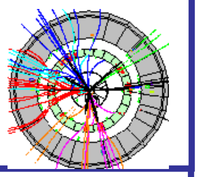
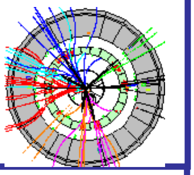


Beyond Standard Model

- Dark matter: WIMPs & axions
- Energy scales & couplings
- Electroweak symmetry breaking
- Grand unified theories
- Supersymmetry
- Extra dimensional models



The Standard Model is a very successful theory of particle interactions

- Electroweak interaction tested at 0.1-1 % level
- Basic strong interaction ingredients are confirmed, however perturbative calculations get very difficult at low Q^2 due to large α_s.
- SM describes well all observed phenomena upto now with the exception of ν oscillations
- Observed interactions are a dynamical consequence of symmetries (”gauge principle”)

Is the Standard Model ”the final theory”? **No, most probably!**

At least since SM doesn’t include gravity!!

Gravity is extremely weak: $m_Z^2 / m_{Pl}^2 \approx 10^{-38}$. WHY?

Why the observed hierarchy of fermion masses ?
Why 3 families of fermions ?
Why this bizarre gauge group combination ?

gauge group ⇒ $G = SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$

matter fermions ⇒

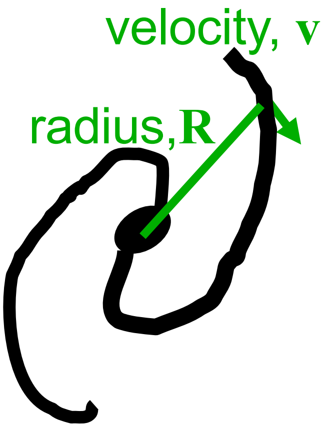
$q_L = (2 , 1/3 , 3)$
$u_R = (1 , 4/3 , 3)$
$d_R = (1 , -2/3 , 3)$
$l_L = (2 , -1 , 1)$
$e_R = (1 , -2 , 1)$

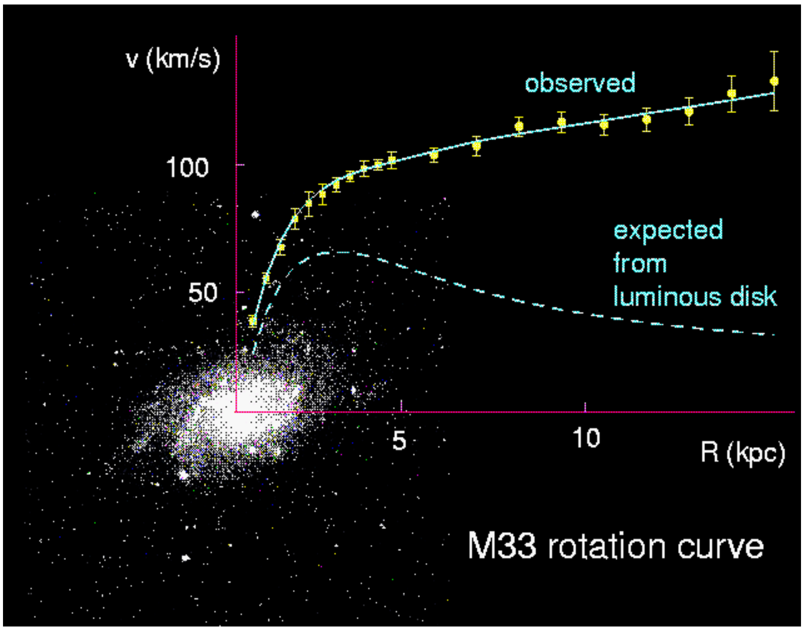
Dark matter

Possible indications of physics beyond SM ?

Dark (i.e. non-luminous & non EM radiation absorbing) **matter** in the universe see e.g. PDG review on dark matter

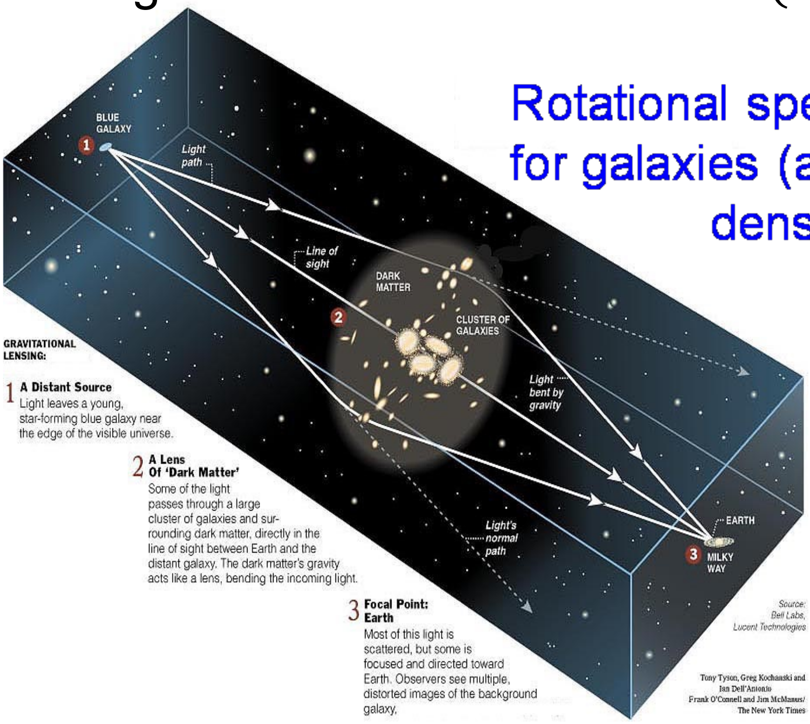
Seen e.g. in galaxy mass distribution





$\mathrm{v}(\mathrm{R}) \propto \sqrt{\mathrm{M}(\mathrm{R})/\mathrm{R}}$,
$M(\mathrm{R})$ enclosed mass

Stars & gas predicts $\mathrm{v}(\mathrm{R}) \propto 1/\sqrt{\mathrm{R}}$ but $\mathrm{v}(\mathrm{R}) \approx$ const. for most galaxies $\Longrightarrow$ dark halo with $M(\mathrm{R}) \propto \mathrm{R}$ or $\rho(\mathrm{R}) \propto \mathrm{R}^{-2}$



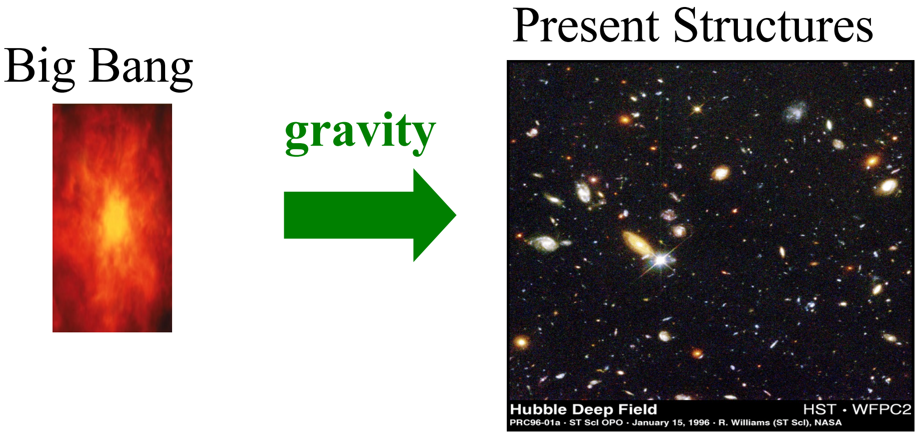
Source: Bell Labs, Lucent Technologies

Tony Tyson, Greg Kochanski and Ian Dell'Antonio
Frank O'Connell and Jim McManus/ The New York Times

Rotational speed measurements for galaxies (above) and mass density measurements from gravitational lensing (left) indicate $\Rightarrow$

Luminous stars contain only small fraction of total galaxy mass

Dark matter & structure formation



Big Bang

gravity

Present Structures

Dark matter dominates matter in our universe ⇒ governs structure formation

2 extreme forms of dark matter possible:

hot (relativistic) and cold (non-relativistic)

Relativistic particles escape from structure formation ⇒ galaxy formation indicate most dark matter cold: CDM

Dark matter must be stable on cosmological time scales, interact weakly with radiation (“electrically neutral”) & matter (no strong interaction) plus have right relic density

Baryonic candidates: primordial black holes e.g. Massive Compact Halo Objects (MACHOs) – not sufficient density, stranglets e.g. a uuddss-quark particle with mass < $2m_{\Lambda_0}$.

Non-baryonic candidates: sterile singlet neutrino (ν mixing angle $\theta \ll 1$), dark photons (vector boson with mass < $2m_e$ & only decay to 3γ possible), weakly interacting massive particles (WIMPs), axions – particle physics discoverables

An obvious WIMP would be a heavy neutrino but a SU(2) doublet neutrino ($m_\nu > m_Z/2$) gives too small relic density. Hotest candidate: lightest supersymmetric particle (LSP)

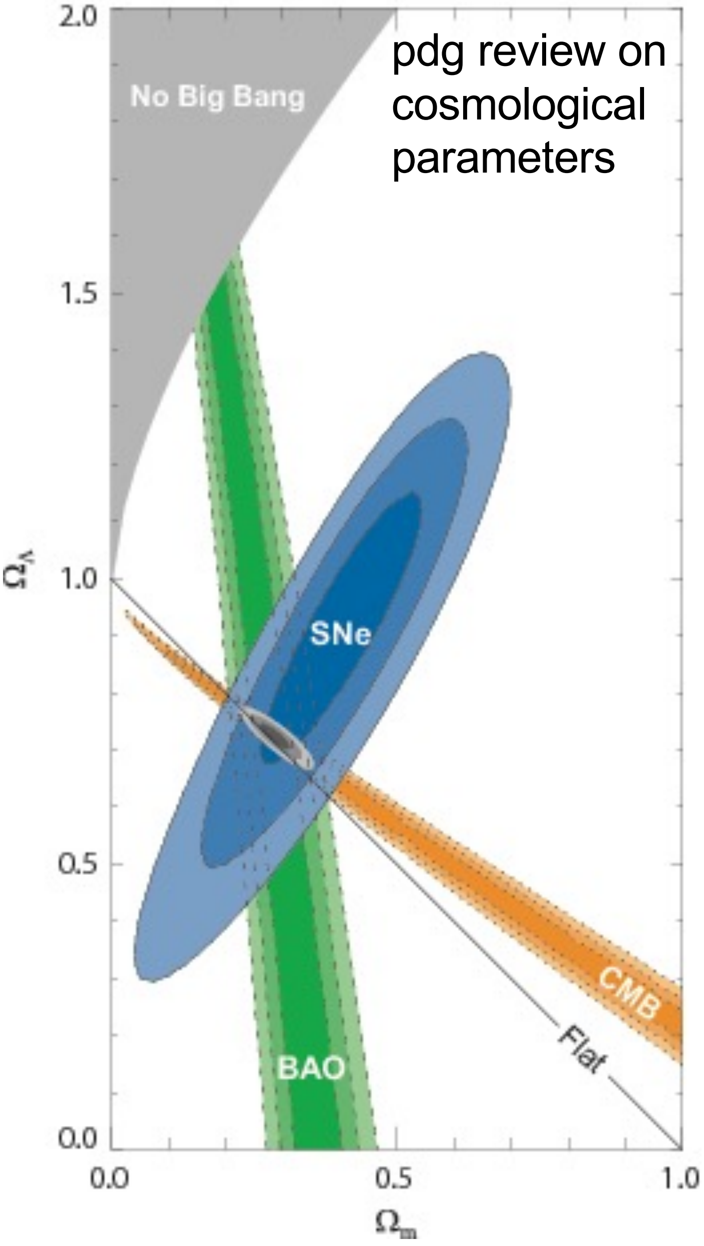
Matter & energy in the universe

Supernova measurements (SNe):

measure brightness → distance: $B = L/4\pi d^2$
measure host galaxy redshift → recesion velocity
test nonlinearity of Hubbles law at large distances

Cosmic microwave background (CMB):

measure size of CMB anisotropy (last baryon-γ scattering surface) → estimate of energy/matter density of universe



pdg review on cosmological parameters

Galaxy clustering, baryonic acustic oscillations (BAO):

measure galaxy clustering as ”tracer” of dark matter distribution vs redshift → estimate of matter density

$\Omega_x = \rho_x / \rho_{critical}$

critical density
for flat universe

$\rho_{critical} = 3H^2/8\pi G_N$

$H = h \cdot 100$ km/s/Mpc

CMB (Planck):

$h = 0.674 \pm 0.005$
$\Omega_{tot} = 1.011 \pm 0.006$ so agrees with flat universe

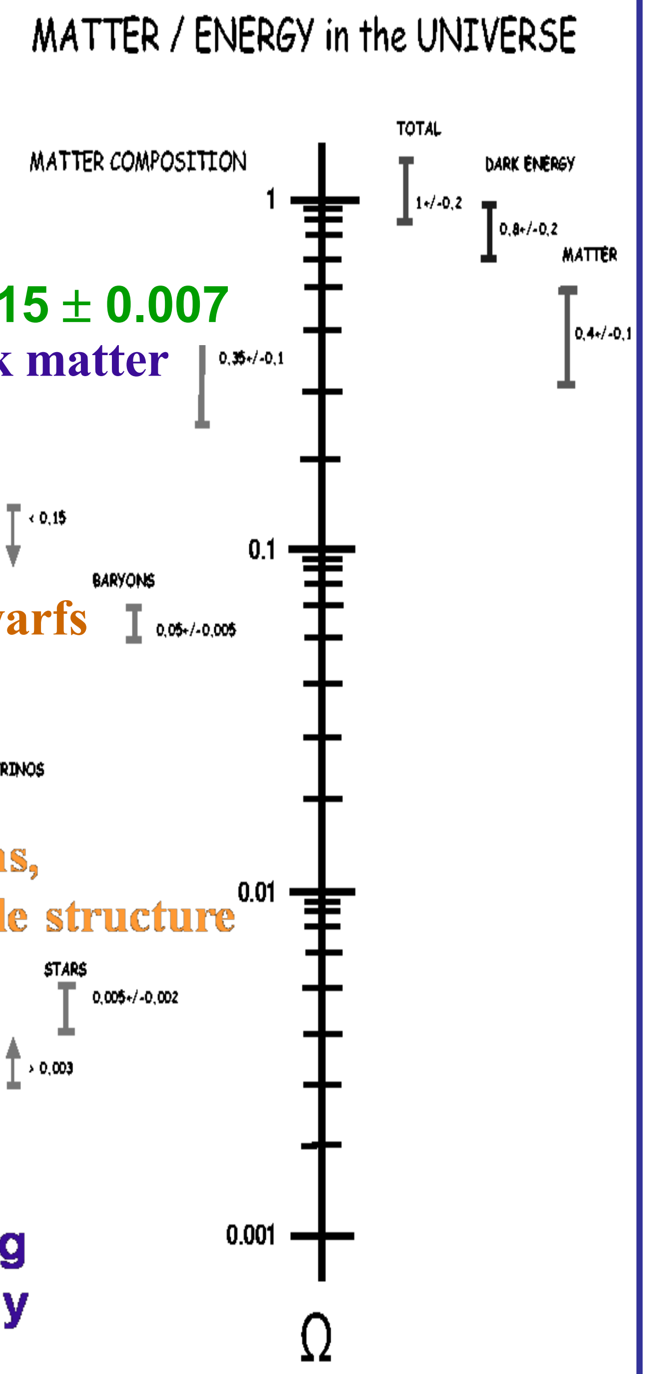
Cosmological constant:
$\Omega_\Lambda = 0.685 \pm 0.007$
$\Omega_m = 0.315 \pm 0.007$

Matter & energy in the universe

Dark energy (≡ cosmological constant) constitutes the largest fraction of energy/matter in our universe !!

$$\Omega_{total} = \Omega_M + \Omega_\Lambda \sim 1$$

matter dark energy



Matter:

$$\Omega_M = \Omega_b + \Omega_\nu + \Omega_{CDM} = 0.315 \pm 0.007$$

baryons neutrinos cold dark matter

Baryonic matter :

$$\Omega_b = 0.0492 \pm 0.0006$$

stars, gas, brown & white dwarfs

Neutrinos:

$$0.001 < \Omega_\nu < 0.004$$

lower bound from oscillations, upper bound from large-scale structure

Cold Dark Matter :

$$\Omega_{CDM} \sim 0.264 \pm 0.004$$

WIMPs (Weakly Interacting Massive Particles), usually neutralinos, or axions

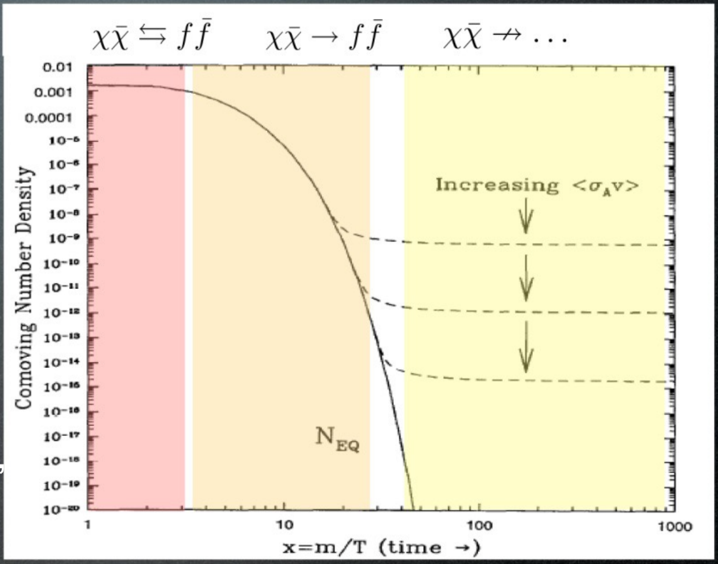
NB! If Planck & BAO is combined, Ω_Λ & h (Ω_{CDM} & Ω_M) increase (decrease) slightly but overall picture looks similar.

