approx 0$$

$\nu_e \equiv$ neutrino $\qquad \bar{\nu}_e \equiv$ anti-neutrino

- They penetrate earth & humans without interacting

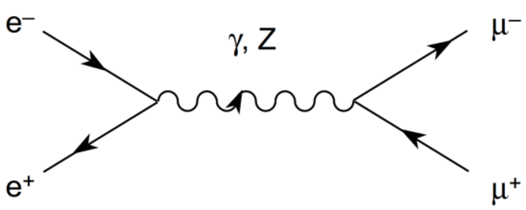


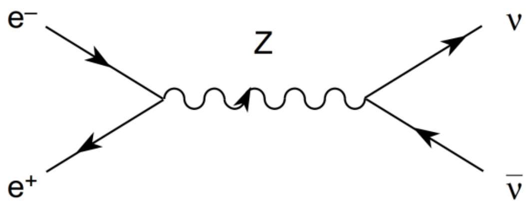
Each second $\sim 10^{14}$ ν_e from the sun pass through your body

($pp \rightarrow d e^+ \nu_e$, ...)

- Artificially produced in nuclear reactors
- Mostly seen in experiments as "missing momentum"
- ν experiments show that different ν flavours turn into each other ("oscillate") and this implies that ν's must have mass, a very small one though ($\leq 10^{-2}$ eV). More about this later.

NEUTRAL CURRENTS





- Flavour Conserving

$$\mu \nrightarrow e\gamma \;\&\; Z \nrightarrow e^{\pm}\mu^{\mp}$$

- $g_{\gamma} \propto q_l \qquad (q_e = q_{\mu} = q_{\tau} = 1 ; q_{\nu} = 0)$

- Same γ interaction for both lepton helicities

- NC Universality:

$$g_{Ze+e-} = g_{Z\mu+\mu-} = g_{Z\tau+\tau-} \neq g_{Z\nu\bar{\nu}}$$

- Different Z coupling to l_L and l_R

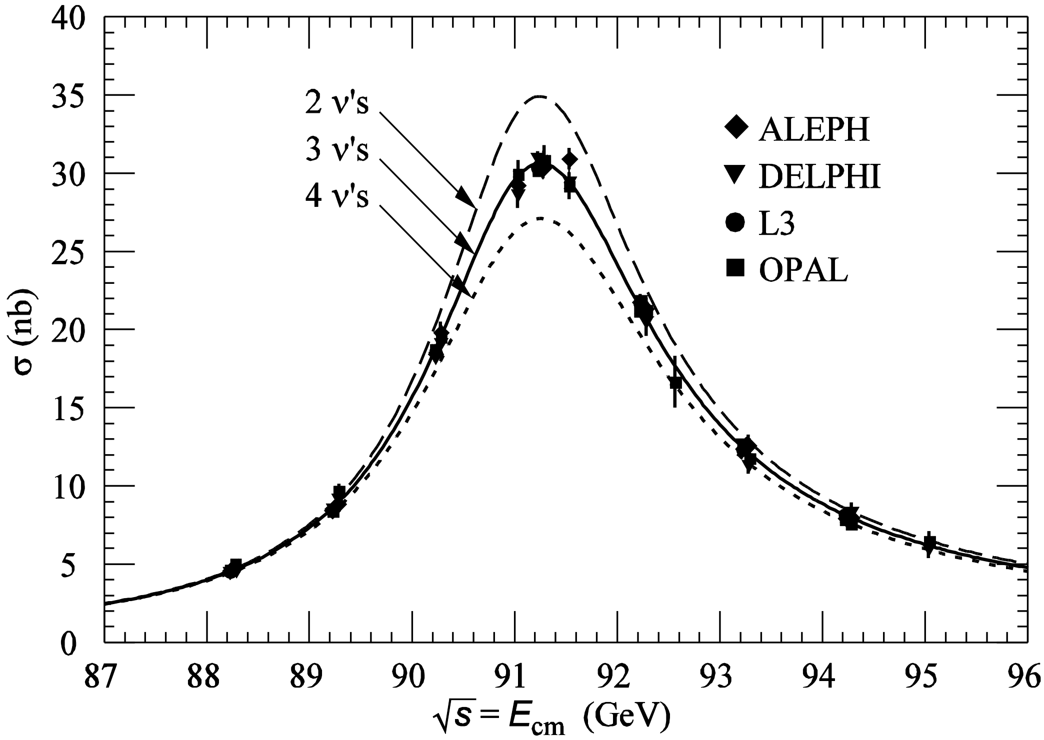
- Left-handed neutrinos only

- 3 Families with light neutrinos

HOW MANY NEUTRINOS ?

$\Delta E \cdot \Delta t \leq \hbar \Rightarrow$ small lifetime t leads to large ΔE

$$\sigma(Z \to hadrons)$$



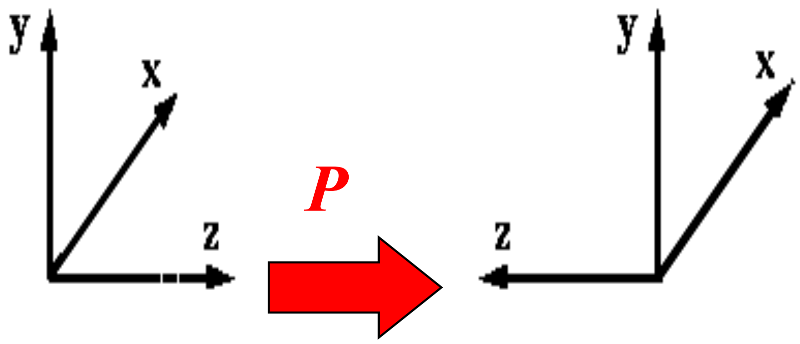
$$\Gamma(Z \to invisible) = \Gamma(Z \to all) - \Gamma(Z \to visible)$$

$$\boxed{N_\nu = \frac{\Gamma(Z \to invisible)}{\Gamma(Z \to \nu_i \bar{\nu}_i)_{theory}} = 2.9963 \pm 0.0074}$$

P. Janot and S. Jadach, PLB 803 (2020) 135319;
G. Voutsinas et al., PLB 800 (2020) 135068

Bucket analogy: the more "holes" (decay channels) → the "faster" (smaller Δt) the bucket becomes empty → the more "uncertain" (larger ΔE) the particle mass is

Parity (P): reversal of all three axes in a reference frame, P transformation equivalent to a mirror reflection.



(first, rotate by 180° around z–axis ; then reverse all 3 axes)

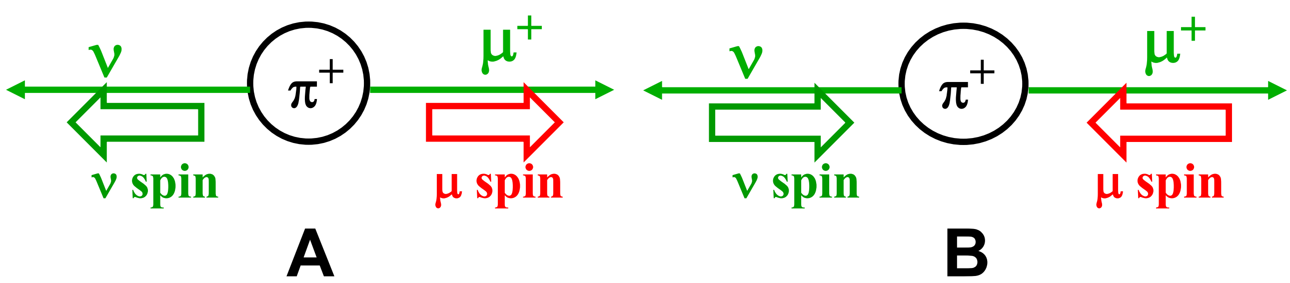
most physics laws invariant w.r.t. a P transformation i.e. Nature does not know the difference between right & left.

Angular momentum & spin doesn't change sign under P
$\Rightarrow$ transition prob. dependence on $\bar{s} \cdot \bar{p}$ indicate P violation.

Lee & Yang (1956): weak interaction violates P invariance

Consequence for e.g. $\pi^+ \rightarrow \mu^+ \nu$ decay ($S_\pi = 0$, $S_{\mu/\nu} = ½$):

P invariance require that the 2 states

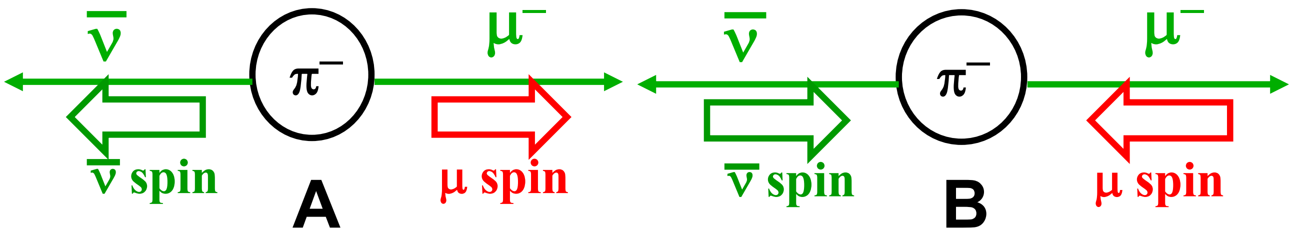


are produced with equal probabilities. Experiments find μ^+ always polarized opposite to the momentum direction (B)