A thermal relic from the Early Universe

Consider a particle χ:

- subject to $\chi\bar{\chi} \to \ldots$
- ‘heavy’ (e.g. 100 GeV)
- ‘stable’
- in an expanding Universe
- symmetric abundance

”neutral”, very long lived (life time ~cosmological scale), weakly interacting particle, limited self-interactions



Kolb,Turner, The Early Universe, 1995

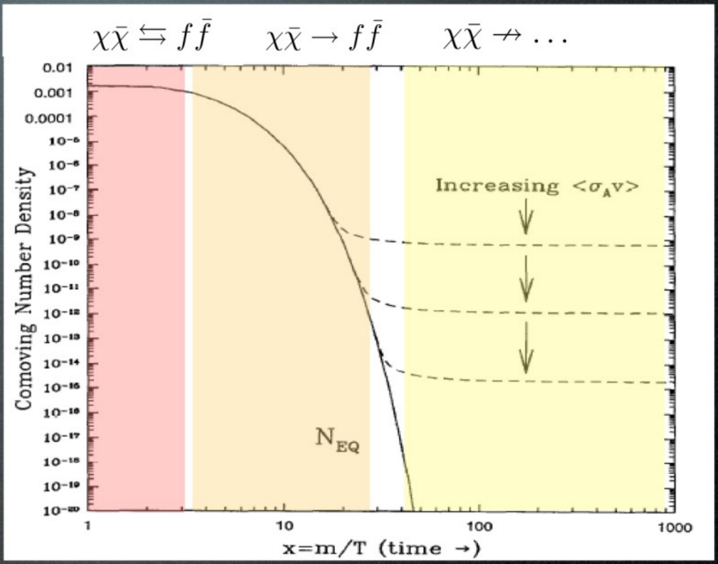
A thermal relic from the Early Universe

Boltzmann equation in the Early Universe:

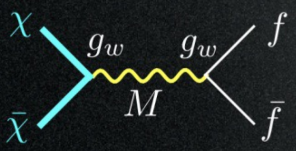
$$\Omega_X \approx \frac{6\ 10^{-27}\mathrm{cm}^3\mathrm{s}^{-1}}{\langle\sigma_{\mathrm{ann}}v\rangle}$$

Relic $\Omega_{\mathrm{DM}} \simeq 0.23$ for

$$\langle\sigma_{\mathrm{ann}}v\rangle = 3 \cdot 10^{-26}\mathrm{cm}^3/\mathrm{sec}$$



Weak cross section:



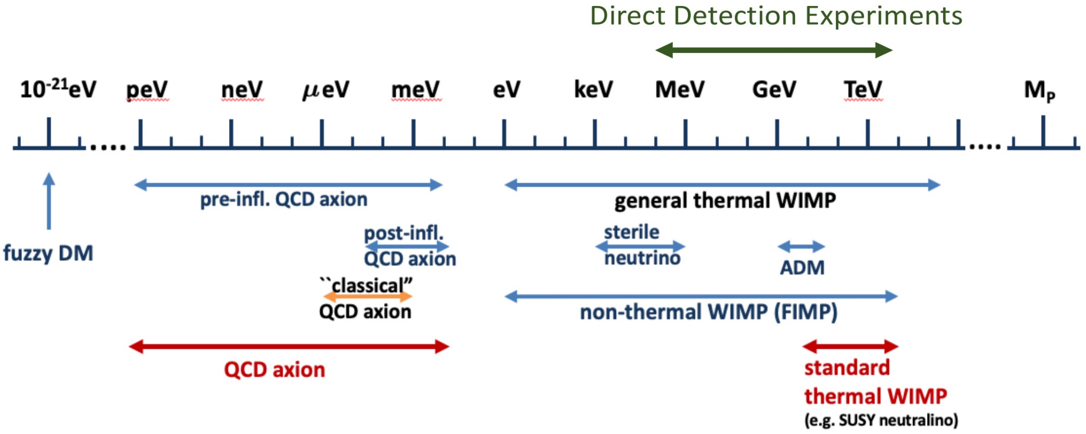
$$\langle\sigma_{\mathrm{ann}}v\rangle \approx \frac{(g_w^2/4\pi)^2}{M^2} \approx 3 \cdot 10^{-26}\mathrm{cm}^3/\mathrm{sec}$$

WIMP miracle!

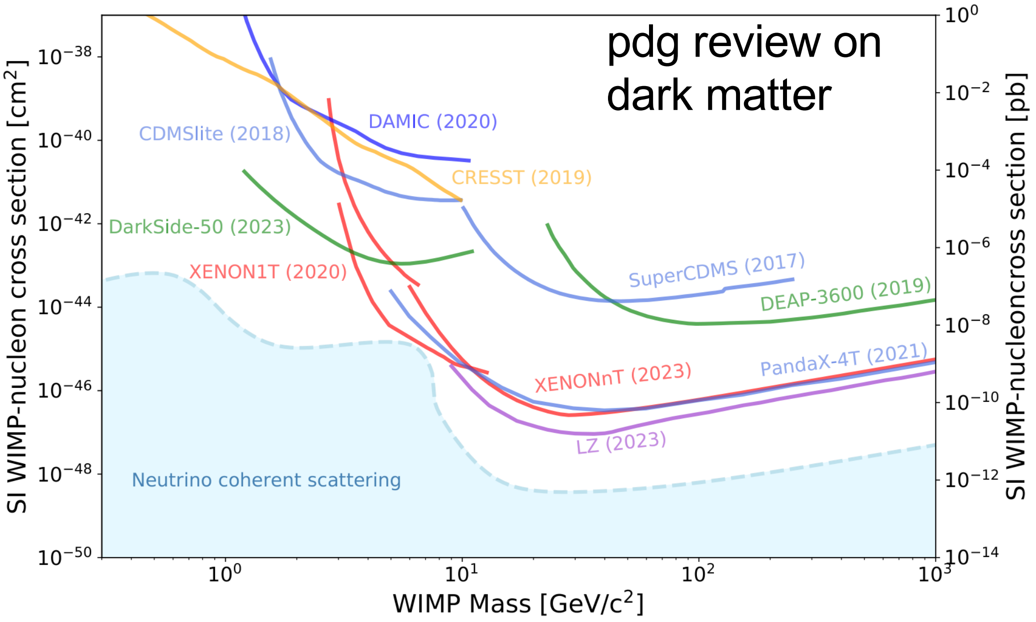
WIMP searches

Searches for WIMPs (or other dark matter candidates):

- accelerator-based: (a) missing (transverse) momentum due to WIMP production; (b) excess or bump of jet or lepton pairs produced by a dark mediator
- direct detection of WIMPs from galactic WIMP halo in terrastial detectors; (a) (in)elastic scattering off a target nucleus giving rise to measurable nuclear recoil (b) scattering off bound electrons (for WIMP masses < GeV giving WIMP-nucleus scattering energy transfers below detection threshold) or absorption via “axioelectric” effect (for axions or dark photons, analogous to photoelectric effect)



No WIMP signal observed ⇒ WIMP direct detection limits



pdg review on dark matter

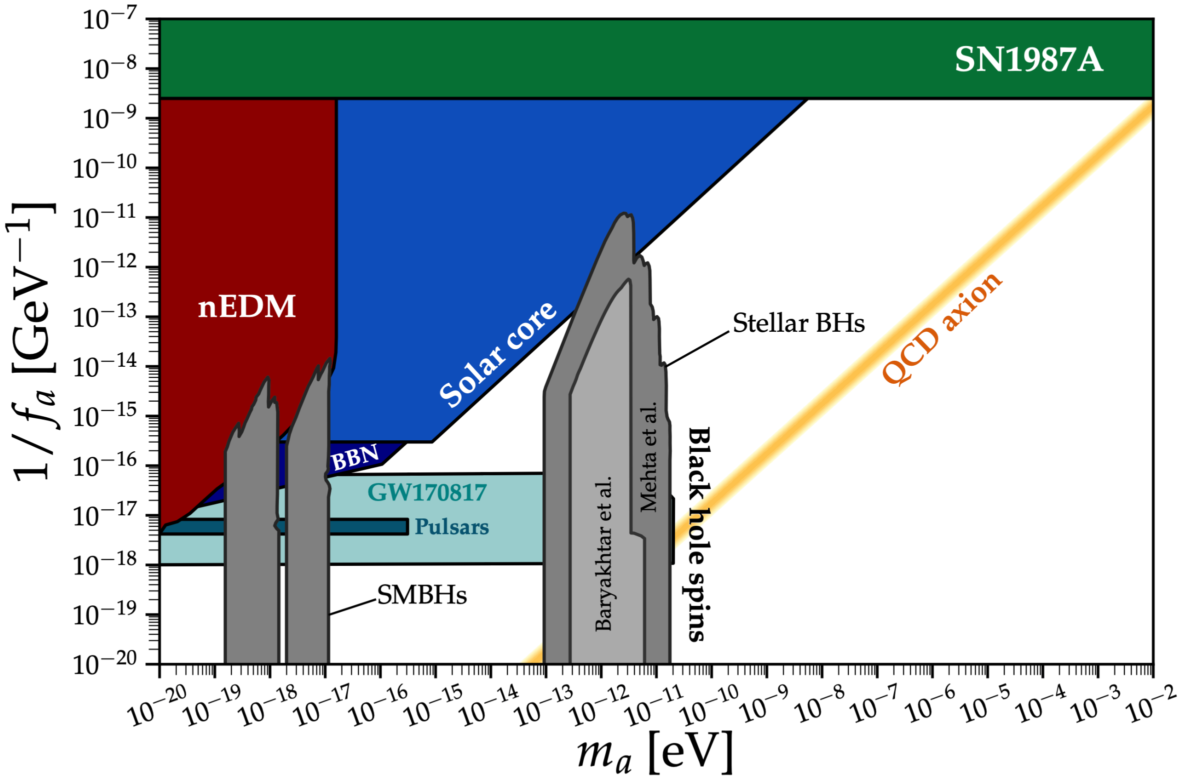
Axion

A very light neutral scalar boson, originally proposed by Peccei & Quinn: pseudo-Goldstone boson from a broken U(1) symmetry introduced to cure CP problem of QCD.

Motivation: CP violation in QCD not observed, stringent limits (< 10^{-10}) from neutron electric dipole moment. Axion field term in the QCD Lagrangian would compensate the CP violating term Θ_i. Coupling $\propto f_A^{-1}$ very small to matter due to the high scale of the U(1) symmetry breaking.

$$\mathcal{L}_{\text{QCD,CPviol}} = (\alpha_s/8\pi)\left(\Theta_i - \phi_A/f_A\right)G^{\mu\nu a}\tilde{G}^a_{\mu\nu} \approx 0$$

Axion density contribution (post-inflation symmetry breaking)**:** $\boldsymbol{\Omega_A h^2 \sim 0.12\cdot(30\mu\text{eV}/m_A)^{1.165}} \Rightarrow$ masses 10^{-5} to 10^{-3} eV most interesting as dark matter (assume $\Theta_i \sim 1$ in axion potential); Axions can constitute CDM due to their non-thermal production).



Axions

axion would interact weakly with matter
(coupling $\propto f_A^{-1}$; very small if $f_A \gg v \rightarrow m_A \ll m_\pi$).

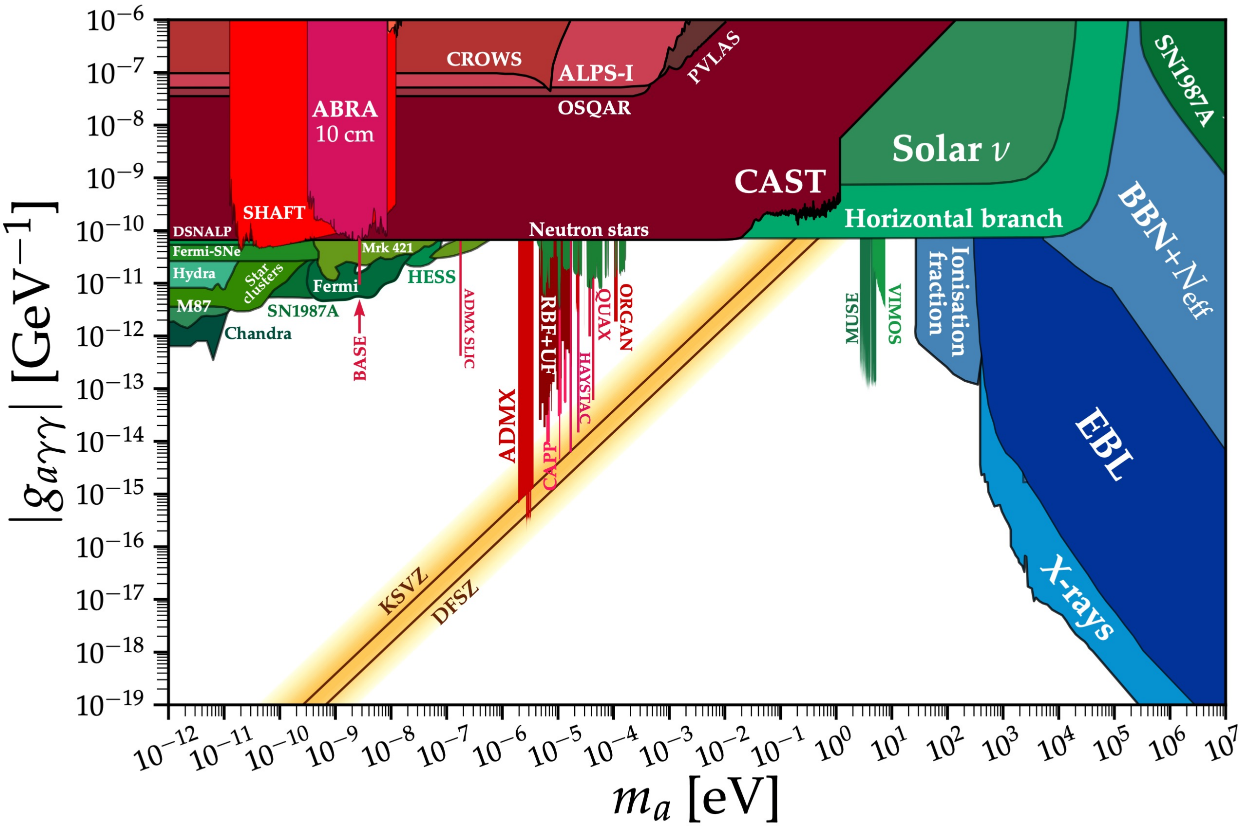
$$\mathcal{L}_{\text{QCD,CP viol}} \propto \left(\Theta_i - \phi_A / f_A\right) G^{\mu\nu a} \widetilde{G}^a_{\mu\nu} \approx 0 \xrightarrow{\Theta_i \sim 1} m_A f_A \approx m_\pi f_\pi$$

Next-to-next-to-leading (NNLO) order correction in chiral perturbation theory gives: $m_A = 5.691 \left(\frac{10^9 \text{ GeV}}{f_A}\right)$ meV

Predicted decay A → $\gamma\gamma$ in external E/B field presence (coupling $g_{A\gamma\gamma}$ very model dependent).

$$\mathcal{L}_{A\gamma\gamma} = \left(g_{A\gamma\gamma} / 4\right) F_{\mu\nu} \widetilde{F}^{\mu\nu} \phi_A = -g_{A\gamma\gamma} \mathbf{E} \cdot \mathbf{B} \phi_A$$

Also very small fermion (i.e. electron) coupling g_{Aff} possible



for more details see e.g. PDG review on ”Axions”

Energy scales & couplings

Physical Energy Scales & Couplings

Ex// scalar field (particle) φ

- most general Lagrangian

$$\mathcal{L} = \partial_\mu \varphi\, \partial^\mu \varphi - m^2\varphi^2 + \lambda_3 \varphi^3 + \lambda_4 \varphi^4 + \lambda_5 \varphi^5 + \lambda_6 \varphi^6 + \dots$$

dimensions

$$\left.\begin{aligned} [\mathcal{L}] &= \frac{E}{L^3} = E^4 \\ [\partial_\mu] &= \frac{1}{L} = E \end{aligned}\right\} \Rightarrow [\varphi] = E$$

couplings

$$[m^2] = E^2$$

$$[\lambda_3] = E$$

$$[\lambda_4] = E^0 \leftarrow \underline{\text{dimensionless}}$$

$$[\lambda_5] = E^{-1}$$

$$[\lambda_6] = E^{-2}$$

Energy scales & couplings

Energy dependence of amplitudes:

$\lambda_4 \rightarrow$

$\mathcal{A}(2\rightarrow 2) \propto \lambda_4$

$$\Rightarrow \boxed{\sigma_{2\rightarrow 2} \sim \frac{\lambda_4^2}{s}}$$

$\lambda_6 \rightarrow$

$\mathcal{A}(2\rightarrow 4) \sim \lambda_6 s$

$$\Rightarrow \boxed{\sigma_{2\rightarrow 4} \sim \lambda_6^2 s}$$

$\sigma \propto |\mathcal{A}|^2/\mathcal{F}$, where $\mathcal{F} \propto s$

... more generally

$$[\lambda_i] = E^{d_i} \Rightarrow d_i = 4 - i$$

(here)

dimensionless quantity ruling perturbative expansion is

$$\boxed{\bar{\lambda}_i = \lambda_i E^{-d_i}}$$

- weak coupling $\Leftrightarrow$ $\bar{\lambda}_i << 1$

$*$ $d_i > 0$ $\Rightarrow$ relevant at small E

Ex//

m

$*$ $d_i = 0$ $\Rightarrow$ relevant at all E

$*$ $d_i < 0$ $\Rightarrow$ suppressed at small E

perturbative expansion

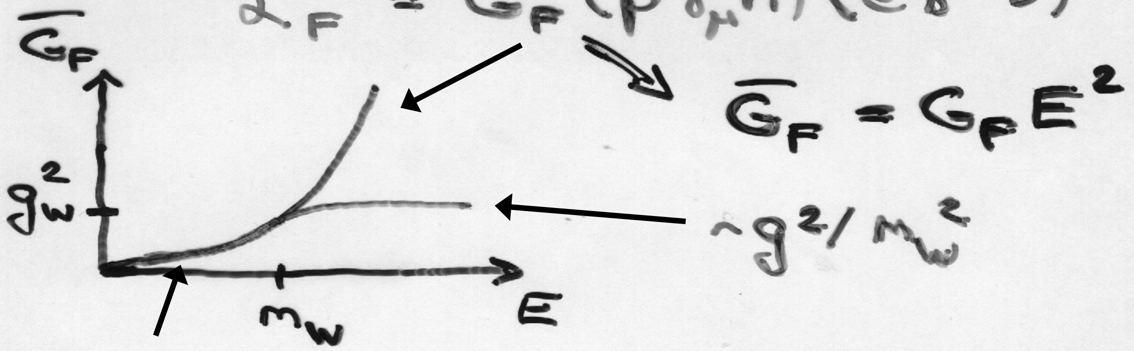
breaks down at high E

Beyond SM

Energy scales & couplings

▲ Ex// Fermi Lagrangian

$$\mathcal{L}_F = G_F (\bar{p}\gamma_\mu n)(\bar{e}\gamma^\mu \nu)$$



both work at low E

Imagine all couplings with $d_i < 0$ to scale like

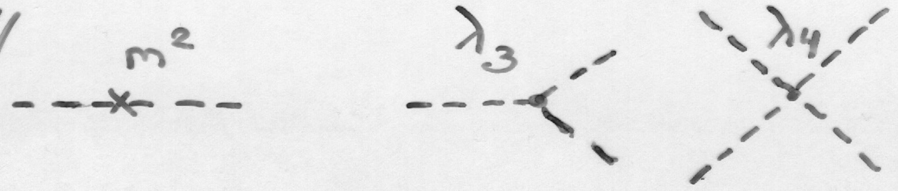
$$\lambda_i \sim \Lambda^{d_i}$$

Ex// $\lambda_5 \sim \frac{1}{\Lambda}$, $\lambda_6 \sim \frac{1}{\Lambda^2}$ . . .

⇓

at $E \ll \Lambda$ the dynamics is accurately described by a finite set of couplings with $d_i \geq 0$

Ex//



- $(m^2, \lambda_3, \lambda_4)$ fully describe an elementary (pointlike) particle

Energy scales & couplings

- $\lambda_5, \lambda_6 \ldots$ corresponds to inner structure
- to probe structure $E \sim \Lambda$ is needed

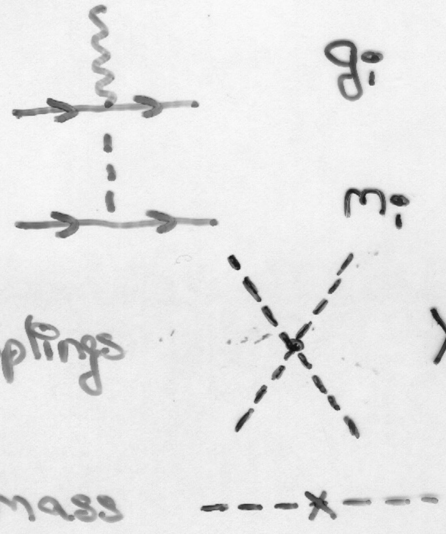
* Ex// φ is bound state of size $\sim \frac{1}{\Lambda}$

NB! Customary to assume New Physics couplings to be $g^n_{\mathrm{SM}}/\Lambda^n$, where n depends on order of term

▲ $\lambda_i = 0$ for $d_i < 0$ ⇒ Theory renormalizable (divergencies can be delt with)

Physical meaning: particles have the minimal amount of internal structure

Standard Model couplings:



gauge g_i

higgs m_i

self-couplings λ_i

} $d_i = 0$

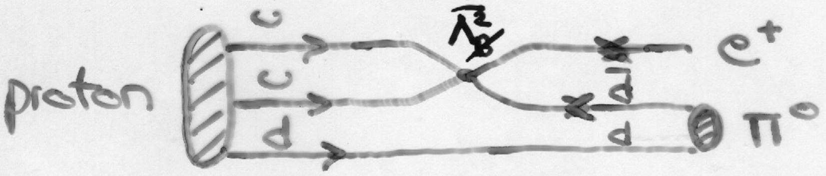
Higgs mass μ^2 $d_i = 2$

Energy scales & couplings

▨ Allowing $d_i < 0$ we would have

i) B- & L-number violation

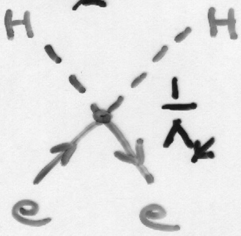
* $\frac{1}{\Lambda^2_{\not B}} (u_\alpha C \gamma_\mu u_\beta)(e C \gamma^\mu d_\delta)\epsilon^{\alpha\beta\delta}$



$p \rightarrow e^+ \pi^0$

$\tau_p \geq 24 \cdot 10^{33}$ years $\Rightarrow$ $\Lambda_{\not B} \geq 10^{16}$ GeV

* $\frac{1}{\Lambda_{\not L}} (e_i^A C e_j^B) H_A H_B$

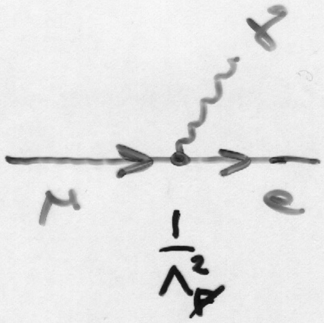


$H \leftrightarrow v_F$

$\longrightarrow$ $m_\nu \sim \frac{v_F^2}{\Lambda_{\not L}}$

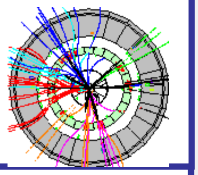
ν-oscillations: $m_\nu \sim 0.1$ eV $\Rightarrow$ $\Lambda_{\not L} \sim 10^{14}$ GeV

* $\frac{m_\mu}{\Lambda^2_{\not F}} (\bar e \gamma_\mu \gamma_\nu \mu) F^{\mu\nu}$



$Br(\mu \rightarrow e\gamma) < 4.2 \cdot 10^{-13}$

$\Rightarrow \Lambda_{\not F} \geq 10^7$ GeV



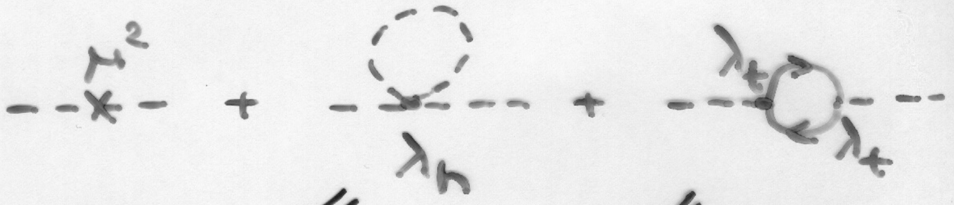
Energy scales & couplings

▲ It is tempting to conclude that the scale of "compositness" Λ in the SM is extremely high ... but can we

- $\langle H \rangle \sim$ 125 GeV $\Rightarrow \mu^2 \sim (125 \text{ GeV})^2$
- can $|\mu| \text{ be } \ll \Lambda$
- have to consider quantum corrections

leading:

$\mu^2_{\text{eff}} =$



cut-off $\int d^4p$ at $p \sim \Lambda$

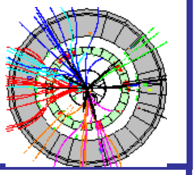
$$+\frac{\lambda_H}{16\pi^2}\Lambda^2 \qquad -\frac{\lambda_t^2}{16\pi^2}\Lambda^2$$

$$\Rightarrow \mu^2_{\text{eff}} = \mu^2 + c\Lambda^2$$

▨ μ^2_{eff} does not like to stay small when $\Lambda \to \infty$!!

large $\Lambda \Rightarrow \mu^2$ must be tuned to make μ^2_{eff} (fine-tuning 10^{-34})

This is the hierarchy problem!



Energy scales & couplings

$p \to e \pi^0$
$\mu \to e \gamma$ $\sim \frac{1}{\Lambda^2}$

effective Higgs mass $\sim \mu^2 + c\Lambda^2$

▨ 2 possibilities

1) $\Lambda \gg \mu$
- → B, L conservation naturally follows
- → separation of mass scales mystery

2) SM is not valid for energy $\gtrsim \mu$: it is replaced by more fundamental theory

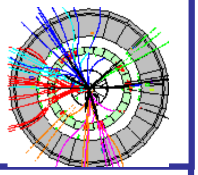
In New Theory
- no Λ^2 corrections to Higgs mass
- must preserve as much as possible good features of SM

How solve Λ^2 corrections to Higgs mass?

additional

loops cancels if $\lambda_t^2 = \lambda_{\tilde{t}}$ ⇒ Need symmetry relating boson to fermions

SUPERSYMMETRY



Top quark

- top quark (t) discovered by the CDF & D∅ experiments at the Tevatron in 1995 ($SU(2)_L$ partner of the b quark)
- a most intriguing fermion : $m_{top} \approx 172.7$ GeV (heaviest known fundamental particle, × 40 heavier than b quark) → clues about origin of particle masses ($h_{top} = m_{top}/v_{EW} \sim 1$)
- top decays instantaneously and almost exclusively to W boson + b quark, $\Gamma\,(t \to Wb) \sim 1.4$ GeV » Λ_{QCD}

- no hadronization (no toponium or T mesons)
- top decay purely an electroweak process

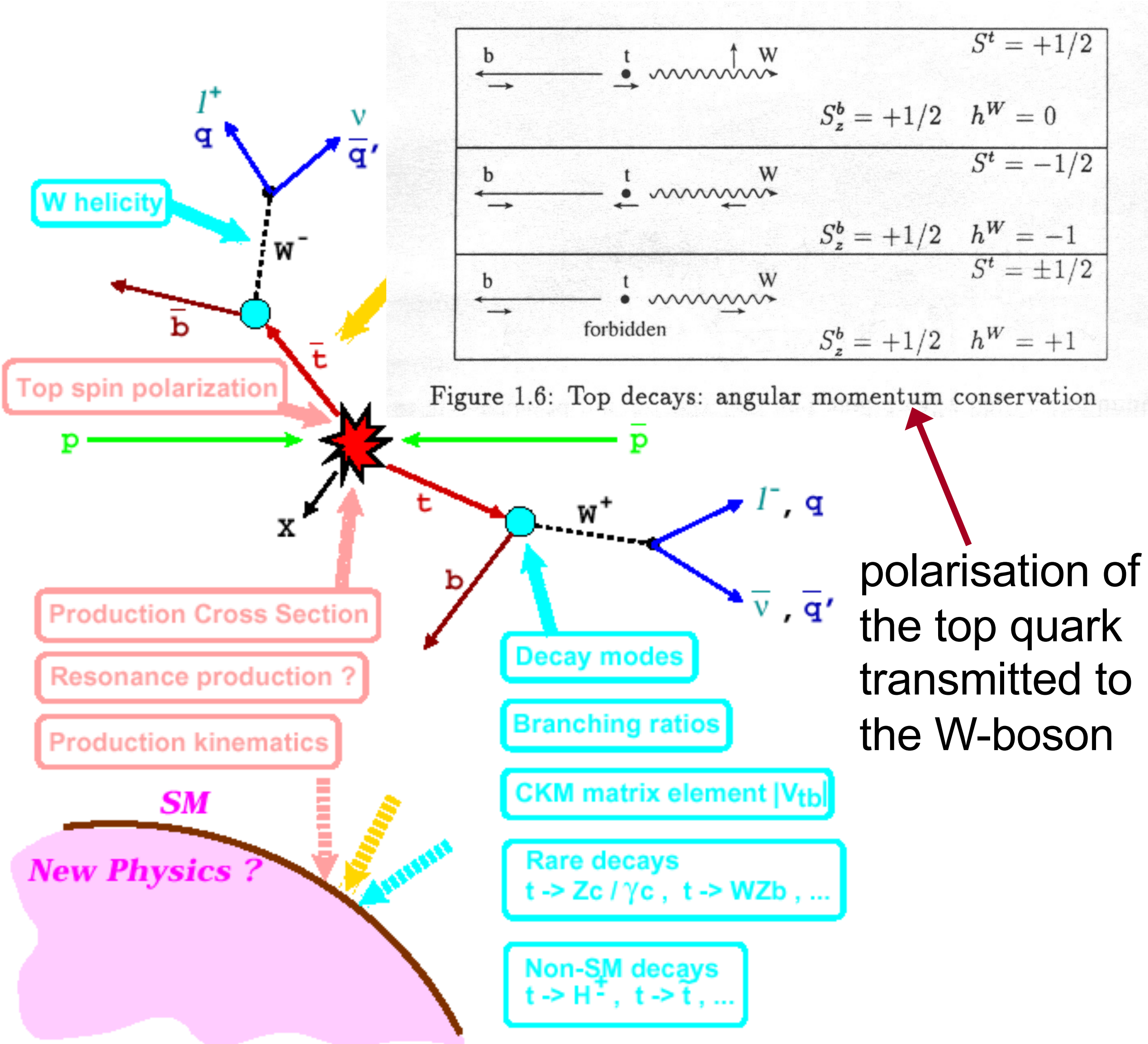


Figure 1.6: Top decays: angular momentum conservation

W helicity

Top spin polarization

Production Cross Section

Resonance production ?

Production kinematics

Decay modes

Branching ratios

CKM matrix element $|V_{tb}|$

Rare decays
t -> Zc / γc , t -> WZb , ...

Non-SM decays
t -> H^+ , t -> $\tilde{t}$, ...

polarisation of the top quark transmitted to the W-boson

Electroweak symmetry breaking

In Standard Model we have the Higgs mechanism to explain the masses of W & Z

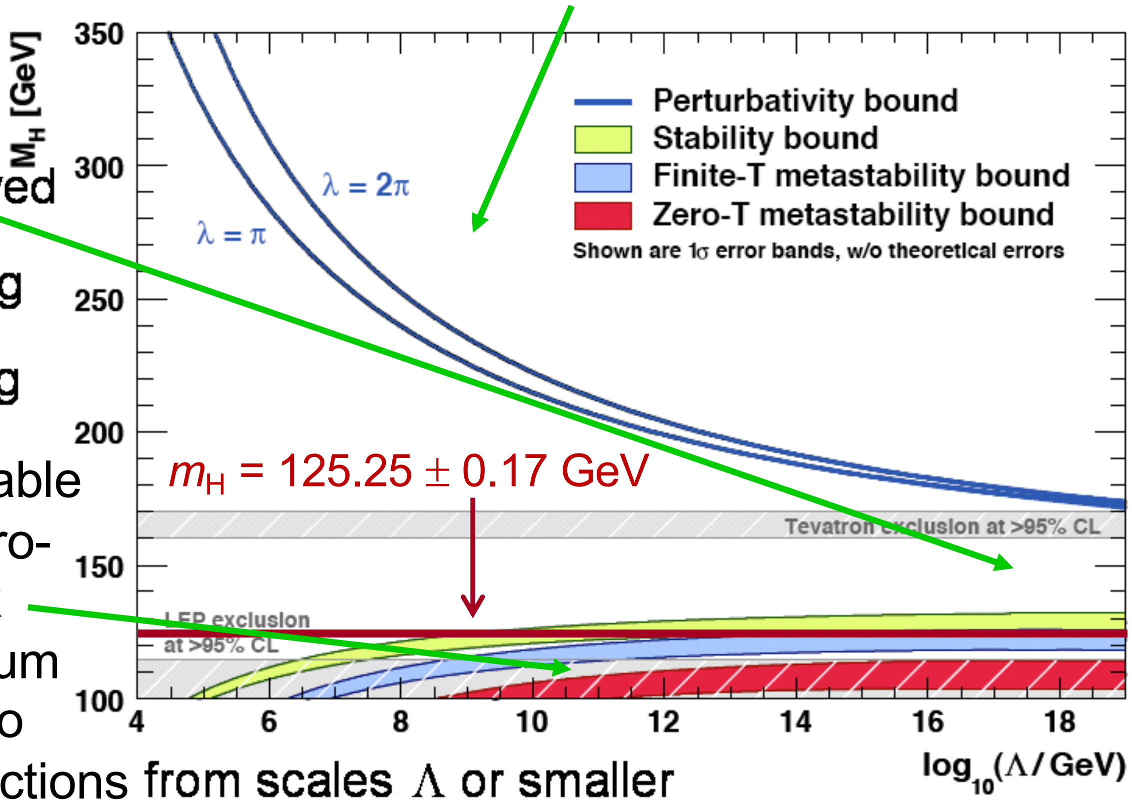
Direct observation of Higgs-like particle by ATLAS & CMS experiments @LHC; $m_H = 125.25 \pm 0.17$ GeV

Current wisdom: Electroweak symmetry breaking generates longitudinal degrees of freedom for W & Z

Assume there is nothing beyond SM; will that work?