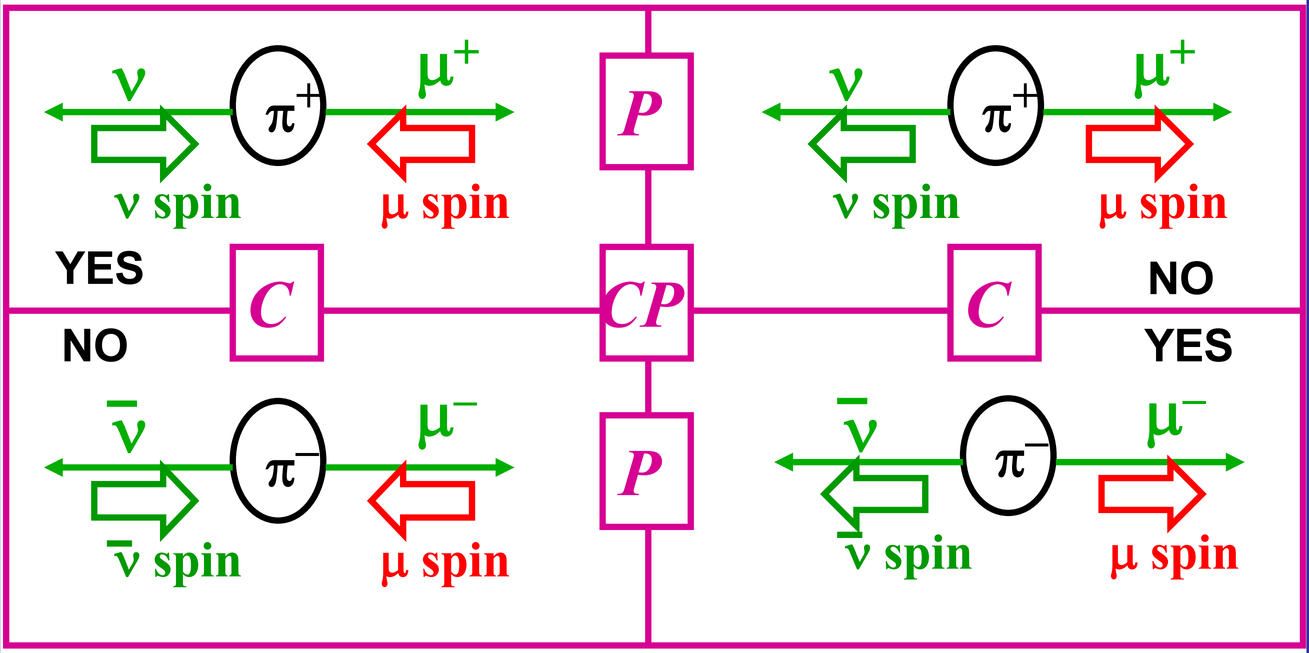
$\Rightarrow$ maximal violation of parity invariance in weak decays !!

Charge conjugation (C): particle ⇔ antiparticle transformation (quantum state defined only for neutrals).

$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ decay: experiments find only decays of type A



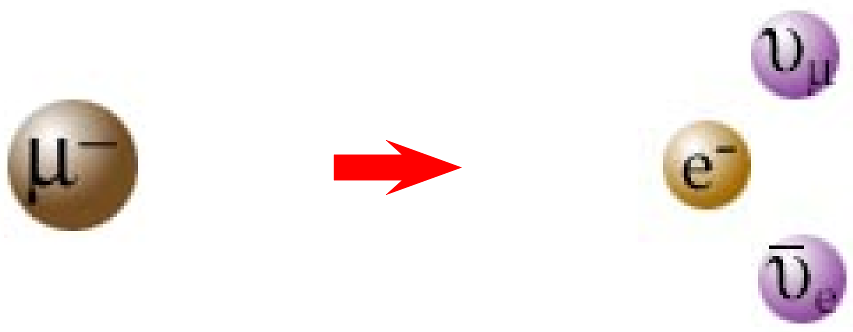
Conclusion for weak (e.g. $\pi^\pm$) decays:

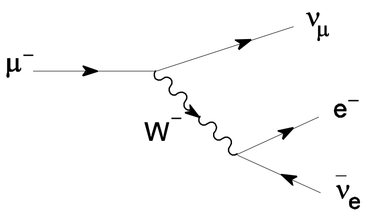


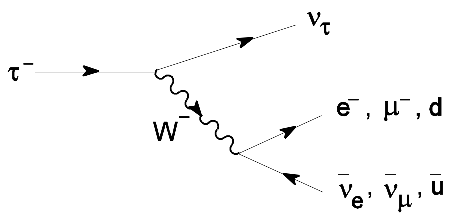
Weak decays violate maximally both P & C invariance but are invariant under CP. In reality CP symmetry is only approximative in weak decays (more later on the slight CP violation in weak decays of neutral K, D & B mesons).

Decay of charged leptons

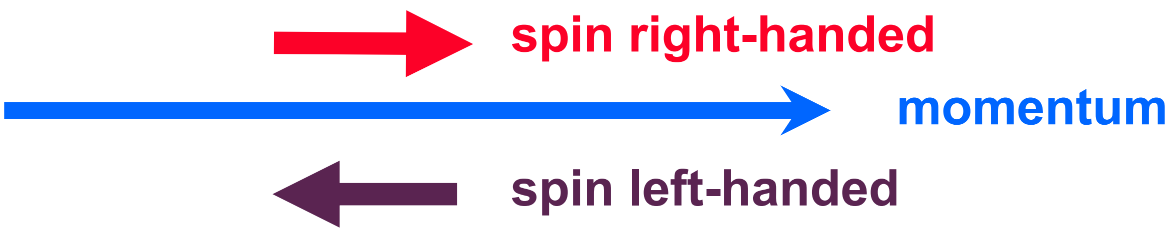
The heavier leptons μ and τ are unstable



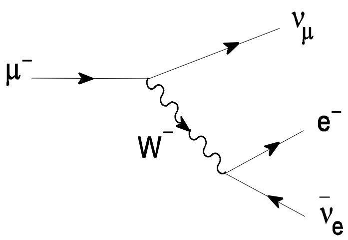


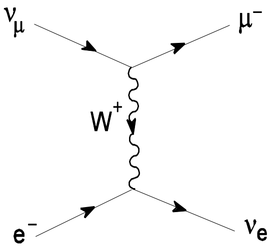


- same process as in the case of radioactivity. Note that the W boson virtual ($Q^2 \ll m_W^2$) in the process.
- only left-handed leptons (right-handed antileptons) (determined from direction of spin) take part in the reaction.



CHARGED CURRENTS





- Left-handed leptons (Right-handed antileptons)

$$l^-, \nu_l \qquad\qquad l^+, \overline{\nu_l}$$

$$\Leftarrow \qquad \vec{J} \qquad \Rightarrow$$

$$\longrightarrow \qquad \vec{p} \qquad \longrightarrow$$

- Doublet partners: $l^- \Leftrightarrow \nu_l$

$$\nu_\mu X \to \mu^- X' \qquad \nu_\mu X \not\to e^- X'$$

- Universal Strength

$$M(l \to \nu_l l' \overline{\nu_{l'}}) \sim \frac{g_W^2}{M_W^2 - Q^2} \xrightarrow{Q^2 \ll M_W^2} \frac{g_W^2}{M_W^2} \sim G_F$$

$$\Rightarrow \quad \Gamma(l \to \nu_l l' \overline{\nu_{l'}}) \sim G_F^2 m_l^5$$

"lepton universality"

$$\Gamma(\tau \to \nu_\tau e \overline{\nu_e}) / \Gamma(\mu \to \nu_\mu e \overline{\nu_e}) = (m_\tau / m_\mu)^5$$

Quarks

- Strong interaction bind them into hadrons
- Spin ½ ; pointlike (at least for dist $\geq$ few $\cdot 10^{-19}$ m)
- Not observed as free particles $\Rightarrow$ **confinement**
- $q_u = 2/3$; $q_d = -1/3$

Family Structure: **Why are there 3?**

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L , \begin{pmatrix} c \\ s' \end{pmatrix}_L , \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

(2,1/3,3)

$(SU(2)_L, U(1)_Y, SU(3)_C)$

Mass eigenstates $\neq$ Weak eigenstates

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V_{CKM} \cdot V_{CKM}^{\dagger} = \mathbf{1}$$

More later !

"Wolfenstein" parametrisation of the CKM matrix. Decay rates depend on the CKM elements e.g. $\Gamma(t \to bW^+) \propto |V_{tb}|^2 \sim 1$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$V_{td}, V_{ub} \ll V_{ts}, V_{cb} \ll V_{cd}, V_{us} \ll V_{ud}, V_{cs}, V_{tb} \approx 1$

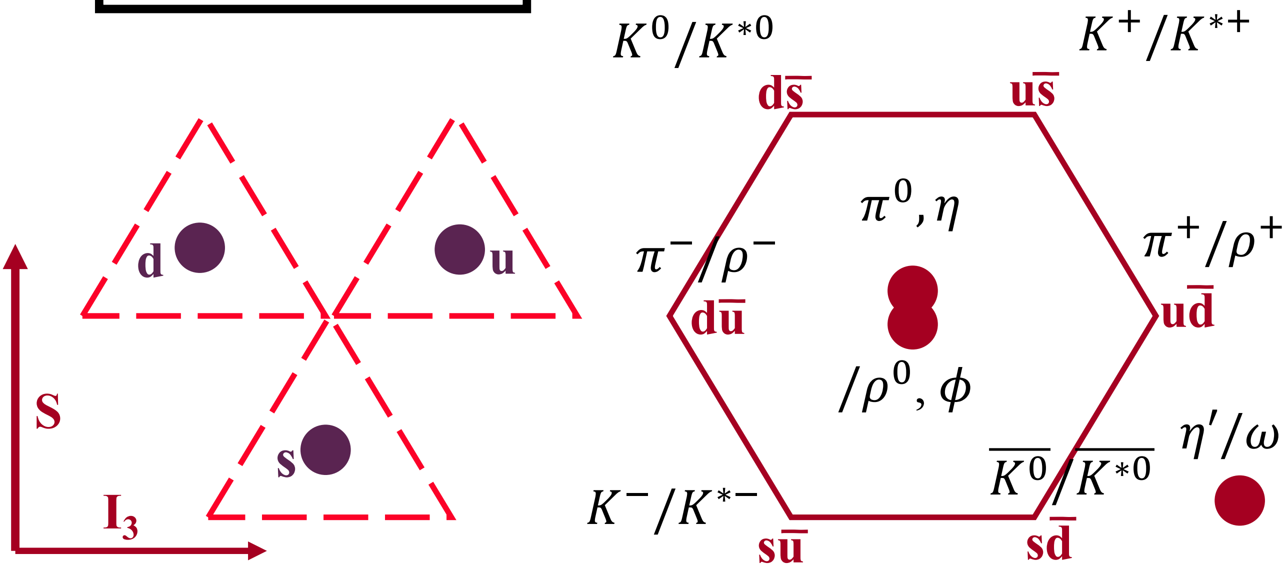
$\lambda \approx 0.225, A \approx 0.83, \bar{\eta} \approx 0.35, \bar{\rho} \approx 0.16$