Apparently not due to unstable electroweak vacuum!!

Unsatisfactory high-energy behaviour of Higgs quartic coupling λ if M_H is too large.



Allowed with strong fine-tuning

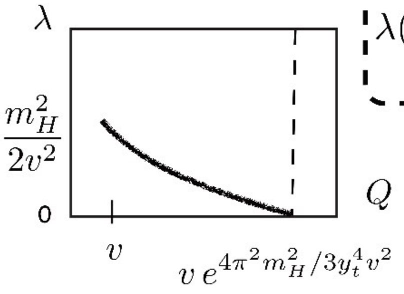
Unstable electroweak vacuum due to corrections from scales Λ or smaller

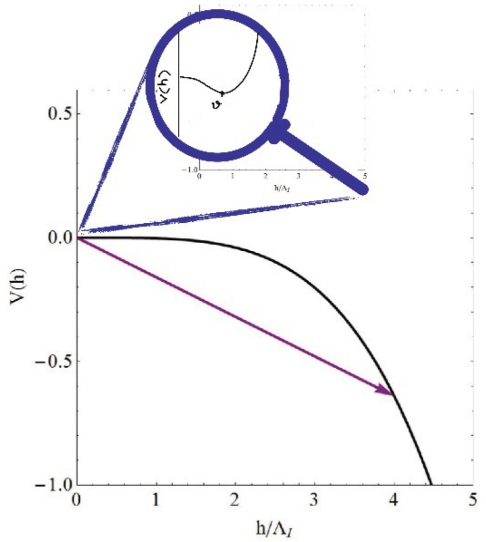
Higgs Stability

Small mass (y_t dominated RGE)

$$\lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln \frac{Q}{Q_0}}{1 - \frac{9}{16\pi^2} y_0^2 \ln \frac{Q}{Q_0}}$$

Linde '76, '80
Weinberg '76
Maini et al '78, '79
Politzer, Wolfram '79
Lindner '86
+...





$\lambda < 0$ ⇛ potential unbounded from below

$$\Lambda \leq v\, e^{4\pi^2 m_H^2/3y_t^4 v^2}$$

New physics should appear before
that point to restore stability

Small mass (y_t dominated RGE)

$$\lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln \frac{Q}{Q_0}}{1 - \frac{9}{16\pi^2} y_0^2 \ln \frac{Q}{Q_0}}$$

Linde '76, '80
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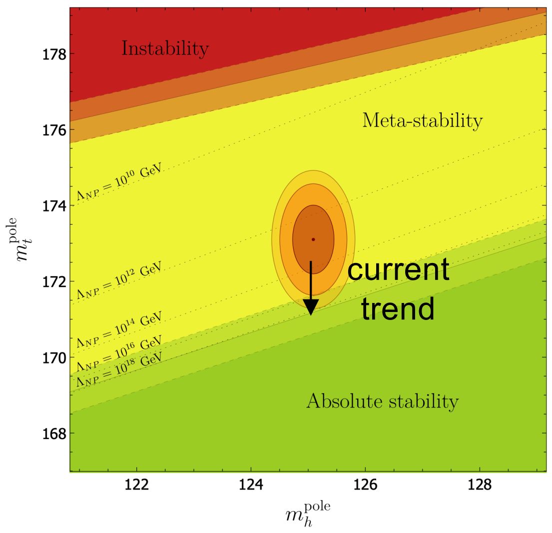
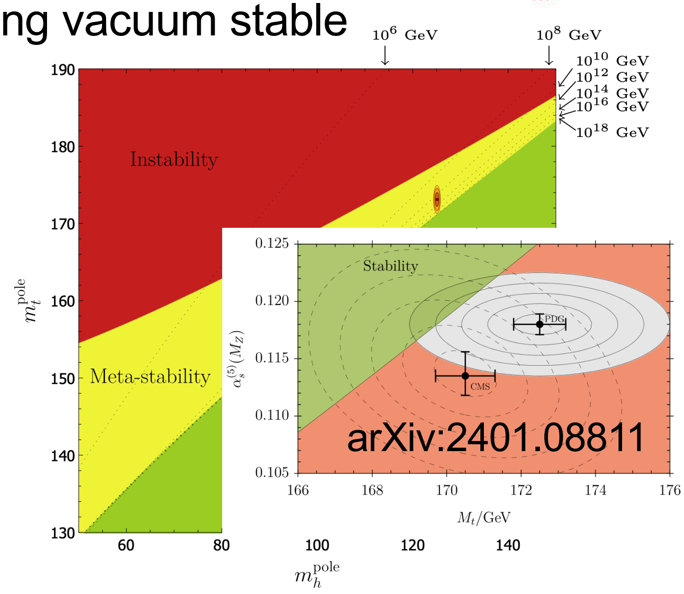
Lindner '86
+...

A. Andreassen et al., PRD 97 (2018) 056006

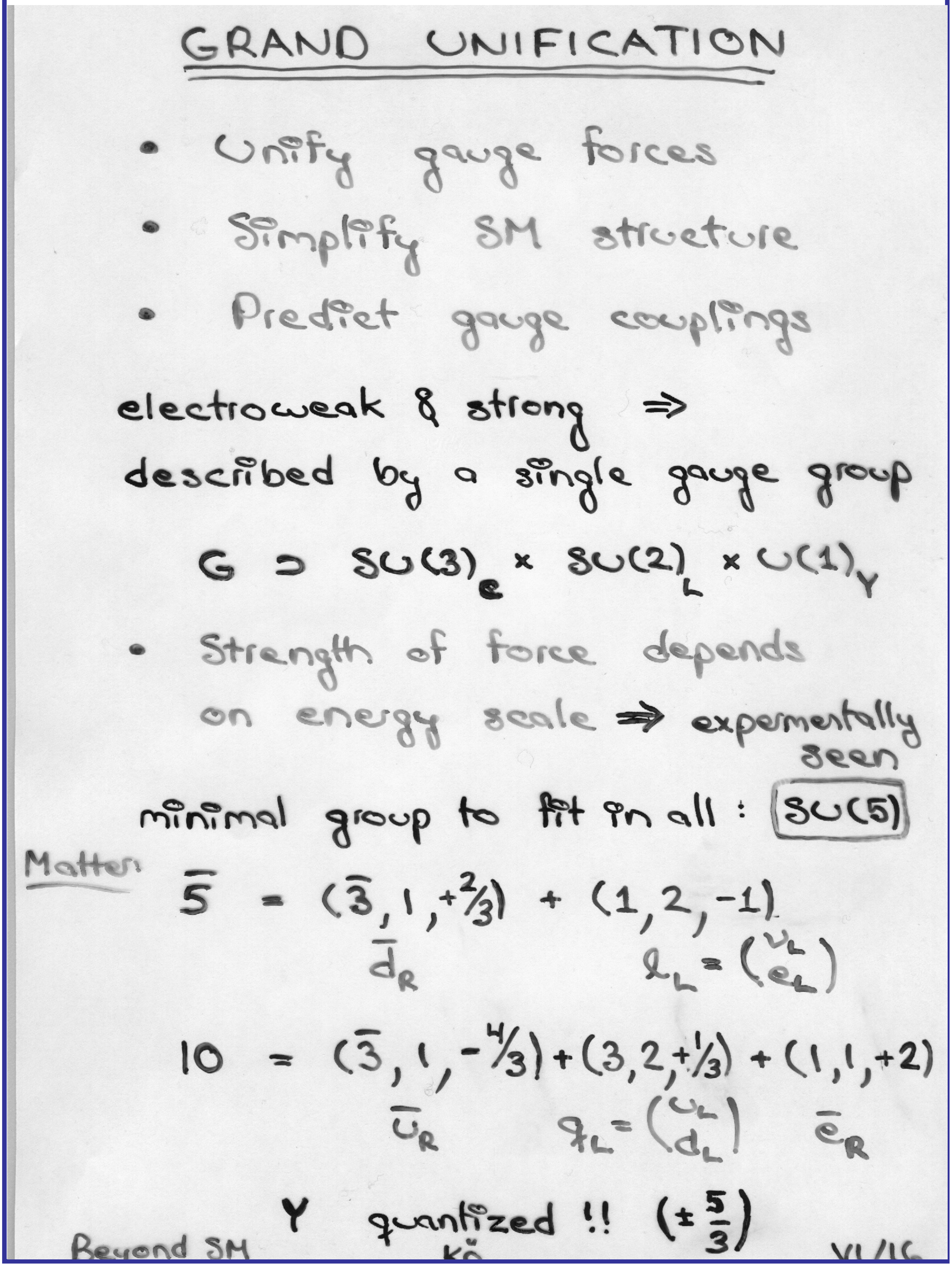
Λ_{NP}= new physics scale making vacuum stable





arXiv:2401.08811

Grand unified theories



GRAND UNIFICATION

- Unify gauge forces
- Simplify SM structure
- Predict gauge couplings

electroweak & strong ⇒ described by a single gauge group

$$G \supset SU(3)_c \times SU(2)_L \times U(1)_Y$$

- Strength of force depends on energy scale ⇒ expermentally seen

minimal group to fit in all : $\boxed{SU(5)}$

Matter:

$$\bar{5} = \underset{\bar{d}_R}{(\bar{3},1,+2/3)} + \underset{\ell_L = \binom{\nu_L}{e_L}}{(1,2,-1)}$$

$$10 = \underset{\bar{u}_R}{(\bar{3},1,-4/3)} + \underset{q_L = \binom{u_L}{d_L}}{(3,2,+1/3)} + \underset{\bar{e}_R}{(1,1,+2)}$$

Y quantized !! $(\pm \frac{5}{3})$

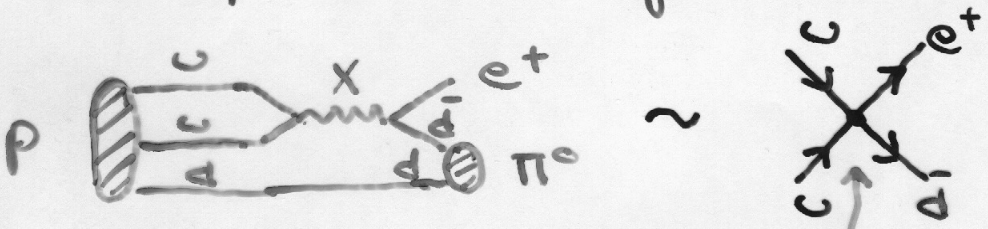
Beyond SM KÖ VI/16

Grand unified theories

- Quarks & leptons belong to same representation ⇒ X and Y violate B-, L-number

New Phenomena not in SM ⇒ p-decay, $n-\bar{n}$ oscillations, ν masses ...

▲ proton decay



$$\tau_p \sim \frac{M_X^4}{m_p^5} \qquad \frac{1}{\Lambda^2_{\not B}} = \frac{1}{2}\frac{g_5^2}{M_X^2}$$

$\tau_p \geq 24 \cdot 10^{33}$ years ⇒ $M_X \geq 10^{16}$ GeV

NB! $M_X \sim 10^{16}$ GeV ≡ "GUT" scale

⌜ X, Y might also help baryogenesis if they slightly violate CP ⌟

Grand unified theories

Gauge:

$$24 = \overset{g}{(8,1,0)} + \overset{W^{\pm},W^{0}}{(1,3,0)} + \overset{B^{0}}{(1,1,0)}$$

$$+ \underset{X,Y}{(3,2,-\tfrac{5}{3})} + \underset{Y,X}{(\bar{3},2,\tfrac{5}{3})}$$

X, Y new gauge boson with colour and weak charge!

- Unification of forces and (partial) unification of matter

- Evidence: purely suggestive + gauge couplings

New symmetry ⇒ relation between couplings

▲ SU(5) Higgs mechanism

$$SU(5) \xrightarrow{\text{broken}} SU(3)_c \times SU(2)_L \times U(1)_Y$$

⇒ X and Y get (large) mass

For more details see PDG review on Grand unified theories
https://pdg.lbl.gov/2023/reviews/rpp2023-rev-guts.pdf

Gauge couplings

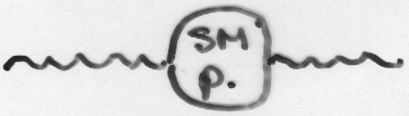
- ▲ unbroken SU(5) ⇒

$$\boxed{\begin{array}{cccc} g_3 = & g_2 = & \underbrace{\sqrt{\tfrac{5}{3}}\, \overset{g'}{\overset{\shortparallel}{g_Y}}}_{} & = g_5 \\ \shortparallel & \shortparallel & & \\ g_s & g & g_1 & \end{array}}$$

$$\sin^2\Theta_W(M_{GUT}) = \frac{g_Y}{g_2+g_Y} = \frac{3/5}{1+3/5} = 0.375$$

but couplings depend on energy

- ▲ must compare SU(5) prediction at $E \sim M_X$ with what we observe at $E \sim m_Z$
  - Standard Model



⇒ $g_i(E)$

$$* \quad \left.\begin{array}{l} \alpha_{em}^{-1}(m_Z) = 127.8 \pm 0.1 \\ g_3^2(m_Z) = 1.50 \pm 0.05 \end{array}\right\} \Rightarrow$$

$$\sin^2\Theta_W = 0.210 \pm 0.003 \neq$$

$$\sin^2\Theta_W^{exp} = 0.2315 \pm 0.005$$

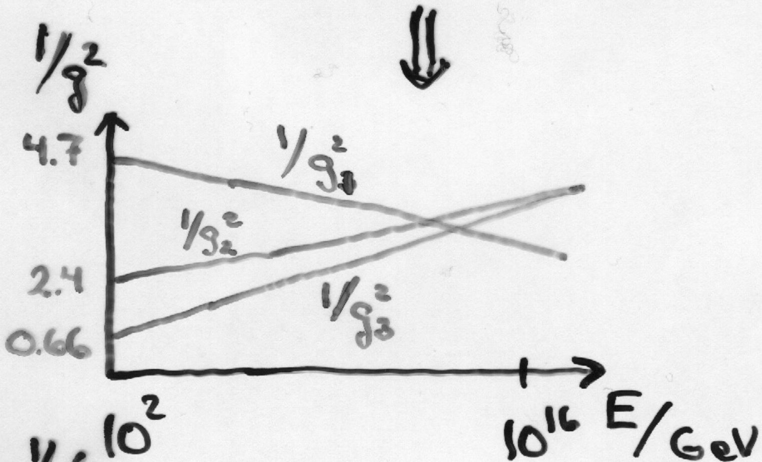
Grand unified theories

- Supersymmetric Standard Model

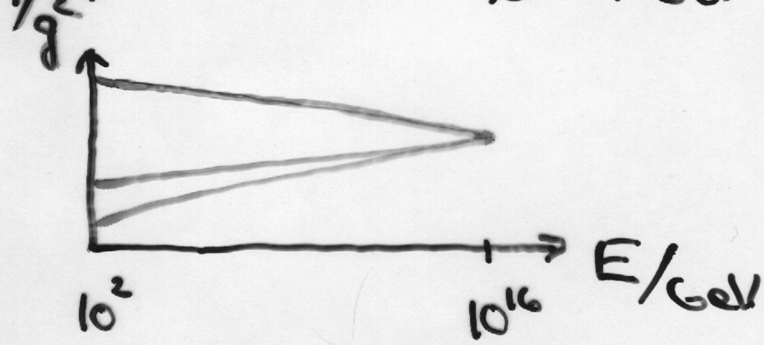
$$\left.\begin{array}{l}\alpha_{em}(m_Z)\\ g_s^2(m_Z)\end{array}\right\} \quad \sin^2\theta_W = 0.2334 \pm 0.005$$

$$\approx \sin^2\theta_W^{exp}$$

$$\Downarrow$$

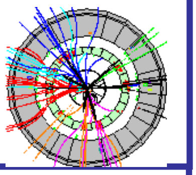


SM



SSM

$$\frac{1}{g^2(Q^2)} - \frac{1}{g^2(m_Z^2)} \propto b \log \frac{Q^2}{m_Z^2}$$



Motivation for supersymmetry

MOTIVATION

Why to go Beyond the Standard Model?

- Standard Model is an effective theory: ~20 parameters to be fixed by experiments.

- SM includes only part of the fundamental interactions: Gravity is missing.

$\Longrightarrow$

Hierarchy problem: $m_W/m_{Planck} \sim 10^{-17}$.

- Quantum corrections to particle masses:

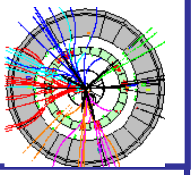
Fermions

E.g. QED:

$$L_{electron} = \bar{e}\not{D}e + m_e\bar{e}_L e_R + m_e\bar{e}_R e_L.$$

- $m_e = 0 \rightarrow$ chiral symmetry:
  $e_L \rightarrow e_L,\ e_R \rightarrow e^{i\alpha}e_R$
- $m_e \neq 0$, quantum corrections:

$\delta m_e = 3\alpha_{em}\log(E/\Lambda)/4\pi\cdot m_e$ (small even if $\Lambda = M_P$)



Motivation for supersymmetry

Photon

Mass term $m_\gamma^2 A_\mu A^\mu$ not invariant under gauge symmetry.
$\Longrightarrow m_\gamma = 0$

Symmetries keep fermions and γ light.

Scalars

BSM physics comes to play at Λ = physical upper limit in the quantum corrections.