$\bar{\rho} \approx \rho(1 - \lambda^2/2)$

$\bar{\eta} \approx \eta(1 - \lambda^2/2)$

(Nearly?) all observed **hadrons** can be explained within the **quark model** by constituting **colourless** combinations, each (anti)quark having a (anti)colour:

Mesons ≡ $q_1\bar{q}_2$ (colour & anticolour combination)



$$3 \otimes \bar{3} = 8 \oplus 1$$

$J^{PC} = 0^{-+}/1^{--}$ mesons

Baryons ≡ $q_1q_2q_3$ (combination of R B G)

Antibaryons ≡ $\bar{q}_1\bar{q}_2\bar{q}_3$ (combination of $\bar{R}$ $\bar{B}$ $\bar{G}$)

Most hadrons very short-lived (”resonances”). Heisenberg uncertainty principle implies that their mass is not precisely defined (~ħ/Δt). Their mass M_R is described by a Breit-Wigner.

$$f(M) \propto \frac{(\Gamma_R^{tot})^2}{(M-M_R)^2 + (\Gamma_R^{tot}/2)^2},$$

Γ_R^{tot} is the total decay width = 1/τ, the lifetime.

Hadronization scale $\Lambda_{had} \sim 1$ GeV

Isospin: u,d: $m_{u,d} \ll \Lambda_{had}$

$M_p = 938.3$ MeV $M_n = 939.6$ MeV (p = uud, n = udd)

accidental but gives rather good predictions (~ 1 %) !!

Quark mass not well defined since quarks confined in hadrons:

- "**constituent**" mass. The mass which a quark appears to have in a hadron in terms of properties, such as magnetic moments. For u and d quarks, ~1/3 of the nucleon mass.
- "**current**" mass. The "bare" mass, the one inserted into a fundamental theory, i.e. difference between constituent & current masses due to strong interactions effects.Theoretical arguments applied to observed hadron masses, suggest the u and d current masses to be very small compared to proton mass: $m_u \approx 2$ MeV & $m_d \approx 5$ MeV. Thats why isospin works !! NB! In current mass picture, masses "runs" with energy scale μ.

NB2! small u-d mass difference results in stable p & unstable n.

Special case: top since $M_t \approx 173$ GeV

$$\tau_t \equiv \quad 1/\Gamma(t \to \text{anything}) \approx 1/\Gamma(t \to bW^+) << 1/\Lambda_{\text{had}}$$

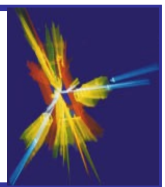
Top quark has no time to hadronize !!!

Summary of Quarks

Masses definied below as current masses at a certain Q^2

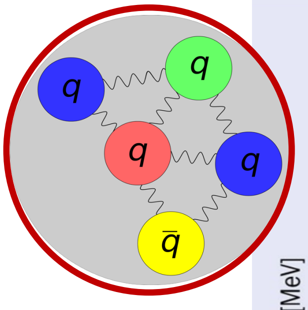
$m_q = m_q(2\ \text{GeV}^2)$; $M_q = m_q(m_q^2)$

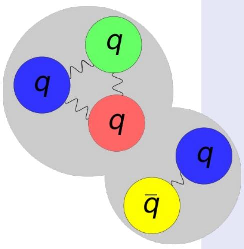
Light (MeV)	$m_u \sim 2$	$m_d \sim 5$	$m_s \sim 93$
Heavy (GeV)	$M_c \sim 1.3$	$M_b \sim 4.2$	$M_t \sim 173$

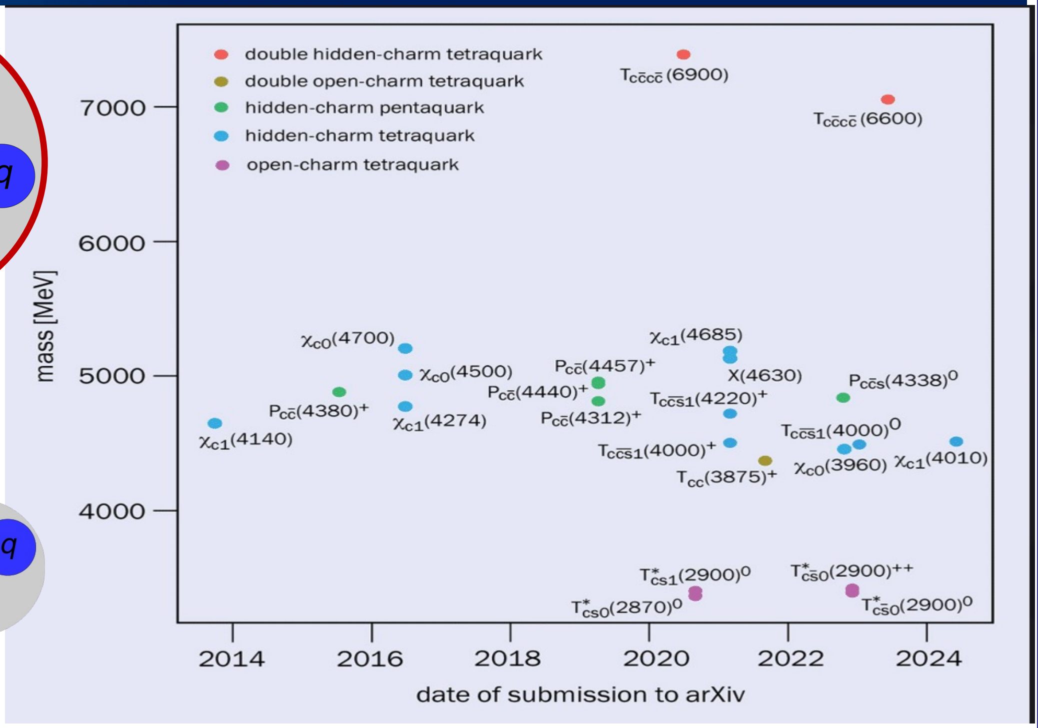


Tetraquarks and Pentaquarks

- Currently one of the unresolved questions about tetraquarks concerns the arrangement of their structure. We know that they are made up of 4 quarks but we do not know how tight the bond of these components is. According to some physicists, the tetraquark can be thought of as a compact object, like the proton or the neutron. Another hypothesis represents them as molecular states, such as structure composed of 2 meson substructures. In a similar way for pentaquarks we can think of them as compact 5 quarks or as baryon –meson molecular states. Data seems to favour the first option.

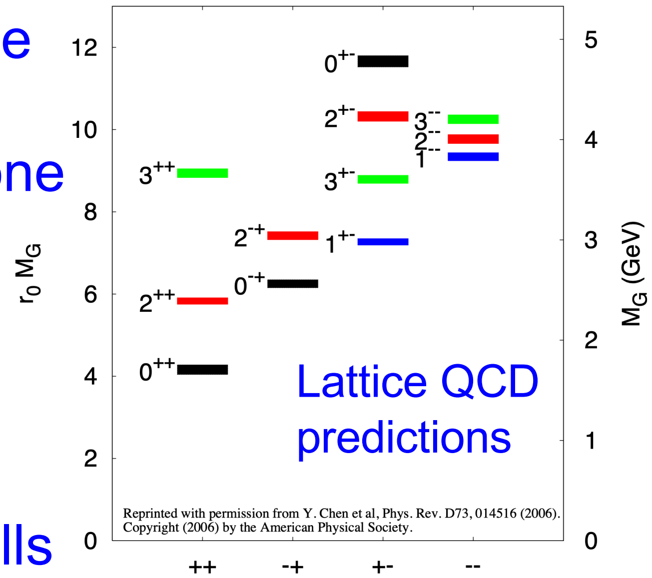






Glueballs (particles made up of only gluons): only candidates upto now, none definitely yet confirmed

However Pomeron (~ 2 gluons) & Odderon (~ 3 gluons) existence imply also existence of glueballs



QUARKS HAVE COLOUR

$\Delta^{++} \sim u^{\uparrow} u^{\uparrow} u^{\uparrow}$ $\quad (J = \frac{3}{2}\,,\; J_3 = \frac{3}{2})$

Pauli (Fermi statistics) ➡ $\Delta^{++} \sim \varepsilon^{\alpha\beta\gamma}\, u^{\uparrow}_{\alpha}\, u^{\uparrow}_{\beta}\, u^{\uparrow}_{\gamma}$

NEW QUANTUM NUMBER: **COLOUR**

baryons $\quad B \sim \varepsilon^{\alpha\beta\gamma}\, q^{i}_{\alpha}\, q^{j}_{\beta}\, q^{k}_{\gamma} \quad ; \quad M \sim \delta^{\alpha\beta}\, q^{i}_{\alpha}\, \bar{q}^{j}_{\beta}$ mesons

$$(i, j, k = u, d, s, \ldots \quad ; \quad \alpha, \beta, \gamma = 1, \ldots, N_C)$$

$\boxed{N_C = 3}$ ➡ $q^i\, q^i\, q^i$

We don't see Colour Multiplets

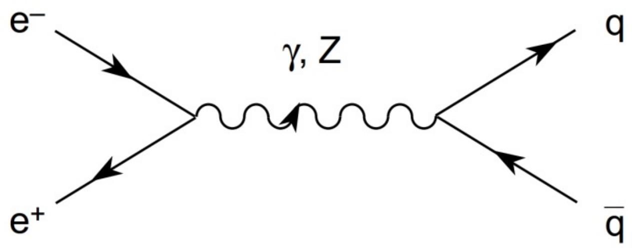
➡ Hadrons are Colour Singlets

$(q\,q\,q,\;\; \bar{q}\,\bar{q}\,\bar{q}$ and $q\,\bar{q}$; BUT NOT $q\,q$ and $q\,q\,q\,q)$

We don't see Quarks

➡
- Don't exist ?
- CONFINEMENT

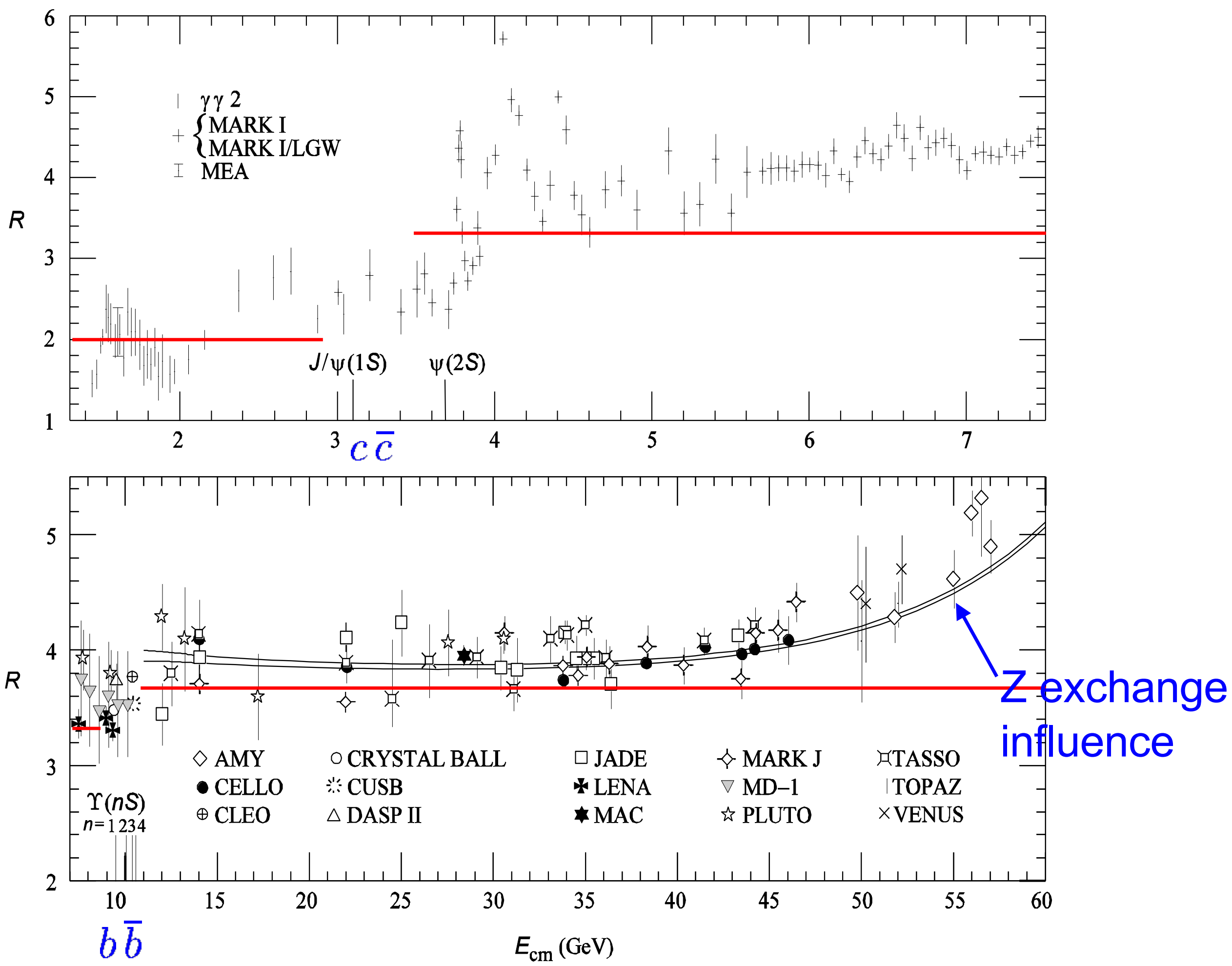
R_{ee}



$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \approx \frac{\sum_q \sigma(e^+e^- \to q\,\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} \approx N_C \sum_q Q_q^2$$

$$= \begin{cases} \frac{2}{3} N_C & , \quad (u,d,s) \\ \frac{10}{9} N_C & , \quad (u,d,s,c) \\ \frac{11}{9} N_C & , \quad (u,d,s,c,b) \end{cases}$$

circumstancial evidence for $N_C = 3$



Group Theory

- An example of a group is the set of rotations of a system (2 successive rotations described by 1, inverse rotations & identity elements (no rotation) exist). The rotation group is a Lie group i.e. each rotation can be expressed as a set of successive infinitesimal rotations $1\text{-}i\varepsilon J_k$.

- The result of the experiment should not depend on the specific laboratory orientation hence the rotation group a symmetry group & the physics invariant under rotations.

- Assume $|\Psi\rangle \rightarrow |\Psi'\rangle = U|\Psi\rangle$ under a rotation; probability that system $|\Psi\rangle$ is in state $|\phi\rangle$ must be unchanged $|\langle\phi' | \Psi'\rangle|^2 = |\langle\phi | \Psi\rangle|^2 \Rightarrow U^\dagger U = 1$ U is a unitary operator

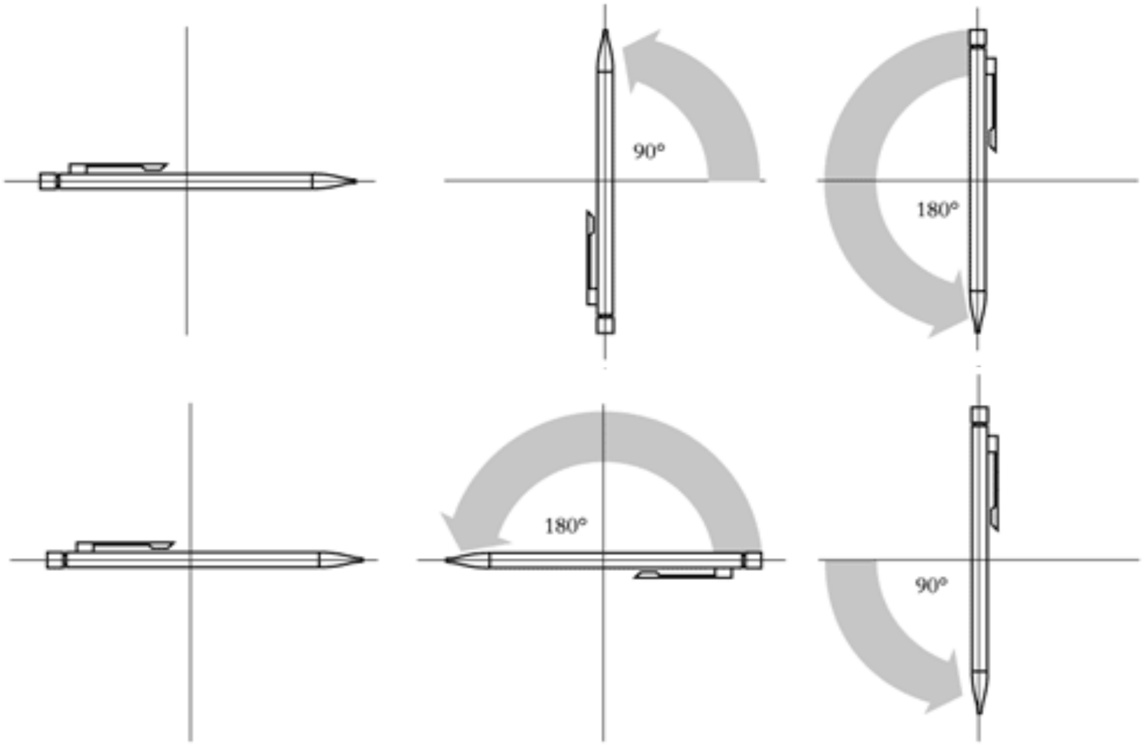
- Hamiltonian unchanged: $\langle\phi' | H | \Psi'\rangle = \langle\phi | H | \Psi\rangle \Rightarrow [U,H] = 0$ & a constant of motion i.e. there is a conserved quantity. Rotations $\rightarrow$ angular momentum conservation.

- For rotations the J_k's are called <u>generators</u>.
Since $U^\dagger U = 1$, they are hermitian $J_k^\dagger = J_k$.
They are related by $[J_i, J_j] = i\varepsilon_{ijk} J_k$.
Due to this the rotational group is a non-abelian group.

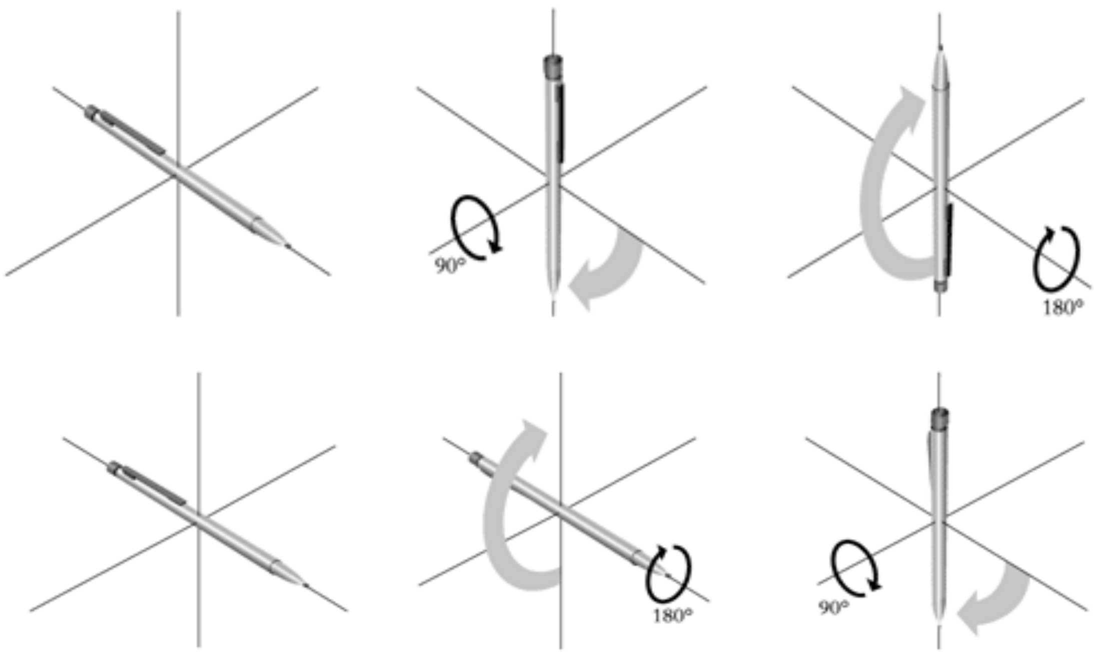
- The lowest-dimensional non-trivial generators of the rotational group are the Pauli matrices. The basis they describe is a spin ½ particle (spin up & down).