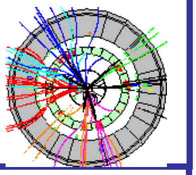
For scalar particles:

$$\delta m^2 \sim \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \left\{ \qquad - \qquad \right\}$$

$$\sim \frac{\lambda_B}{16\pi^2}\Lambda^2 - \frac{h_f^2}{16\pi^2}\Lambda^2 \sim \Lambda^2.$$

Should be $m_H \sim$ electroweak scale.

Motivation for supersymmetry



- With suitable symmetry bosonic and fermionic contributions cancel!
$\Longrightarrow$ Supersymmetry

- Volkov, Akulov, 1973;
Wess, Zumino, 1974

- In nature supersymmetric partners of the SM particles have not been seen $\Rightarrow$ SUSY must be broken. To solve the naturalness problem, must be

$$|m_B^2 - m_F^2| < \mathcal{O}(1\ \mathrm{TeV}^2).$$

$\Rightarrow$ low-energy supersymmetry

- Generators of supersymmetry, translations and Lorentz-transformations satisfy a common algebra.
Consistency $\longrightarrow$ supersymmetry is local.
$\Longrightarrow$ Supergravity (which includes gravity)

- Important ingredient in superstring theories.

For more details see PDG review on Supersymmetry: theory
https://pdg.lbl.gov/2023/reviews/rpp2023-rev-susy-1-theory.pdf

SYPERSYMMETRY MODELS

Bosons: commutation relations
Fermions: anticommutation relations.

- An indefinite number of bosons can exist at the same place at the same time, whereas only one fermion can be in any given place at a given time.
- The matter is made of fermions, while the forces are associated with bosons.

Symmetries come in two types: external (or space-time) and internal symmetries.

- Internal symmetries include the Standard Model symmetry.
- External symmetries include invariance under Lorentz transformations.
- Particle spin is an external symmetry, while isospin is not based on Lorentz invariance and is an internal symmetry.

Supersymmetric models

Translations

$$x^\mu \longrightarrow x^\mu + a^\mu \Longrightarrow$$
$$\Phi(x) \longrightarrow \Phi(x+a);\ \delta\Phi = a^\mu \partial_\mu \Phi$$

Lorentz

$$x^\mu \longrightarrow A^\mu_\nu x^\nu;\ x^\mu x_\mu \text{ invariant} \Longrightarrow$$
$$\delta\Phi = A^\mu_\nu x^\nu \partial_\mu \Phi$$

- SUSY is a space-time symmetry.
- A supersymmetry operation alters particle spin by 1/2, changing bosons into fermions and vice versa.
- Supersymmetry is the first symmetry that can unify matter and force.

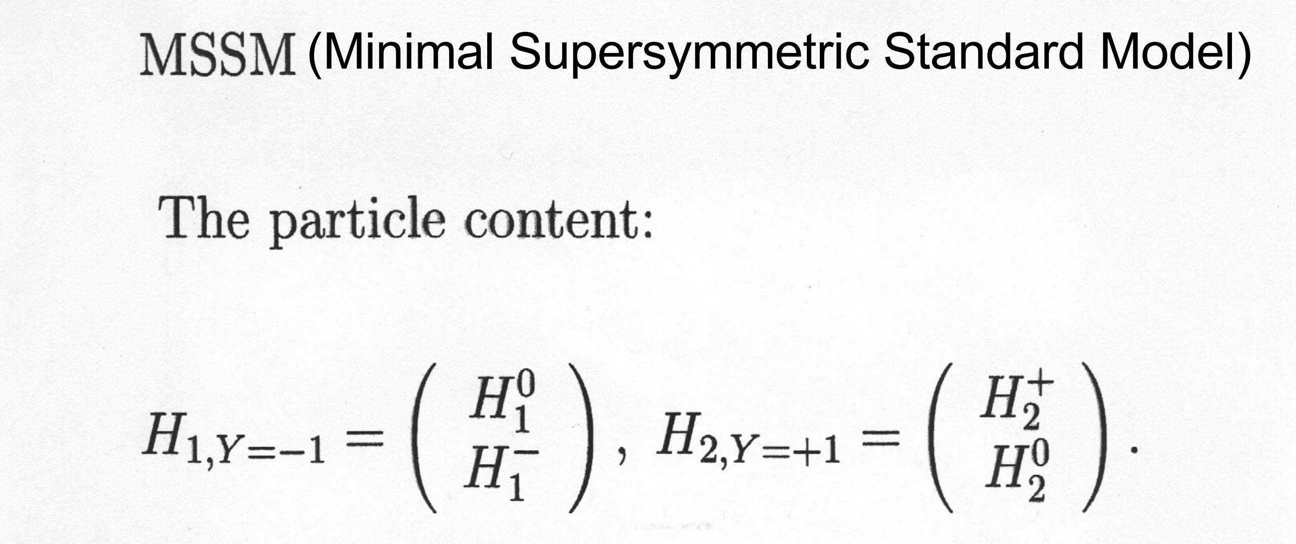
Supersymmetry

$$\begin{aligned} \delta\phi &= \bar{\xi}(1-\gamma_5)\psi \\ \delta\psi &= -i\gamma^\mu(1+\gamma_5)\xi\partial_\mu\phi \end{aligned}$$

ξ is fermionic, analogue of a^μ and A^μ_ν

$$\delta_2\delta_1\phi = \underbrace{\bar{\xi}_1\gamma^\mu(1+\gamma_5)\xi_2}_{\equiv a^\mu \ \Rightarrow\ (\delta\text{susy})^2 \sim \text{translation}}\partial^\mu\phi$$

Particles in supersymmetric models



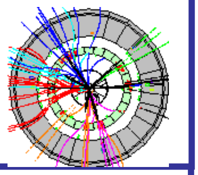
MSSM (Minimal Supersymmetric Standard Model)

The particle content:

$$H_{1,Y=-1} = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \; H_{2,Y=+1} = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}.$$

Field Content of the MSSM

Super-multiplets	Super-field	Bosonic fields	Fermionic partners	SU(3)	SU(2)	U(1)
gluon/gluino	$\widehat{V}_8$	g	$\widetilde{g}$	8	1	0
gauge/	$\widehat{V}$	$W^{\pm}, W^0$	$\widetilde{W}^{\pm}, \widetilde{W}^0$	1	3	0
gaugino	$\widehat{V}'$	B	$\widetilde{B}$	1	1	0
slepton/	$\widehat{L}$	$(\widetilde{\nu}_L, \widetilde{e}^-_L)$	$(\nu, e^-)_L$	1	2	-1
lepton	$\widehat{E}^c$	$\widetilde{e}^-_R$	e^-_R	1	1	-2
squark/	$\widehat{Q}$	$(\widetilde{u}_L, \widetilde{d}_L)$	$(u, d)_L$	3	2	1/3
quark	$\widehat{U}^c$	$\widetilde{u}_R$	u_R	3	1	4/3
	$\widehat{D}^c$	$\widetilde{d}_R$	d_R	3	1	$-2/3$
Higgs/	$\widehat{H}_d$	(H^0_d, H^-_d)	$(\widetilde{H}^0_d, \widetilde{H}^-_d)$	1	2	-1
higgsino	$\widehat{H}_u$	(H^+_u, H^0_u)	$(\widetilde{H}^+_u, \widetilde{H}^0_u)$	1	2	1



Particles in supersymmetric models

MSSM (Minimal Supersymmetric Standard Model)

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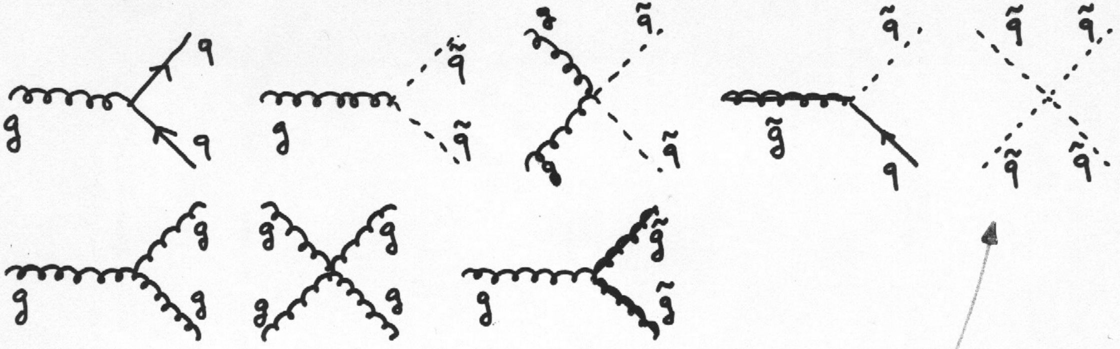
particle	sparticle			
	weak interaction eigenstate		mass eigenstate	
$q = u, d, c, s, t, b$	$\tilde{q}_L, \tilde{q}_R$	squark	$\tilde{q}_1, \tilde{q}_2$	squark
$l = e, \mu, \tau$	$\tilde{l}_L, \tilde{l}_R$	slepton	$\tilde{l}_1, \tilde{l}_2$	slepton
$\nu = \nu_e, \nu_\mu, \nu_\tau$	$\tilde{\nu}$	sneutrino	$\tilde{\nu}$	sneutrino
g	$\tilde{g}$	gluino	$\tilde{g}$	gluino
$W^\pm$	$\tilde{W}^\pm$	wino		
H_1^+	$\tilde{H}_1^+$	higgsino	$\tilde{\chi}^\pm_{1,2}$	chargino ($\chi^\pm_1$ lightest, $\chi^\pm_2$ next lightest …)
H_2^-	$\tilde{H}_2^-$	higgsino		
γ	$\tilde{\gamma}$	photino		
Z	$\tilde{Z}$	zino		
H_1^0	$\tilde{H}_1^0$	higgsino	$\tilde{\chi}^0_{1,2,3,4}$	neutralino (χ^0_1 lightest, χ^0_2 next lightest …)
H_2^0	$\tilde{H}_2^0$	higgsino		
g_2	$\tilde{g}_{3/2}$	gravitino	(only in supergravity)	

Supersymmetric gauge interactions

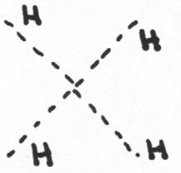
Supersymmetric gauge interactions

SU(3):



- all vertices controlled by g_s
- quartic scalar vertex $\propto i g_s^2$

* Higgs quartic coupling determined by gauge interactions



$\sim i\,(g_2^2 + g_Y^2)$

- In the Standard Model

$= i\lambda_H$ = free parameter

$m_H = \sqrt{\lambda_H}\, v_F$

* in SUSY $m_H \propto \sqrt{g_2^2 + g_Y^2}\; v = m_Z$

Both vertex couplings & supersymmetric diagrams (to first order) identical to Standard Model ones, only ”dressed” by supersymmetric partners NB! Higgs vertex couplings modified.

Supersymmetric model parameters

Parameters of the models

Supersymmetric parameters:

- $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge couplings g_3, g_2, g_1
- Higgs-fermion coupling matrices h_u, h_d, h_e.
- Higgs mixing parameter μ

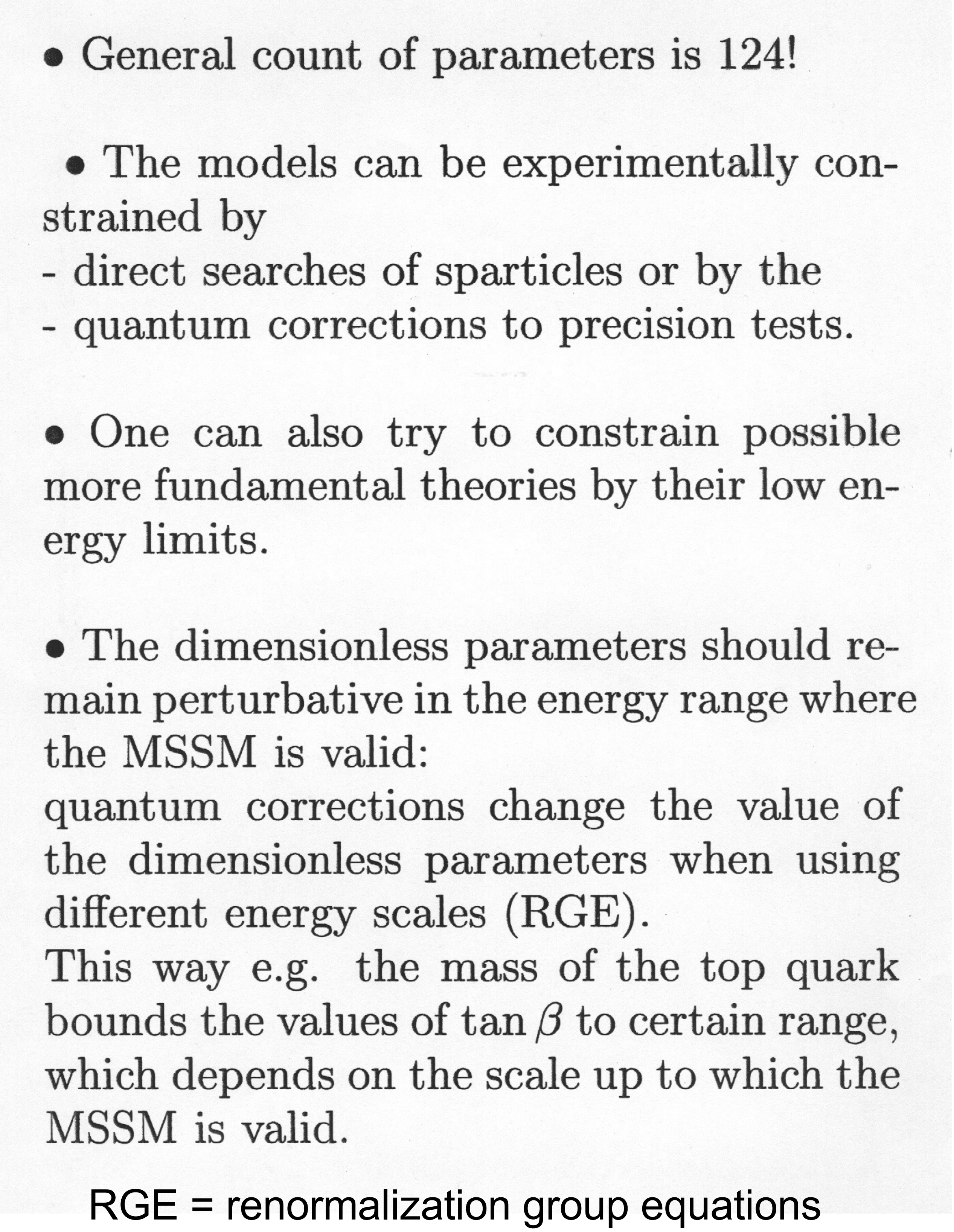
Soft breaking parameters (these do not bring back the quadratic divergences):

- gaugino masses M_3, M_2, M_1 (= M of SU(3), SU(2), U(1) gauginos i.e. gluinos, wino/zino, bino)
- $\tilde{q}$, $\tilde{l}$ masses $M^2_{\tilde{Q}}$, $M^2_{\tilde{U}}$, $M^2_{\tilde{D}}$, $M^2_{\tilde{L}}$, $M^2_{\tilde{E}}$
- Higgs masses m_1^2, m_2^2, b (b = soft Higgs mixing parameter)
- $H - \tilde{q} - \tilde{q}$, $H - \tilde{l} - \tilde{l}$ interaction parameters A_U, A_D, A_E

Instead of m_i^2, use Higgs VEVs v_1, v_2 and mass of one neutral Higgs (m_A).

From the known m_W, $v_1^2 + v_2^2 = (246\,\text{GeV})^2$
$\Longrightarrow$ the free parameter is $\tan\beta = v_2/v_1$.

Supersymmetric model parameters



- General count of parameters is 124!

- The models can be experimentally constrained by
  - direct searches of sparticles or by the
  - quantum corrections to precision tests.

- One can also try to constrain possible more fundamental theories by their low energy limits.

- The dimensionless parameters should remain perturbative in the energy range where the MSSM is valid:
quantum corrections change the value of the dimensionless parameters when using different energy scales (RGE).
This way e.g. the mass of the top quark bounds the values of tan β to certain range, which depends on the scale up to which the MSSM is valid.

RGE = renormalization group equations

Supersymmetric model parameters

allowed mass range for top quark as function of tanβ assuming that SUSY valid up to $\Lambda = 10^{16}$ GeV

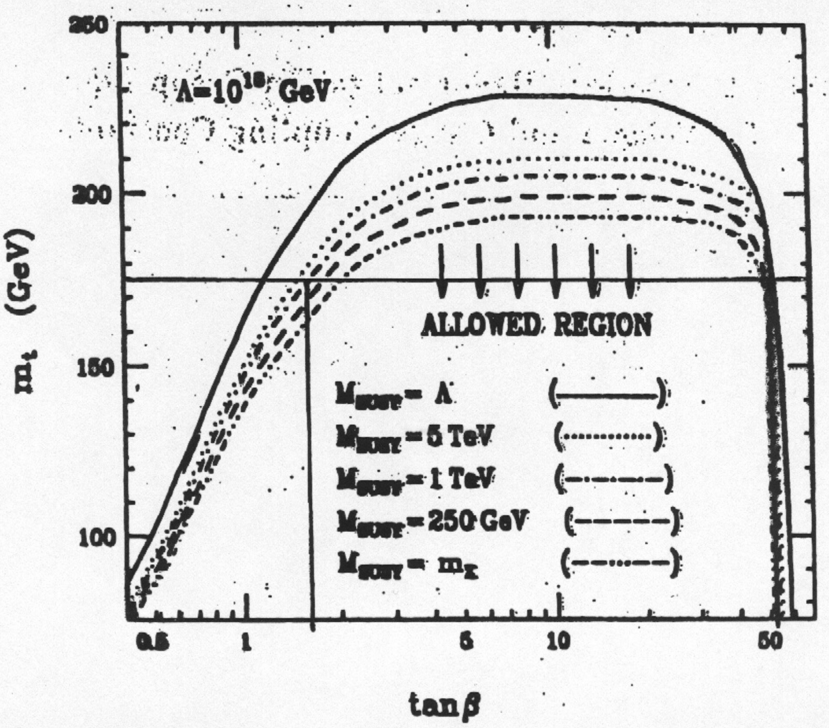
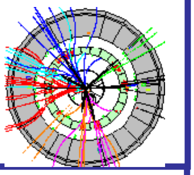


Fig. 1. The region of tanβ–m_t parameter space in which all running Higgs-fermion Yukawa couplings remain finite at all energy scales, μ, from m_Z to $\Lambda = 10^{16}$ GeV [79]. Non-supersymmetric two-Higgs-doublet (one-loop) renormalization group equations (RGEs) are used for $m_Z \le \mu \le M_{SUSY}$ and the RGEs of the minimal supersymmetric model are used for $M_{SUSY} \le \mu \le \Lambda$ (see table 2). Five different values of M_{SUSY} are shown; the allowed parameter space lies below the respective curves.