- The Pauli matrices are also traceless hence called special unitary matrices and form the group SU(2). They also constitute its <u>fundamental representation</u>, basis for the building of all other representations.

Abelian group: rotation on a plane



Non-abelian group: rotation in space



LAGRANGIAN FORMALISM

$$S = \int d^4x \; \mathcal{L}\,(\phi, \partial_\mu \phi)$$

$$\boxed{\delta S = 0 \quad \longrightarrow \quad \frac{\partial \mathcal{L}}{\partial \phi} - \partial^\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi)} \right) = 0}$$

Eq. Motion

Klein–Gordon: (spin 0)

$$\mathcal{L} = \partial^\mu \phi^* \, \partial_\mu \phi - m^2 \, \phi^* \, \phi \quad \longrightarrow \quad \left(\Box + m^2\right) \phi = 0$$

Dirac: (spin $\frac{1}{2}$) $\qquad \bar{\psi} \equiv \psi^\dagger \, \gamma^0$

$$\mathcal{L} = \bar{\psi} \, (i \, \gamma^\mu \, \partial_\mu - m) \, \psi \quad \longrightarrow \quad (i \, \gamma^\mu \, \partial_\mu - m) \, \psi = 0$$

Gauge Invariance

Quantum Mechanics

- A QM state is described by a complex wave function $\Psi(x)$

- Observables: $< O >= \int dx \Psi^*(x) O \Psi(x)$

$O = O^\dagger$

$\Psi \rightarrow e^{i\theta}\,\Psi,\ \Psi^* \rightarrow \Psi^* e^{-i\theta}$

- The ”global” gauge θ is abritrary & non-measurable
- Make a position dependent change of phase (”local” gauge): $\Psi \rightarrow e^{i\alpha(x)}\,\Psi$
- Schrödinger equation involves derivates,

$(-\nabla^2\Psi / 2m = i\, \partial\Psi/\partial t)$

$\partial_\mu \Psi \rightarrow \partial_\mu[e^{i\alpha(x)}\,\Psi(x)] = e^{i\alpha(x)}\,[\partial_\mu\Psi(x) + i\,\Psi(x)\partial_\mu\alpha(x)]$

$\Rightarrow$ Local phase invariance is spoiled

How to proceed ?

- Add a four vector to remove the extra stuff :

$$\partial_\mu \to D_\mu \equiv \partial_\mu - ieA_\mu \quad (e = \text{e.m. charge})$$

- D_μ transformation properties require

$$D_\mu\psi \to e^{i\alpha(x)} D_\mu\psi(x)$$

- Thus $\quad (\partial_\mu - ieA'_\mu)(e^{i\alpha(x)}\psi(x)) = D'_\mu\psi'(x)$

$$= e^{i\alpha(x)} [\partial_\mu + i\,(\partial_\mu\alpha(x)) - ieA'_\mu]\psi(x)$$

$$= e^{i\alpha(x)} [\partial_\mu - ieA_\mu]\psi(x)$$

- This is true IF $\quad A_\mu \to A'_\mu = A_\mu + 1/e\, \partial_\mu\alpha(x)$

$\Rightarrow \psi^* D_\mu\psi$ is locally phase invariant

Momentum operator $p_\mu = i\hbar\, \partial_\mu \to i\hbar\, D_\mu$

(= covariant derivative)

Mass of the photon ?

A mass term L = $\frac{1}{2} m_\gamma^2 A_\mu A^\mu$ violates local phase invariance:

$$A'_\mu A'^\mu = [A_\mu + 1/e\, \partial_\mu\alpha]\, [A^\mu + 1/e\, \partial^\mu\alpha] \neq A_\mu A^\mu$$

- Local phase invariance $\Rightarrow m_\gamma = 0$

Gauge principle: Phase invariance should hold locally !!

$m_\gamma = 0$ OK for photon but what about $W^\pm$, Z^0? More later !!

Gauge Principle

- Symmetries has always played an important role in physics especially in particle physics
- Noethers theorem: If an action is invariant under some group transformations, there exists one or more conserved quantities. Example: rotational invariance $\Rightarrow$ angular momentum conservation
- Salam & Ward (1961): Possible to generate interaction terms for forces by requiring local gauge transformation invariance of the free Lagrangian

What does that mean e.g. for a U(1) group?

- Start from the Dirac free Lagrangian:

$$L_\psi = \bar{\psi}(i\not{\partial} - m)\psi \quad ; \not{\partial} = \partial_\mu \gamma^\mu$$

- In local gauge transformation:

$$\psi \rightarrow e^{i\alpha(x)}\,\psi \; ; \qquad L_\psi \rightarrow L_\psi + \bar{\psi}\gamma_\mu\psi\partial^\mu\alpha$$

- Introducing a gauge field A_μ through minimal coupling: $D_\mu = \partial_\mu - ieA_\mu$ and require:

$$A_\mu \rightarrow A_\mu + 1/e\,\partial_\mu\alpha(x)$$

- Then $L_\psi \rightarrow L_\psi + e\bar{\psi}\gamma_\mu\psi A^\mu$

conservation of electric charge!

- The Lagrangian for the free gauge field is:

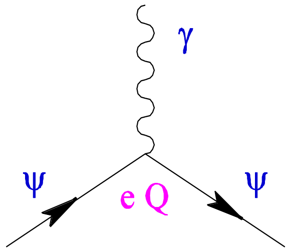
$$L_A = -\tfrac{1}{4}\,F_{\mu\nu}F^{\mu\nu} \; ; \; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

QUANTUM ELECTRODYNAMICS

based on U(1) gauge invariance

$$\mathcal{L} = \bar{\psi}\,(i\,\gamma^{\mu}\,D_{\mu} - m)\,\psi = \bar{\psi}\,(i\,\gamma^{\mu}\,\partial_{\mu} - m)\,\psi + \mathcal{L}_I$$

$$\mathcal{L}_I = e\,Q\,A_{\mu}\,\left(\bar{\psi}\,\gamma^{\mu}\,\psi\right)$$



conservation of electric charge

Kinetic term: Maxwell

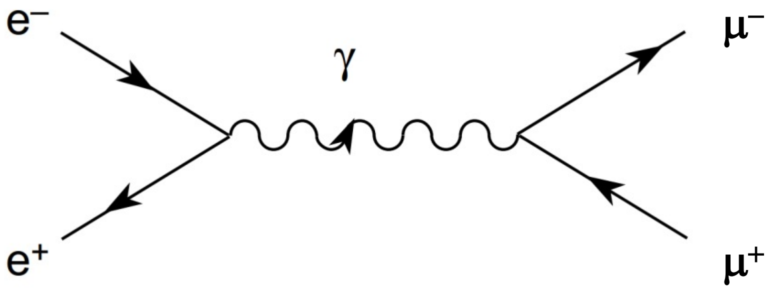
$$\mathcal{L}_K = -\tfrac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu} \quad \Rightarrow \quad \partial_{\nu}F^{\nu\mu} = -e\,Q\,\left(\bar{\psi}\,\gamma^{\mu}\,\psi\right)$$

$$\mathcal{L}_m = \tfrac{1}{2}\,m_{\gamma}^{2}\,A^{\mu}\,A_{\mu}$$ Not Gauge Invariant

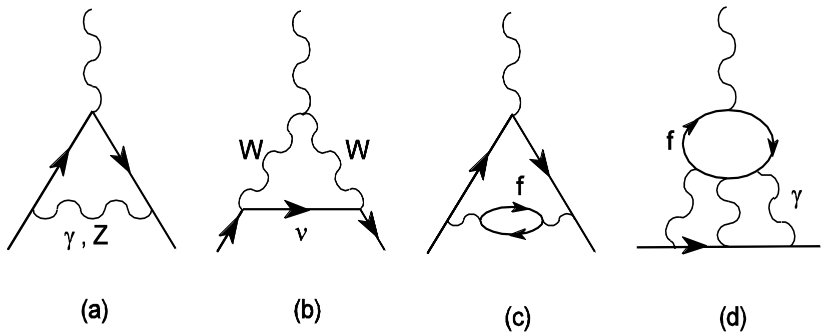
$\Rightarrow$ $m_{\gamma} = 0$ [exp: $m_{\gamma} < 1 \cdot 10^{-18}$ eV]

Gauge Symmetry $\Rightarrow$ QED Dynamics

Successful Theory



Anomalous Magnetic Moment



$$\mu_l \equiv g_l \frac{e}{2m_l} \quad ; \quad a_l \equiv \frac{1}{2}(g_l - 2)$$

$\boldsymbol{a}_{\mathbf{e}} = (115\,965\,218.073 \pm 0.028) \cdot 10^{-11} \Rightarrow$

$1/\boldsymbol{\alpha}_{\mathbf{em}} = 137.035\,999\,084 \pm 0.000\,000\,021 \Rightarrow$

$\boldsymbol{a}_{\boldsymbol{\mu}}^{\mathbf{theory}} = (116\,592\,033 \pm 62) \cdot 10^{-11}$ (new estimate 2025)

[Experiment: $(116\,592\,072 \pm 15) \cdot 10^{-11}$]

Earlier large (> 4σ) discrepancy theory-experiment

Now no discrepancy & precision up to 8th significant digit!

SU(N) ALGEBRA

$N \times N$ matrices: $U\,U^\dagger = U^\dagger U = 1$; $\det U = 1$

$$\rightarrow \quad U = \exp\{i\,T^a\,\theta_a\}$$

$T^a = T^{a\dagger}$; $\mathrm{Tr}\,(T^a) = 0$; $a = 1, \ldots, N^2 - 1$

Commutation Relation:

$$\left[T^a, T^b\right] = i\,f^{abc}\,T^c$$

Structure Constants f^{abc} real , antisymmetric

Fundamental Representation: $T^a_F = \frac{\lambda^a}{2}$ $N \times N$

The local gauge transformation invariance of the Lagrangian for the U(1) group can be generalized to SU(N) groups:

$D_\mu = \partial_\mu + igT^aA^a_\mu$

then $F^{a\mu\nu} = \partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} - g\, f^{abc} A^{b\nu} A^{c\mu}$

and $\mathcal{L}_A = -\,¼\, F^a_{\mu\nu} F^{a\mu\nu}$

SU(2)

2×2 **matrices:** $U\,U^\dagger = U^\dagger U = 1 \quad ; \quad \det U = 1$

$$\rightarrow \qquad U = \exp\{i\,T^a\,\theta_a\}$$

$$T^a = T^{a\dagger} \quad ; \quad \mathrm{Tr}\,(T^a) = 0 \quad ; \quad a = 1,\ldots,3$$

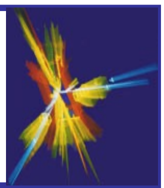
Commutation Relation:

$$\left[T^a, T^b\right] = i\,\epsilon^{abc}\,T^c$$

Fundamental Representation: $T_F^a = \frac{1}{2}\,\sigma^a$ **Pauli**

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; ; \; \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \; ; \; \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\left\{\sigma^a, \sigma^b\right\} = 2\,\delta_{ab}$$



SU(3)

$$\left[T^a, T^b\right] \;=\; i\, f^{abc}\, T^c$$

Fundamental Rep.: $T_F^a = \frac{1}{2}\lambda^a$ **Gell-Mann**

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \;;\; \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \;;\; \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \;;\; \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \;;\; \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\tfrac{1}{2}\, f^{123} = f^{147} = -f^{156} = f^{246} = f^{257} = f^{345}$$

$$= f^{345} = -f^{367} = \tfrac{1}{\sqrt{3}}\, f^{458} = \tfrac{1}{\sqrt{3}}\, f^{678} = \tfrac{1}{2}$$

Quantum chromodynamics

- Basic structure
- QCD Lagrangian
- Quantum corrections
- Running couplings
- α_s measurements

QUANTUM CHROMODYNAMICS

$$N_C = 3 \quad \Rightarrow \quad \mathbf{q} \equiv \begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

FREE QUARKS: $$\mathcal{L} = \bar{\mathbf{q}} \left[i \gamma^\mu \partial_\mu - m \right] \mathbf{q}$$

SU(3) Colour Symmetry:

$$\mathbf{q} \rightarrow \mathbf{U}\, \mathbf{q} = \exp\left\{ -i\, g_s \frac{\lambda^a}{2} \theta_a \right\} \mathbf{q}$$

Gauge Principle: Local Symmetry $\theta_a = \theta_a(x)$

$$\mathbf{D}^\mu \mathbf{q} \equiv (\mathbf{I}_3\, \partial^\mu - i\, g_s\, \mathbf{G}^\mu)\, \mathbf{q} \rightarrow \mathbf{U}\ \mathbf{D}^\mu \mathbf{q}$$

$$\mathbf{D}^\mu \rightarrow \mathbf{U}\, \mathbf{D}^\mu\, \mathbf{U}^\dagger \quad ; \quad \mathbf{G}^\mu \rightarrow \mathbf{U}\, \mathbf{G}^\mu\, \mathbf{U}^\dagger - \frac{i}{g_s} (\partial^\mu \mathbf{U})\, \mathbf{U}^\dagger$$

$$[\mathbf{G}^\mu]_{\alpha\beta} \equiv \frac{(\lambda^a)_{\alpha\beta}}{2} G_a^\mu(x)$$

8 Gluon Fields

Infinitesimal SU(3) Transformation:

$$\mathbf{q}^\alpha \;\rightarrow\; \mathbf{q}^\alpha - i\, g_s\, \delta\theta_a \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} \mathbf{q}^\beta$$

$$G_a^\mu \;\rightarrow\; G_a^\mu - \partial^\mu (\delta\theta_a) + g_s\, f^{abc}\, \delta\theta_b\, G_c^\mu$$

Non Abelian Group: $f^{abc} \neq 0$

- δG_a^μ depends on G_a^μ
- Universal g_s No Colour Charges

Kinetic Term:

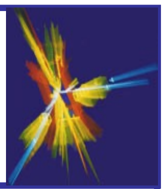
$$\mathbf{G}^{\mu\nu} \;\equiv\; \frac{i}{g_s}\,[\mathbf{D}^\mu, \mathbf{D}^\nu] \;=\; \partial^\mu \mathbf{G}^\nu - \partial^\nu \mathbf{G}^\mu - i\, g_s\, [\mathbf{G}^\mu, \mathbf{G}^\nu]$$

$$\mathbf{G}^{\mu\nu} \;\rightarrow\; \mathbf{U}\,\mathbf{G}^{\mu\nu}\,\mathbf{U}^\dagger \qquad ; \qquad \mathbf{G}^{\mu\nu} \;\equiv\; \frac{\lambda^a}{2}\, G_a^{\mu\nu}$$

$$G_a^{\mu\nu} \;=\; \partial^\mu G_a^\nu - \partial^\nu G_a^\mu + g_s\, f^{abc}\, G_b^\mu\, G_c^\nu$$

$$\boxed{\mathcal{L}_K \;=\; -\frac{1}{2}\,\mathrm{Tr}\,(\mathbf{G}^{\mu\nu}\,\mathbf{G}_{\mu\nu}) \;=\; -\frac{1}{4}\, G_a^{\mu\nu}\, G_{\mu\nu}^a}$$

Mass term for gluons? Not gauge invariant $\Rightarrow \mathrm{m_G} = 0$



QCD LAGRANGIAN

$$\mathcal{L}_{QCD} = -\tfrac{1}{2}\,\mathrm{Tr}\,(\mathrm{G}^{\mu\nu}\,\mathrm{G}_{\mu\nu}) + \bar{\mathrm{q}}\,[i\,\gamma^{\mu}\,\mathrm{D}_{\mu} - m_q]\,\mathrm{q}$$

$$= -\frac{1}{4}\left(\partial^{\mu}G^{\nu}_{a} - \partial^{\nu}G^{\mu}_{a}\right)\left(\partial_{\mu}G^{a}_{\nu} - \partial_{\nu}G^{a}_{\mu}\right)$$

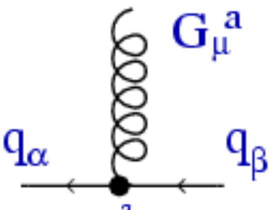
Kinetic

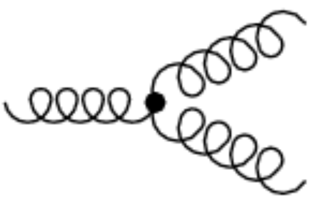
$$+ \sum_{q} \bar{q_{\alpha}}\;[i\,\gamma^{\mu}\,\partial_{\mu} - m_q]\;q_{\alpha}$$

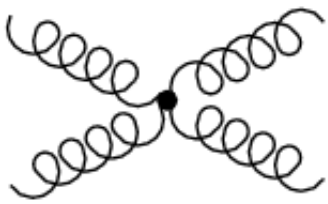
$$+ \frac{1}{2}\sum_{q} g_s\;\left[\bar{q_{\alpha}}\;(\lambda^{a})_{\alpha\beta}\,\gamma^{\mu}\,q_{\beta}\right]\;G^{a}_{\mu}$$

$$- \frac{1}{2}\,g_s\;f_{abc}\;\left(\partial_{\mu}G^{a}_{\nu} - \partial_{\nu}G^{a}_{\mu}\right)\,G^{\mu}_{b}\,G^{\nu}_{c}$$

$$- \frac{1}{4}\,g_s^{2}\;f_{abc}\,f_{ade}\;G^{\mu}_{b}\,G^{\nu}_{c}\,G^{d}_{\mu}\,G^{e}_{\nu}$$

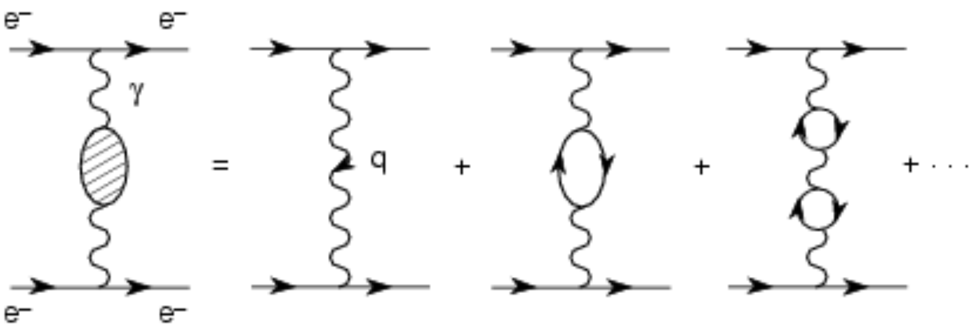






- **Gluon Self-interactions** G^3, G^4
- **Universal Coupling** g_s
- **No Colour Charges**

QUANTUM CORRECTIONS



$$T(Q^2) \sim \frac{\alpha}{Q^2}\left\{1 + \Pi(Q^2) + \Pi(Q^2)^2 + \cdots\right\} \sim \frac{\alpha(Q^2)}{Q^2}$$