Supersymmetric model parameters



- Interestingly in the MSSM the gauge couplings change in such a way that they seem to unite at certain energy scale.
This does not happen in the SM.

- Assume GUT, e.g. SUSY-SU(5):

$$\begin{aligned} M_{susy} &= 10^{3.4}\,\mathrm{GeV}, \\ M_X &= 10^{15.8}\,\mathrm{GeV}, \\ \alpha_U^{-1} &= 26.3. \end{aligned}$$

- It is often assumed that in addition to the gauge coupling constants also the gaugino masses, scalar masses and trilinear A-terms unify at the GUT scale or Planck scale:

”constrained” MSSM (CMSSM):

$$g_1(M_X) = g_2(M_X) = g_3(M_X) = g_U, \quad \text{couplings}$$

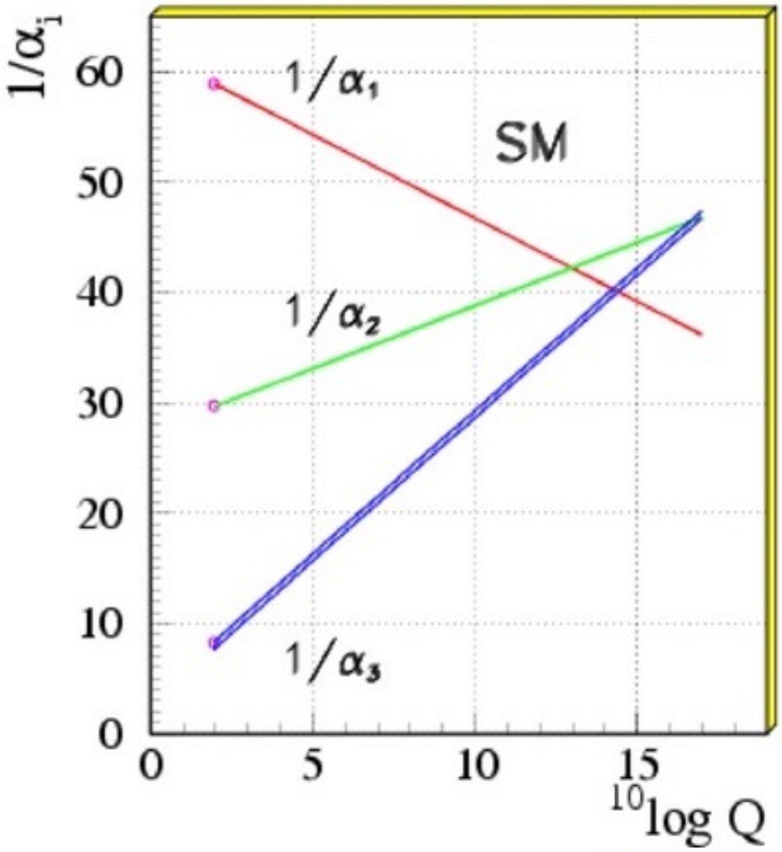
$$M_1(M_X) = M_2(M_X) = M_3(M_X) = m_{1/2}, \quad \text{gaugino masses}$$

$$\left.\begin{aligned} M^2_{\tilde{Q}}(M_X) = \ldots = M^2_{\tilde{E}}(M_X) = m_0^2 1, \\ m_1^2(M_X) = m_2^2(M_X) = m_0^2, \end{aligned}\right\} \quad \text{scalar masses}$$

$$A_U(M_X) = A_D(M_X) = A_E(M_X) = A_0 1. \quad \text{Higgs-sfermion couplings}$$

• couplings unite at m_{GI}

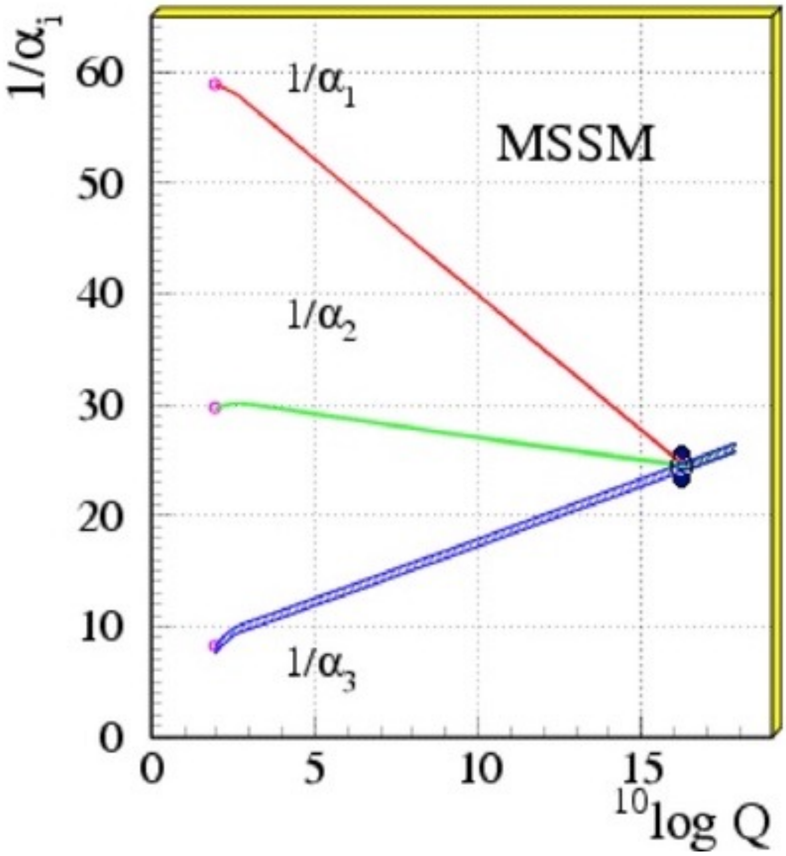
Supersymmetric model parameters

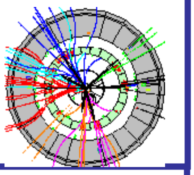


All couplings unite at a specific m_{GUT} in Minimal Supersymmetric Standard Model (MSSM) but not in the Standard Model (SM)

• m(

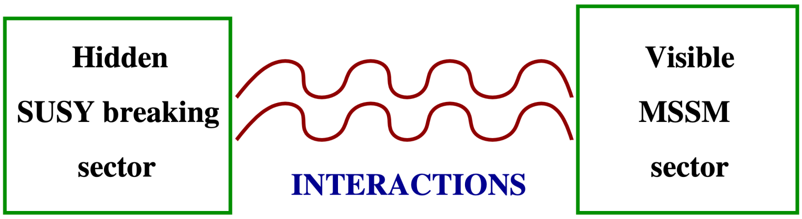
Predicted m_{GUT} in MSSM larger than in SM and therefore more consistent with the lower limits on the proton lifetime from experiments.





SUPERSYMMETRY BREAKING

- spin-3/2 fermion gravitino, whose mass is related to supersymmetry breaking scale $\Lambda_S = \sqrt{3 m_{gravitino} M_P}$
- breaking in hidden sector, messenger transmission with coupling λ_I



Gravity mediated supersymmetry breaking

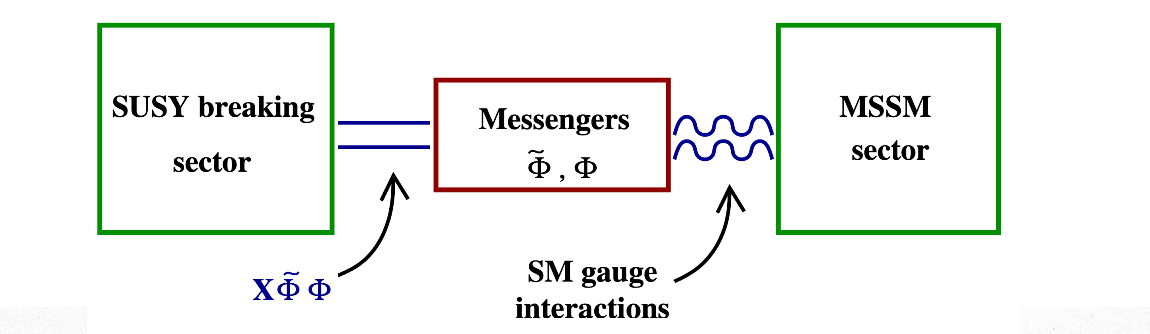
- Supersymmetry breaking is mediated from the hidden sector by interactions which are of gravitational strength.
- Appealing: Gravity exists anyway.
  superpartner mass splitting: $\Delta m^2 \sim \lambda_I \Lambda_S^2$
- $m_{gravitino} \sim$ electroweak scale, but interactions suppressed by M_{Planck}. $\Rightarrow$ large Λ_S + very small λ_I
- mSUGRA (Best studied SUSY model):
  - Radiative symmetry breaking assumed $\Longrightarrow |\mu|$ determined. (through radiative corrections)

$$\{sgn(\mu),\ m_0,\ M_{1/2},\ A_0,\ \tan(\beta)\}$$

- The lightest neutralino (or sneutrino) is the LSP.
  LSP = lightest supersymmetric particle

Supersymmetry breaking

Gauge mediated supersymmetry breaking



SUSY breaking sector

Messengers
$\tilde{\Phi}$, Φ

MSSM sector

$X\tilde{\Phi}\,\Phi$

SM gauge interactions

- Supersymmetry breaking is transmitted to MSSM via gauge interactions.
- Messenger sector has particles with SM quantum numbers.Gaugino masses:

black (green) lines MSSM (messenger) fields

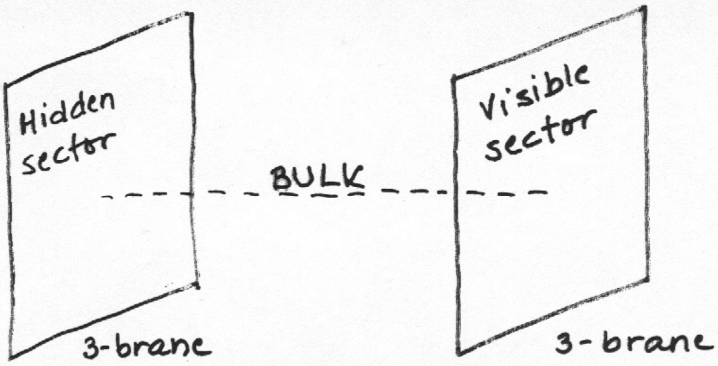
- The parameters are chosen so that the gauge mediated effects dominate over the gravity mediated effects
$\Longrightarrow m_{gravitino} \sim$ eV-keV $\Rightarrow \lambda_I \sim 1$
- Parameters: (mGMSB)

SUSY breaking scale

$$\{\Lambda,\ \underbrace{M_m}_{\text{messenger scale}},\ \tan(\beta),\ \underbrace{n_5}_{\text{\# of complete reps. of SU(5) in messenger sector}},\ sign(\mu)\}.$$

- The gravitino is the LSP.

Scenarios with higher dimensional space time



- Assume that the visible fields are in one 3+1 dimensional brane, while the hidden sector is on another brane.

$\Longrightarrow$ Anomaly mediated and gaugino mediated models. (supersymmetry breaking transmitted through fields that live in the ”bulk”)

Anomaly mediation:

- gravity generates soft terms even if no direct coupling between hidden and observable sectors
- soft terms depend only on ew scale coupling constants
- RGE invariant
- high predictability → negative slepton masses
  → need something more

heavy gravitino $\Rightarrow$
large Λ_S + very small λ_I

Supersymmetry breaking

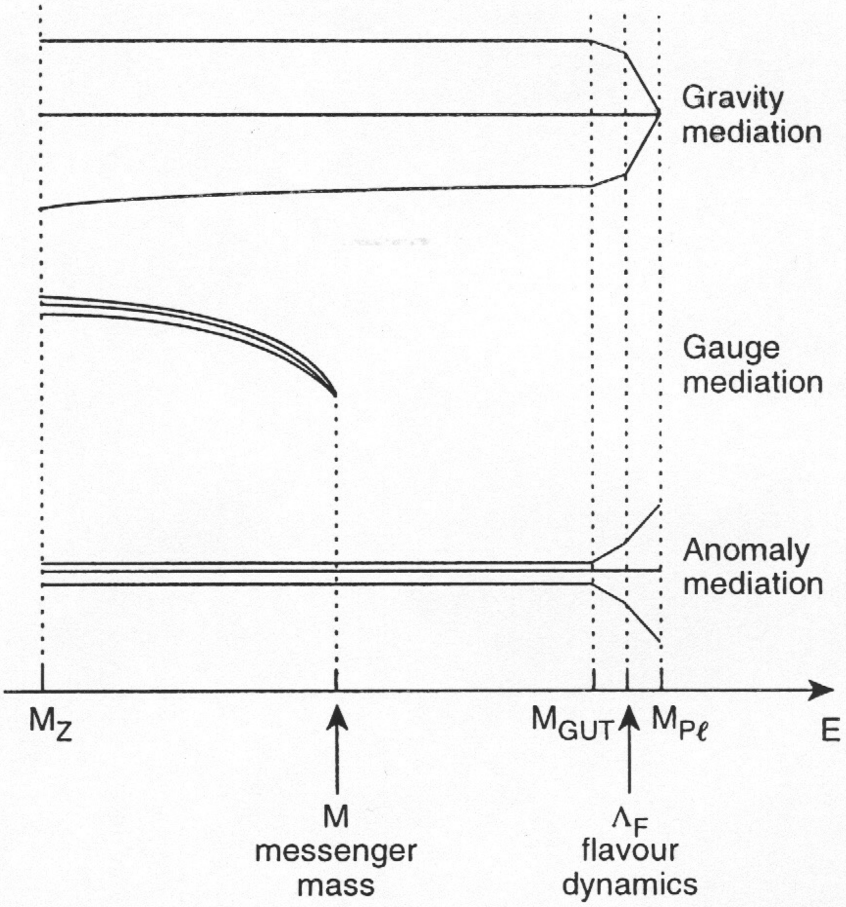
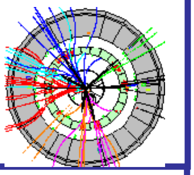


Figure 1: A schematic illustration of the energy dependence of the running squark masses belonging to the three different generations, in the context of the various supersymmetric scenarios discussed in the text. In gravity mediation, new dynamics at the scale Λ_F and GUT physics tend to induce large flavour-breaking effects in the squark spectrum, even if we start from a universality assumption at M_{Pl}. In the case of gauge mediation, the squark masses can be generated at scales sufficiently low to ensure a super-GIM mechanism. In anomaly mediation, the squark spectrum is determined by the low-energy theory and it is insensitive to flavour violations occurring at large scales.