Effective *(Running)* **Coupling:**

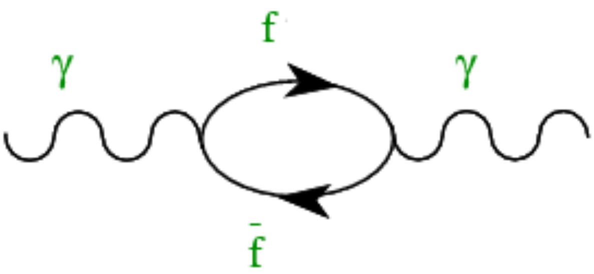
$$\boxed{\alpha(Q^2) = \frac{\alpha}{1 - \Pi(Q^2)} \approx \frac{\alpha}{1 - \frac{\alpha}{3\pi}\ln\left(\frac{Q^2}{m^2}\right)}}$$

$\alpha(Q^2)$ Increases with $Q^2 \equiv -q^2$

$\Longrightarrow$ increases at shorter distances
$\Longrightarrow$ decreases at larger distances

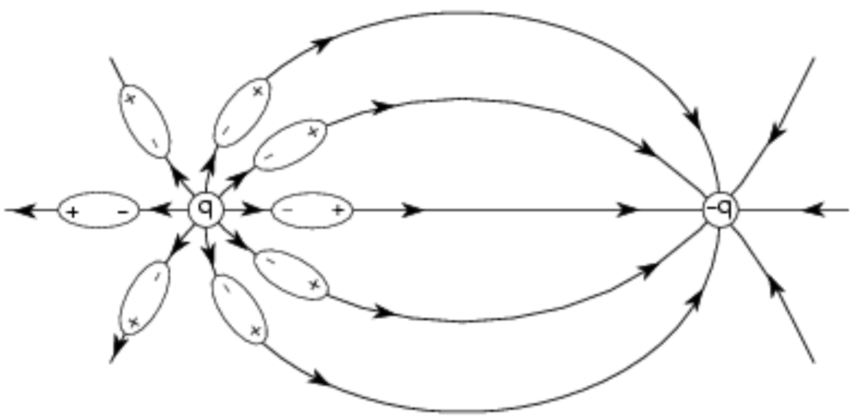
SCREENING

VACUUM POLARIZATION



The Photon Couples to *Virtual* $f\,\bar{f}$ *Pairs*

Vacuum ⟷ Polarized Dielectric Medium



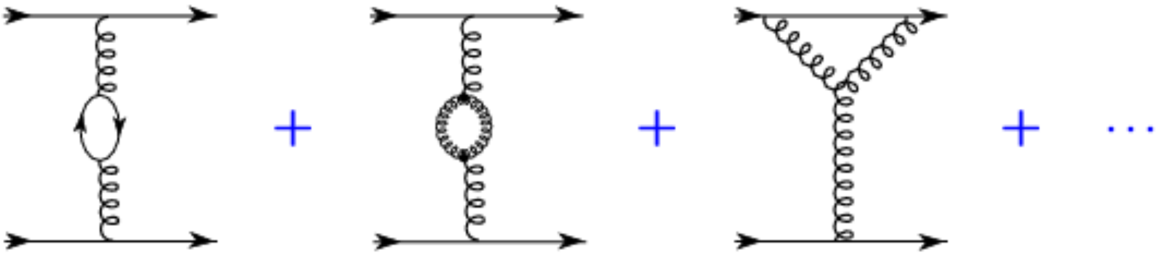
$$1/\alpha = 1/\alpha\,(m_e^2) = 137.035999084\,(21)$$

$$1/\alpha\,(m_Z^2) = 127.930 \pm 0.008$$

($l^- l^+$ and $q\,\bar{q}$ contributions included)

source: PDG review on electro-weak model

QCD RUNNING COUPLING



$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 - \beta_1 \frac{\alpha_s(Q_0^2)}{2\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)}$$

$$\beta_1 = \frac{1}{3} N_F - \frac{11}{6} N_C$$

quarks gluons

$N_C = 3$, $N_F = 6$ ➡ $\beta_1 < 0$

$Q^2 > Q_0^2$ ➡ $\alpha_s(Q^2) < \alpha_s(Q_0^2)$

$\alpha_s(Q^2)$ Decreases at Short Distances

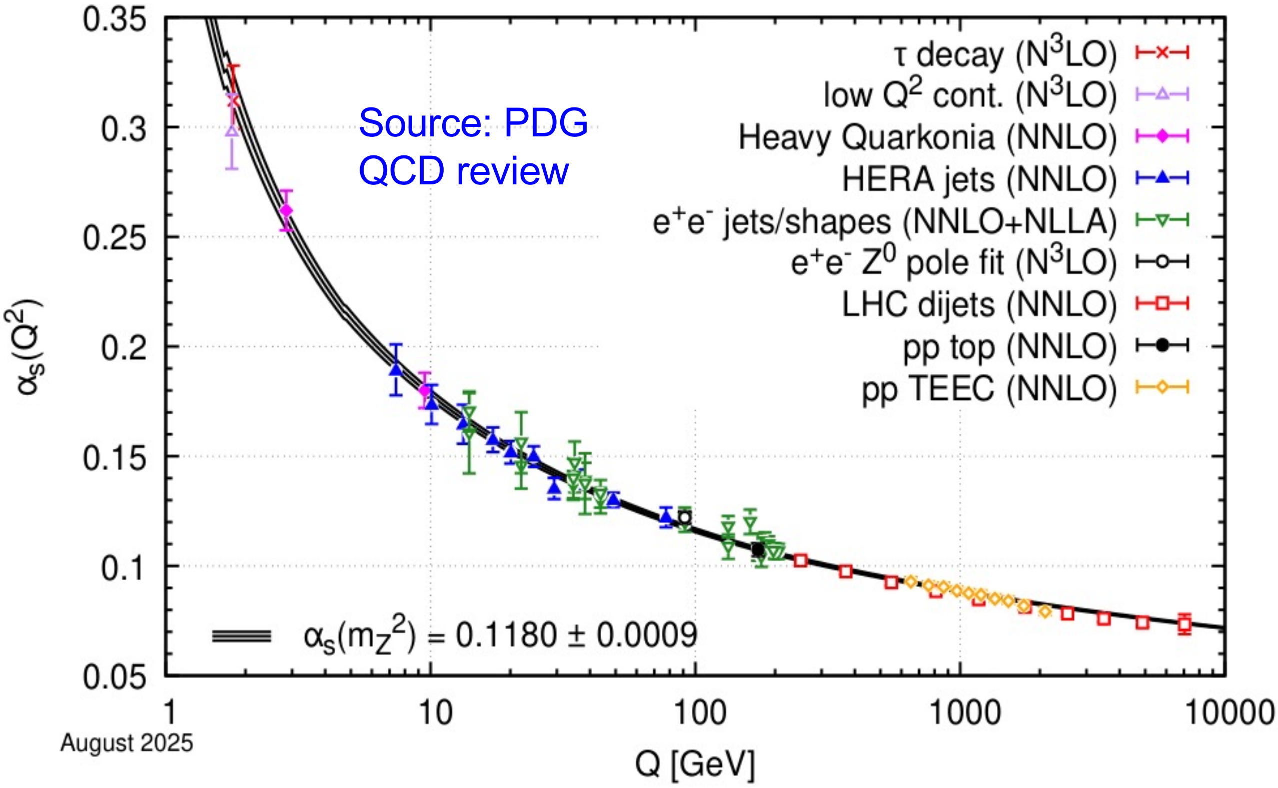
ANTI–SCREENING

Asymptotic freedom & confinement

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 - \beta_1 \frac{\alpha_s(Q_0^2)}{2\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)}$$

$$\beta_1 < 0 \quad \rightarrow \quad \lim_{Q^2 \to \infty} \alpha_s(Q^2) = 0$$

ASYMPTOTIC FREEDOM

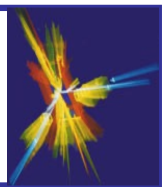


$\alpha_s(Q^2)$ **increases at low energies**

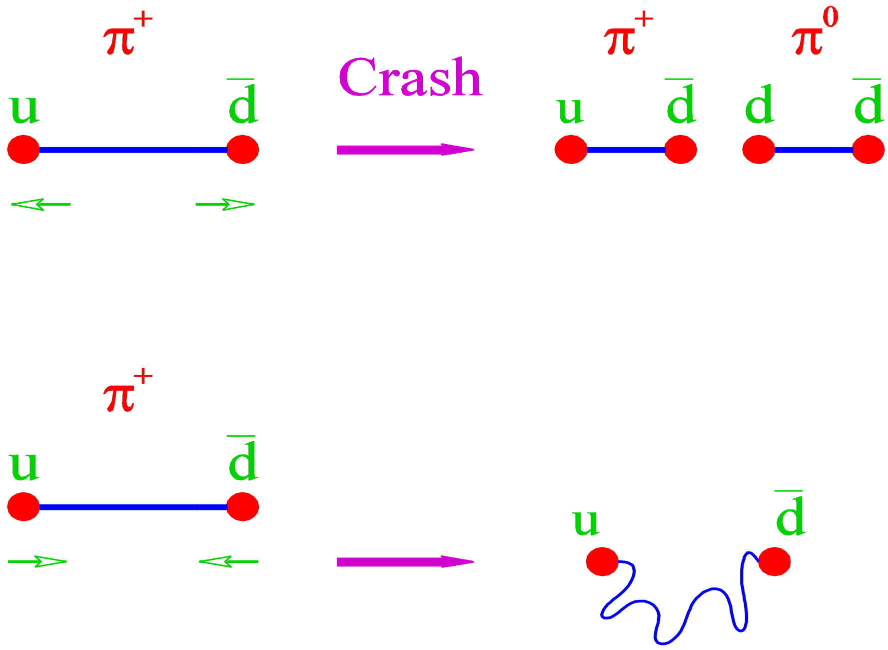
$\alpha_s \sim O(1)$ at 1 GeV Non-Perturbative Region

$\rightarrow$ **CONFINEMENT ?**

Confinement & asymptotic freedom



CONFINEMENT



ASYMPTOTIC FREEDOM

Asymptotic freedom:

$\alpha_s \rightarrow 0$ at large E (short distances)

Confinement:

large α_s at small E (large distances)

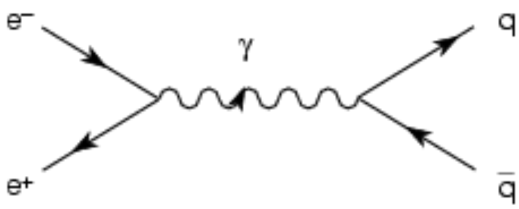
Quark flavour (u,d,s,c,b,t):

strong interactions — flavour independent & conserving

weak interactions — change quark flavour

e^+e^- annihilation

$$T\left(e^+e^- \to q\,\bar{q}\right) =$$



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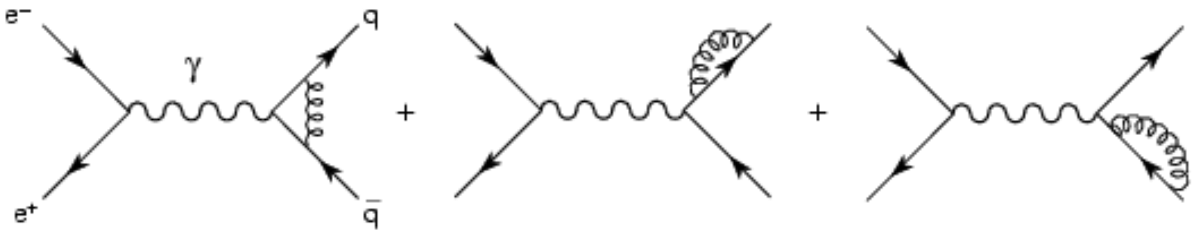
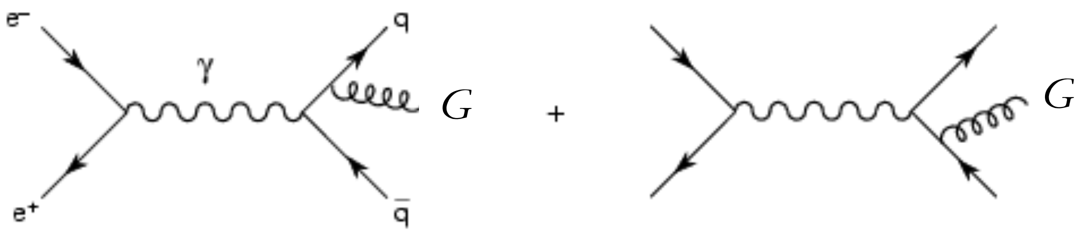
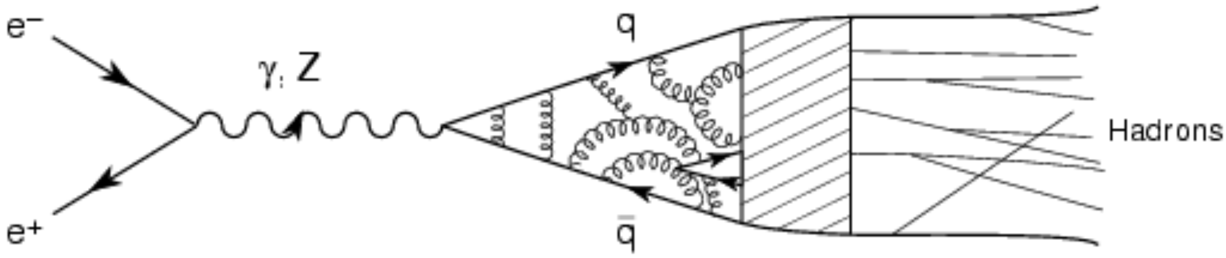


$$T\left(e^+e^- \to q\,\bar{q}\,G\right) =$$



Confinement ⟷ Prob. Hadronization = 1



$$\sigma(e^+e^- \to \text{hadrons}) =$$

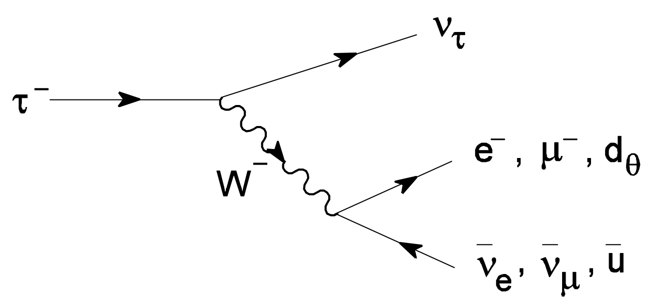
$$\sigma\left(e^+e^- \to q\,\bar{q} + q\,\bar{q}\,G + q\,\bar{q}\,G\,G + q\,\bar{q}\,q\,\bar{q} + \cdots\right)$$

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_q Q_q^2\, N_C \quad \left\{1 + \frac{\alpha_s(s)}{\pi} + \cdots\right\}$$

$$R_Z \equiv \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to e^+e^-)} = R_Z^{EW}\, N_C \quad \left\{1 + \frac{\alpha_s(s)}{\pi} + \cdots\right\}$$

$\tau^- \to \nu_\tau + \text{Hadrons}$

$$d_\theta = \cos\theta_C d + \sin\theta_C s$$



$$B_l \equiv Br(\tau^- \to \nu_\tau l^- \bar{\nu}_l) \approx 1/(2+N_C) = 1/5 = 20\%$$

difference mainly due to QCD

$$B_e = (17.85 \pm 0.04)\ \% \qquad B_\mu = (17.37 \pm 0.04)\ \%$$

QCD :

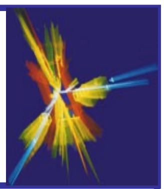
$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \nu_\tau)} = N_C(1+S_{ew})\left\{1 + \frac{\alpha_s(m_\tau^2)}{\pi} + 5.2\frac{\alpha_s(m_\tau^2)^2}{\pi^2} + \ldots\right\}$$

$$R_\tau = 3.629 \pm 0.013 \quad \Rightarrow \quad \alpha_s(m_\tau^2) = 0.315 \pm 0.015$$

$$\alpha_s(m_\tau^2) = 0.315 \pm 0.015 \quad \gg \quad \alpha_s(m_Z^2) = 0.1180 \pm 0.0009$$

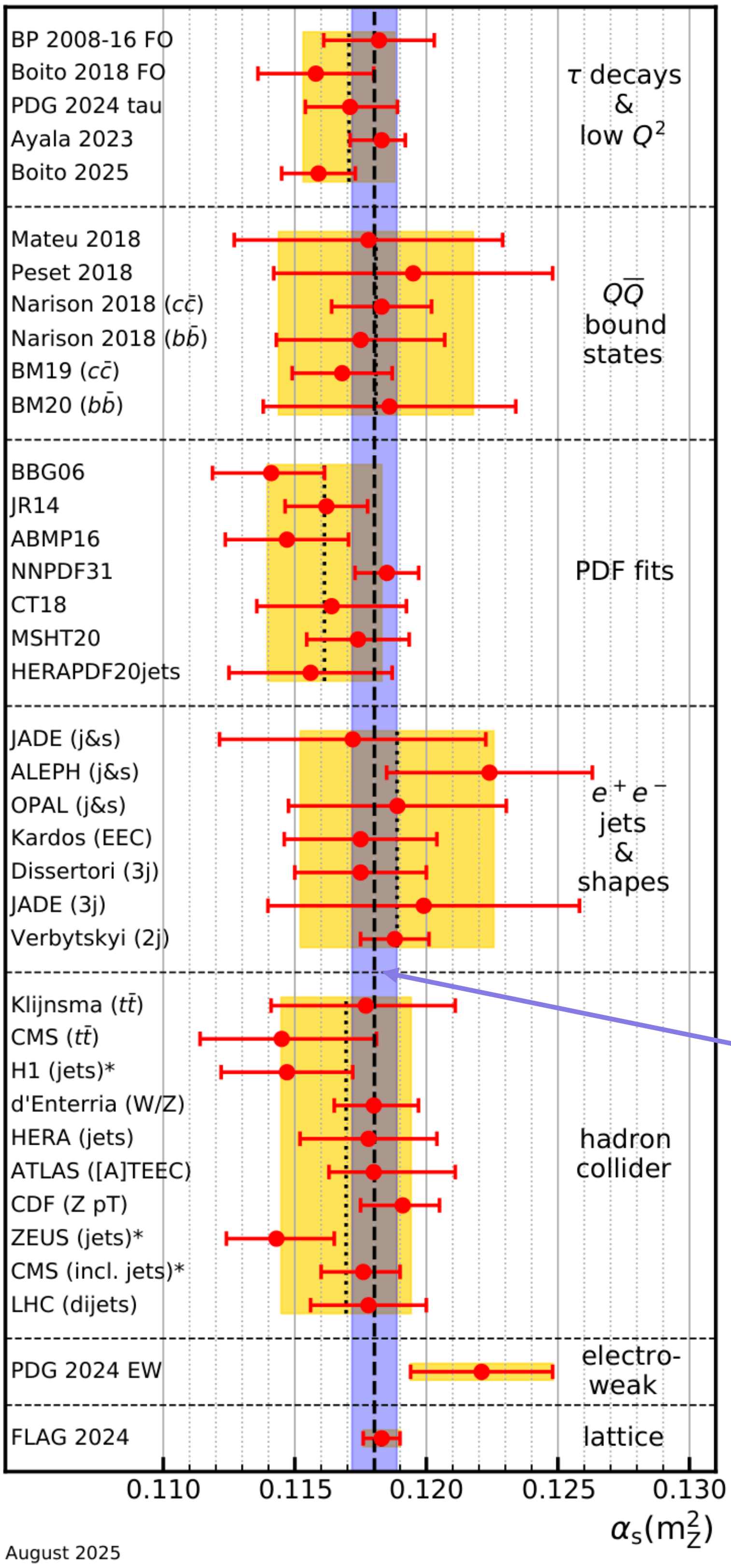
NB! electroweak correction $S_{ew} \approx 0.019$

Sources: PDG QCD & electro-weak reviews



Summary of different α_s measurements:

(for details see Quantum chromodynamics review in PDG)



PDG 2025:
$\alpha_s(m_Z^2) = 0.1180 \pm 0.0009$