R-parity



All renormalizable supersymmetric theories consistent with (global) $B-L$ conservation ⇒ **R-parity invariance**

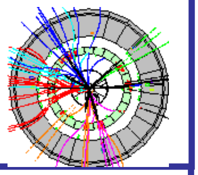
$$R_P = (-1)^{\overset{\text{Baryon Number}}{3B} + \underset{\text{Lepton number}}{L} + \overset{\text{Spin}}{2S}}$$

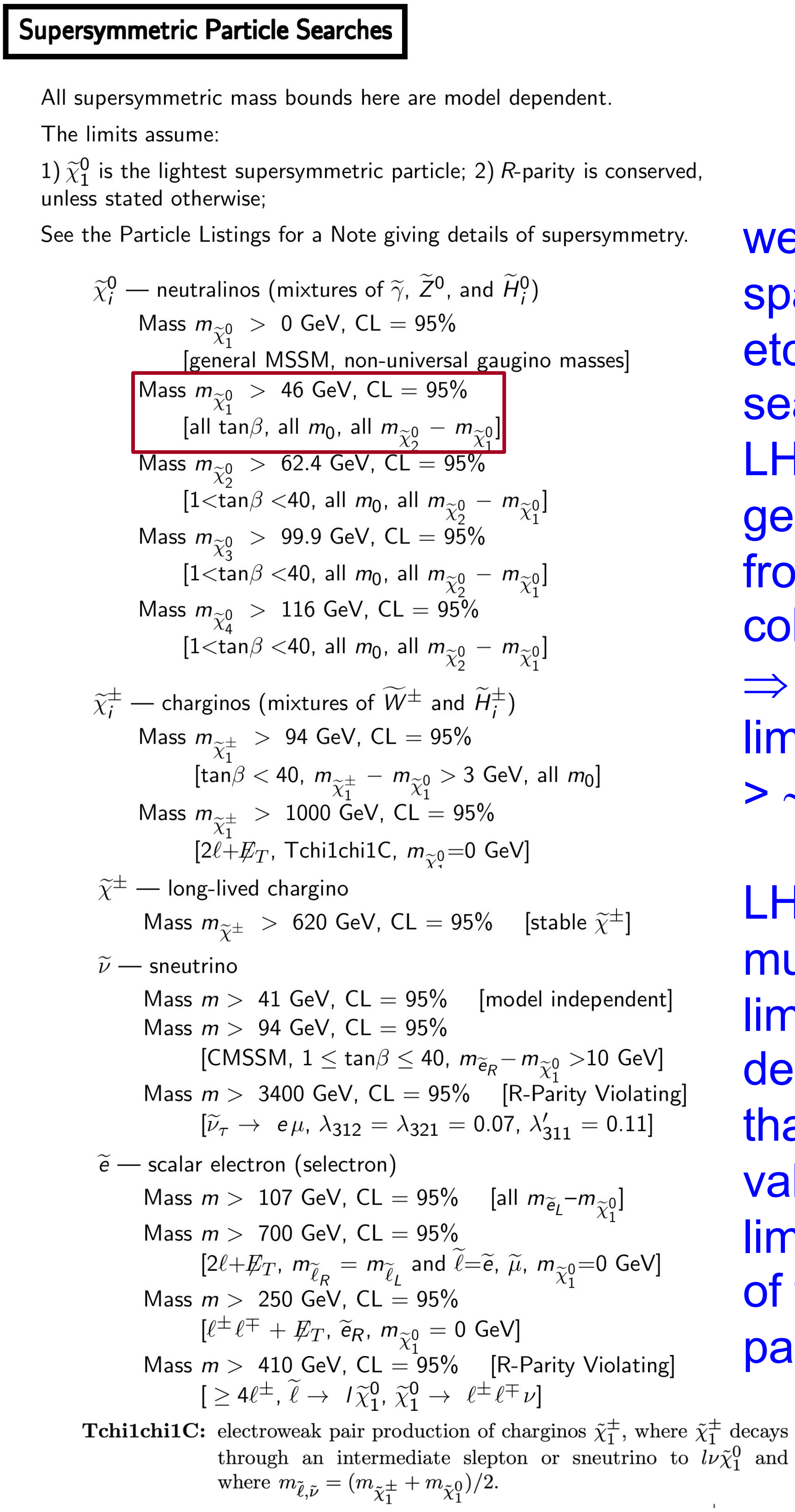
+1 for Standard Particles

-1 for Supersymmetric Partners

R_P Conserved	**R_P Violated**
◆ **SUSY particles are pair-produced** ◆ **The LSP is stable** **(→ neutral,colourless** **→ good dark-matter candidate)** ◆ **All SUSY particles decay into the LSP**	◆ **The LSP decay into standard particles** (no candidate for dark matter) ◆ **And so do all other SUSY particles** **NB! R_P conservation not required by neither SUSY or gauge invariance**
Experimental	**Signature**
◆ **The LSP (neutral, colourless) interacts only weakly with matter: it is invisible.** **→ MISSING ENERGY**	◆ **SUSY particles decay into quarks, leptons, neutrinos.** **→ Multi-jet, multi-leptons final state, not (necessarily) missing energy!!**

Supersymmetric particle searches





Supersymmetric Particle Searches

All supersymmetric mass bounds here are model dependent.

The limits assume:

1) $\tilde\chi^0_1$ is the lightest supersymmetric particle; 2) R-parity is conserved, unless stated otherwise;

See the Particle Listings for a Note giving details of supersymmetry.

$\tilde\chi^0_i$ — neutralinos (mixtures of $\tilde\gamma$, $\tilde Z^0$, and $\tilde H^0_i$)

Mass $m_{\tilde\chi^0_1}$ > 0 GeV, CL = 95%
[general MSSM, non-universal gaugino masses]

Mass $m_{\tilde\chi^0_1}$ > 46 GeV, CL = 95%
[all tanβ, all m_0, all $m_{\tilde\chi^0_2} - m_{\tilde\chi^0_1}$]

Mass $m_{\tilde\chi^0_2}$ > 62.4 GeV, CL = 95%
[1<tanβ <40, all m_0, all $m_{\tilde\chi^0_2} - m_{\tilde\chi^0_1}$]

Mass $m_{\tilde\chi^0_3}$ > 99.9 GeV, CL = 95%
[1<tanβ <40, all m_0, all $m_{\tilde\chi^0_2} - m_{\tilde\chi^0_1}$]

Mass $m_{\tilde\chi^0_4}$ > 116 GeV, CL = 95%
[1<tanβ <40, all m_0, all $m_{\tilde\chi^0_2} - m_{\tilde\chi^0_1}$]

$\tilde\chi^\pm_i$ — charginos (mixtures of $\tilde W^\pm$ and $\tilde H^\pm_i$)

Mass $m_{\tilde\chi^\pm_1}$ > 94 GeV, CL = 95%
[tanβ < 40, $m_{\tilde\chi^\pm_1} - m_{\tilde\chi^0_1}$ > 3 GeV, all m_0]

Mass $m_{\tilde\chi^\pm_1}$ > 1000 GeV, CL = 95%
[$2\ell+\not{E}_T$, Tchi1chi1C, $m_{\tilde\chi^0_1}$=0 GeV]

$\tilde\chi^\pm$ — long-lived chargino

Mass $m_{\tilde\chi^\pm}$ > 620 GeV, CL = 95% [stable $\tilde\chi^\pm$]

$\tilde\nu$ — sneutrino

Mass m > 41 GeV, CL = 95% [model independent]
Mass m > 94 GeV, CL = 95%
[CMSSM, $1 \le$ tan$\beta \le 40$, $m_{\tilde e_R} - m_{\tilde\chi^0_1}$ >10 GeV]

Mass m > 3400 GeV, CL = 95% [R-Parity Violating]
[$\tilde\nu_\tau \to e\mu$, $\lambda_{312} = \lambda_{321} = 0.07$, $\lambda'_{311} = 0.11$]

$\tilde e$ — scalar electron (selectron)

Mass m > 107 GeV, CL = 95% [all $m_{\tilde e_L} - m_{\tilde\chi^0_1}$]

Mass m > 700 GeV, CL = 95%
[$2\ell+\not{E}_T$, $m_{\tilde\ell_R} = m_{\tilde\ell_L}$ and $\tilde\ell = \tilde e$, $\tilde\mu$, $m_{\tilde\chi^0_1}$=0 GeV]

Mass m > 250 GeV, CL = 95%
[$\ell^\pm\ell^\mp + \not{E}_T$, $\tilde e_R$, $m_{\tilde\chi^0_1}$ = 0 GeV]

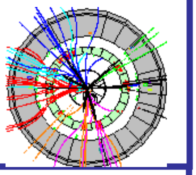
Mass m > 410 GeV, CL = 95% [R-Parity Violating]
[$\ge 4\ell^\pm$, $\tilde\ell \to l\tilde\chi^0_1$, $\tilde\chi^0_1 \to \ell^\pm\ell^\mp\nu$]

Tchi1chi1C: electroweak pair production of charginos $\tilde\chi^\pm_1$, where $\tilde\chi^\pm_1$ decays through an intermediate slepton or sneutrino to $l\nu\tilde\chi^0_1$ and where $m_{\tilde\ell,\tilde\nu} = (m_{\tilde\chi^\pm_1} + m_{\tilde\chi^0_1})/2$.

weakly coupling sparticles ($\tilde\chi$, $\tilde l$, $\tilde\nu$ etc ...): difficult to search for at LHC ⇒ most general limits from e⁺e⁻ collisions (LEP) ⇒ lower mass limits typically > ~ 100 GeV.

LHC provides much higher limits for specific decay modes that are only valid in more limited regions of the SUSY parameter space.

Supersymmetric particle searches



$\tilde{\mu}$ — scalar muon (smuon)

- Mass $m >$ 700 GeV, CL = 95%
  - [$2\ell + \not{E}_T$, $m_{\tilde{\ell}_R} = m_{\tilde{\ell}_L}$ and $\tilde{\ell} = \tilde{e}$, $\tilde{\mu}$, $m_{\tilde{\chi}^0_1}$=0 GeV]
- Mass $m >$ 210, CL = 95%
  - [$\ell^{\pm}\ell^{\mp} + \not{E}_T$, $\tilde{\mu}_R$, $m_{\tilde{\chi}^0_1} = 0$ GeV]
- Mass $m >$ 94 GeV, CL = 95%
  - [CMSSM, $1 \leq \tan\beta \leq 40$, $m_{\tilde{\mu}_R} - m_{\tilde{\chi}^0_1} > 10$ GeV]
- Mass $m >$ 410 GeV, CL = 95% [R-Parity Violating]
  - [$\geq 4\ell^{\pm}$, $\tilde{\ell} \to l\tilde{\chi}^0_1$, $\tilde{\chi}^0_1 \to \ell^{\pm}\ell^{\mp}\nu$]

$\tilde{\tau}$ — scalar tau (stau)

- Mass $m >$ 81.9 GeV, CL = 95%
  - [$m_{\tilde{\tau}_R} - m_{\tilde{\chi}^0_1}$ >15 GeV, all θ_τ, B($\tilde{\tau} \to \tau\tilde{\chi}^0_1$) = 100%]
- Mass $m >$ 90 GeV, CL = 95%
  - [R-Parity Violating, $\tilde{\tau}_R$, indirect, Δm >5 GeV]
- Mass $m >$ 286 GeV, CL = 95% [long-lived $\tilde{\tau}$]

$\tilde{q}$ – squarks of the first two quark generations

- Mass $m >$ 1.220×10^3 GeV, CL = 95%
  - [jets + $\not{E}_T$, Tsqk1, 1 non-degenerate $\tilde{q}$, $m_{\tilde{\chi}^0_1} = 0$ GeV]
- Mass $m >$ 1.600×10^3 GeV, CL = 95% [R-Parity Violating]
  - [$\tilde{q} \to q\tilde{\chi}^0_1$, $\tilde{\chi}^0_1 \to \ell\ell\nu$, λ_{121},$\lambda_{122} \neq 0$, $m_{\tilde{g}}$=2400GeV]

$\tilde{q}$ — long-lived squark

- Mass $m >$ 1340, CL = 95% [$\tilde{t}$ R-hadrons]
- Mass $m >$ 1250, CL = 95% [$\tilde{b}$ R-hadrons]

$\tilde{b}$ — scalar bottom (sbottom)

- Mass $m >$ 1.270×10^3 GeV, CL = 95%
  - [b-jets + $\not{E}_T$, Tsbot1, $m_{\tilde{\chi}^0_1}$=0 GeV]
- Mass $m >$ 307 GeV, CL = 95% [R-Parity Violating]
  - [$\tilde{b} \to td$ or ts, λ''_{332} or λ''_{331} coupling]

$\tilde{t}$ — scalar top (stop)

- Mass $m >$ 1.310×10^3 GeV, CL = 95%
  - [jets + $\not{E}_T$, Tstop1, $m_{\tilde{\chi}^0_1}$ < 300 GeV]
- Mass $m >$ 1100 GeV, CL = 95% [R-Parity Violating]
  - [$\tilde{t} \to be$, Tstop2RPV, prompt]
- Mass $m >$ 460 GeV, CL = 95%
  - [R-Parity Violating, long-lived $\tilde{t}$, $\tilde{t} \to d\bar{\ell}$, 0.01cm < $c\tau$ < 1000 cm]

$\tilde{g}$ — gluino

- Mass $m >$ 2.300×10^3 GeV, CL = 95%
  - [jets + $\not{E}_T$, Tglu1A, $m_{\tilde{\chi}^0_1}$ < 200 GeV]
- Mass $m >$ 2.260×10^3 GeV, CL = 95% [R-Parity Violating]
  - [$\geq 4\ell$, $\lambda_{12k} \neq 0$, $m_{\tilde{\chi}^0_1}$ > 1000 GeV]

Tsbot1: sbottom pair production with $\tilde{b} \to b\tilde{\chi}^0_1$

Tstop1: stop pair production with $\tilde{t} \to t\tilde{\chi}^0_1$

Tstop2RPV: stop pair production with $\tilde{t} \to b\ell$, via RPV coupling λ'_{i33}.

Tglu1A: gluino pair production with $\tilde{g} \to q\bar{q}\tilde{\chi}^0_1$

strongly coupling sparticles ($\tilde{q}$, $\tilde{g}$, etc ...): easy to search for at LHC ⇒ lower mass limits > ~ 1-2 TeV.

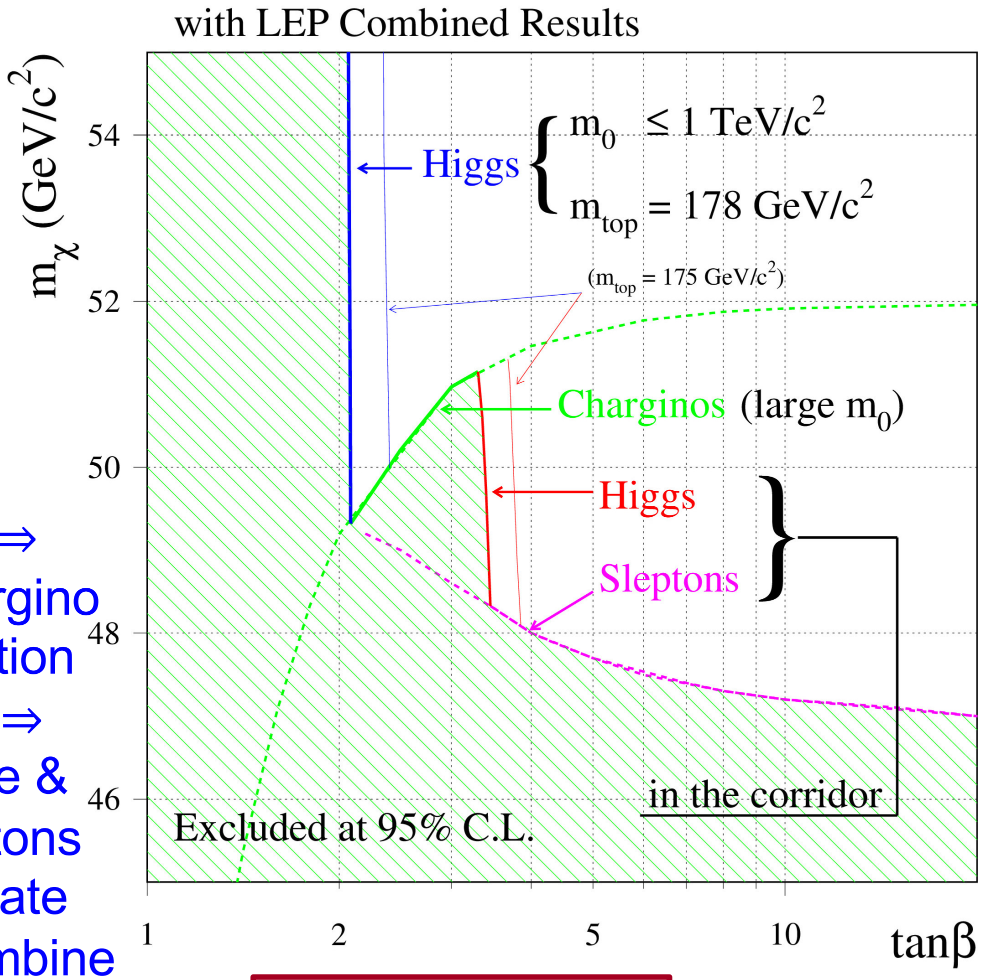
For more details see PDG review on Supersymmetry: experiment https://pdg.lbl.gov/2023/reviews/rpp2023-rev-susy-2-experiment.pdf

Lower bounds on the LSP mass

Lower bounds on the LSP mass can be extracted e.g. in constrained MSSM, where gaugino and sfermion masses separately unify at the GUT scale. Free parameters: $\tan\beta$, $M_{1/2}$ (gaugino masses at GUT scale), m_0 (sfermion & Higgs mass at GUT scale), μ (Higgs mass mixing term), A_t (trilinear coupling in the stop sector) & m_A (pseudoscalar Higgs mass)

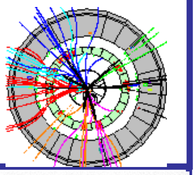
Combining slepton, Higgs & chargino searches to constrain lightest neutralino.

- large $m_0 \Rightarrow$ large chargino cross section
- small $m_0 \Rightarrow$ detectable & light sleptons
- Intermediate $m_0 \Rightarrow$ combine with Higgs search

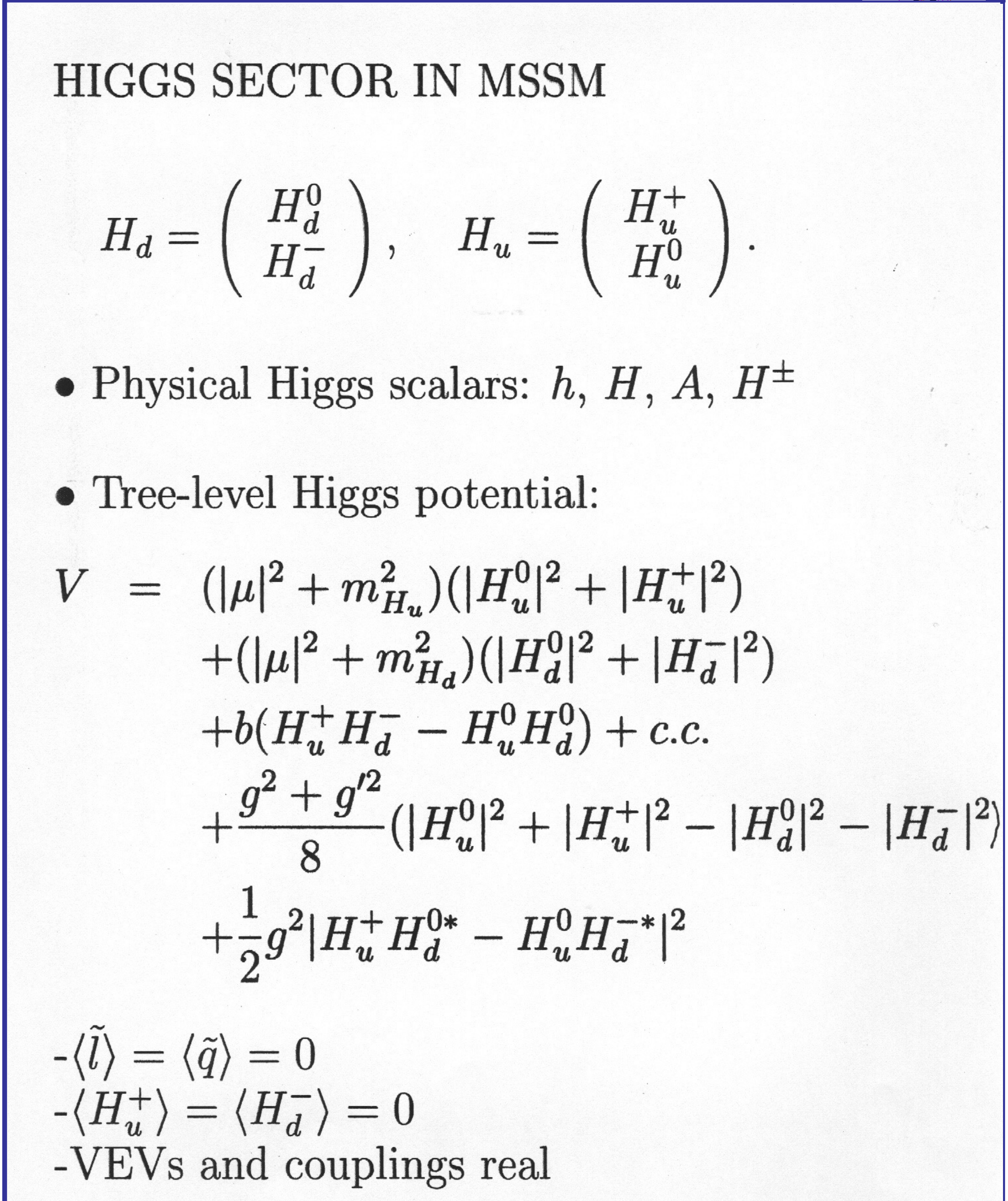


$$\boxed{m_{LSP} > 46 \text{ GeV}}$$

A more elaborate analysis in mSUGRA (only $\tan\beta$, sign(μ), m_0, $M_{1/2}$ & A_0 free) using also stable particle searches & electroweak parameter constraints give m_{LSP} > 50 GeV.



Higgs sector in supersymmetry



HIGGS SECTOR IN MSSM

$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \qquad H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}.$$

- Physical Higgs scalars: h, H, A, $H^\pm$
- Tree-level Higgs potential:

$$\begin{aligned} V \;=\; & (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) \\ & +(|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\ & +b(H_u^+H_d^- - H_u^0H_d^0) + c.c. \\ & +\frac{g^2+g'^2}{8}(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2) \\ & +\frac{1}{2}g^2|H_u^+H_d^{0*} - H_u^0H_d^{-*}|^2 \end{aligned}$$

-$\langle\tilde{l}\rangle = \langle\tilde{q}\rangle = 0$

-$\langle H_u^+\rangle = \langle H_d^-\rangle = 0$

-VEVs and couplings real

Two Higgs doublets are needed in SUSY to (1) cancel gauge anomalies (higgsino contributions in 3 gauge boson diagrams) (2) generate masses for both “up”- and ”down”-type quarks

Higgs sector in supersymmetry

- The masses of physical particles are

$$m_A^2 = 2b/\sin 2\beta, \quad m_{H^\pm}^2 = m_A^2 + m_W^2,$$

$$m_{h,H}^2 = \frac{1}{2}\Big[m_A^2 + m_Z^2 \mp\sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta}\Big] .$$

Here

$$h = (H_u^{0r} - v_2)\cos\alpha - (H_d^{0r} - v_1)\sin\alpha,$$
$$H = (H_d^{0r} - v_1)\cos\alpha + (H_u^{0r} - v_2)\sin\alpha,$$

where

$$\frac{\cos 2\alpha}{\cos 2\beta} = -\frac{m_A^2 - m_Z^2}{m_H^2 - m_h^2}, \quad \frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_H^2 + m_h^2}{m_H^2 - m_h^2}.$$

- At tree-level:

$$m_h^2 < m_{Z,A}^2, \quad m_H^2 > m_{Z,A}^2$$
$$m_h^2 + m_H^2 = m_A^2 + m_Z^2.$$

Final combined LEP limit:

- Experimentally: $m_{H^\pm}$ > 80 GeV

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Higgs sector: SM vs MSSM

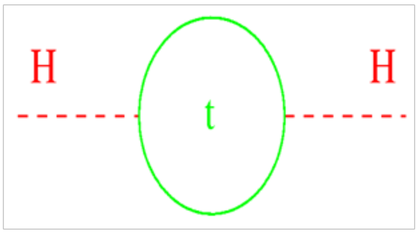
In the Standard Model

- One Higgs doublet
  v.e.v. v

- One physical state
  H

- One parameter
  M_H

- Radiative corrections
  to m_h
  quadratically divergent



In the M.S.S.M

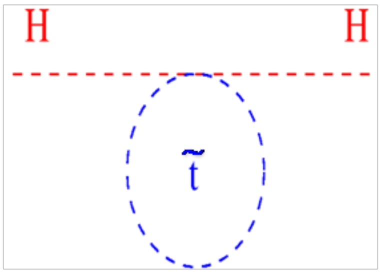
- Two Higgs doublets
  v.e.v.'s v_1 and v_2

- Five physical states

h, H,	A,	H^+, H^-
CP-even	CP-odd	Charged

- Two parameters
  (at tree-level)
  M_h, $\tan\beta = v_2/v_1$

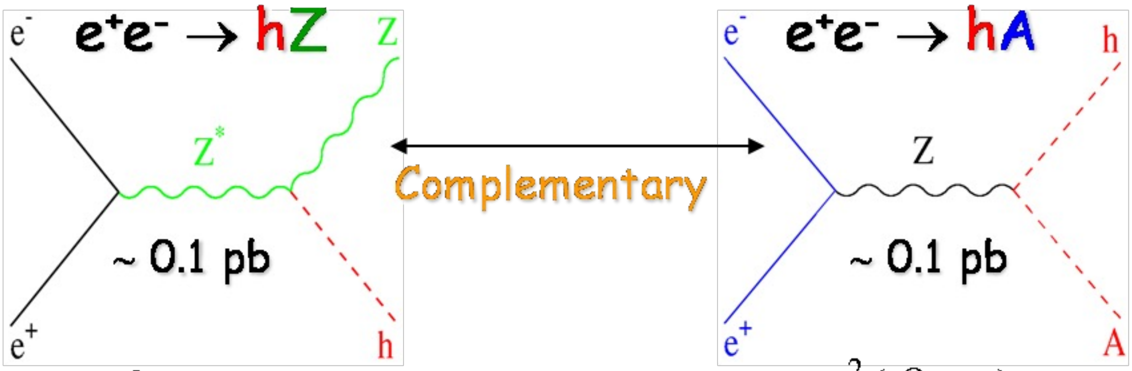
- Radiative corrections
  to m_h, m_H
  stabilized and finite



Depend on M_{top}, $M_{stop(L,R)}$ ....

MSSM Higgs

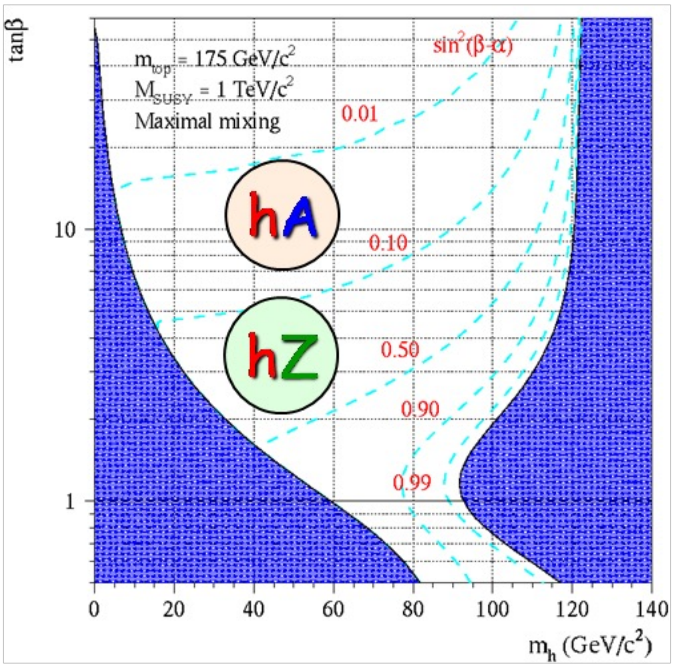
- In SUSY processes very often complementary e.g.



$\propto \sin^2(\beta-\alpha)\sigma_{SM}$

$\propto \cos^2(\beta-\alpha)\sigma_{SM}$

⇒ Look for both processes



Couplings between Higgses & the particles in MSSM:

	$u\bar{u}$	$d\bar{d}$, l^+l^-	VV (V=Z,W)
h	cosα/sinβ	– sinα/cosβ	sin(β-α)
H	sinα/sinβ	cosα/cosβ	cos(β-α)
A	cotβ	tanβ	0

In exact Supersymmetry: $m_h \leq m_Z \left|\cos 2\beta\right|$

In broken Supersymmetry: $m_h^2 \leq m_Z^2 + \Delta m_h^2$

SUSY little hierarchy problem

SUSY needs new (super)particles that haven't been seen (yet?)
SUSY (at least MSSM) predicts a (very) light Higgs

$$V = (|\mu|^2 + m_{H_u}^2)\left|H_u^0\right|^2 + (|\mu|^2 + m_{H_d}^2)\left|H_d^0\right|^2 - B(H_u^0 H_d^0 + c.c.) + \frac{g^2 + g'^2}{8}\left(\left|H_u^0\right|^2 - \left|H_d^0\right|^2\right)^2$$

one-loop level

$$m_h^2 \approx m_Z^2 \cos^2 2\beta + \frac{3 G_F m_t^4}{\sqrt{2}\pi^2} \log \frac{m_{\tilde{t}}^2}{m_t^2}$$

$$m_Z^2/2 = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}$$

$m_H > 115$ GeV $\Rightarrow \tilde{m}_t > 1$ TeV

fine-tuned

$$\delta m_{H_u}^2 = -\frac{3\sqrt{2} G_F m_t^2 m_{\tilde{t}}^2}{4\pi^2} \log \frac{\Lambda}{m_{\tilde{t}}}$$

requires some fine-tuning O(1%) in m_Z

susy little hierarchy problem

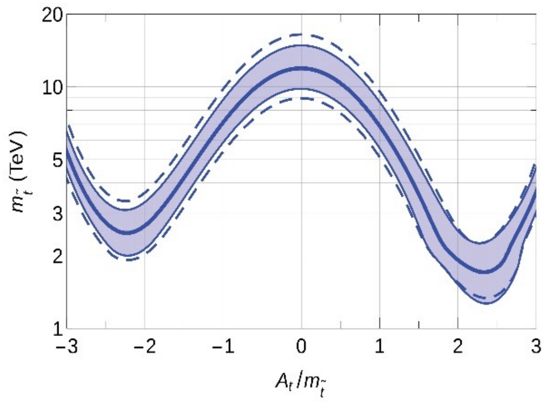
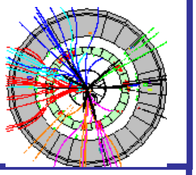


Figure 5: *Allowed values of the OS stop mass reproducing $m_h = 125$ GeV as a function of the stop mixing, with $\tan\beta = 20$, $\mu = 300$ GeV and all the other sparticles at 2 TeV. The band reproduce the theoretical uncertainties while the dashed line the 2σ experimental uncertainty from the top mass. The wiggle around the positive maximal mixing point is due to the physical threshold when $m_{\tilde{t}}$ crosses $M_3 + m_t$.*

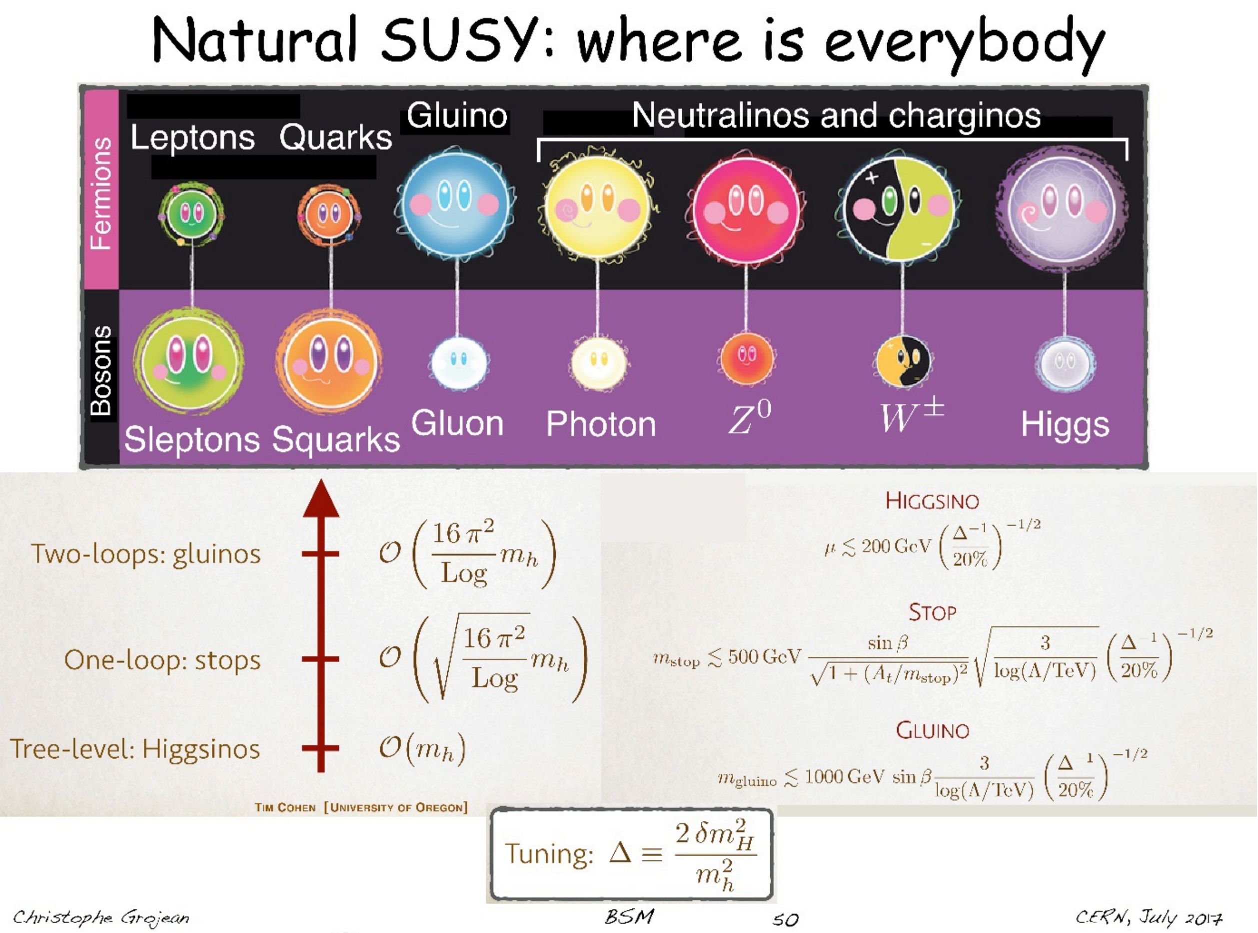
Pardo Vega, Villadoro '15 + many others

⬅ One needs heavy stop(s)
to obtain a 125GeV Higgs
(within the MSSM)

Current lower limits on stop mass: up to 1310 GeV (but very decay & LSP mass dependent)



Supersymmetry summary



Natural SUSY: where is everybody

Two-loops: gluinos $\mathcal{O}\left(\frac{16\,\pi^2}{\text{Log}}\,m_h\right)$

One-loop: stops $\mathcal{O}\left(\sqrt{\frac{16\,\pi^2}{\text{Log}}}\,m_h\right)$

Tree-level: Higgsinos $\mathcal{O}(m_h)$

TIM COHEN [UNIVERSITY OF OREGON]

HIGGSINO

$$\mu \lesssim 200\,\text{GeV}\left(\frac{\Delta^{-1}}{20\%}\right)^{-1/2}$$

STOP

$$m_{\text{stop}} \lesssim 500\,\text{GeV}\,\frac{\sin\beta}{\sqrt{1+(A_t/m_{\text{stop}})^2}}\sqrt{\frac{3}{\log(\Lambda/\text{TeV})}}\left(\frac{\Delta^{-1}}{20\%}\right)^{-1/2}$$

GLUINO

$$m_{\text{gluino}} \lesssim 1000\,\text{GeV}\,\sin\beta\,\frac{3}{\log(\Lambda/\text{TeV})}\left(\frac{\Delta^{-1}}{20\%}\right)^{-1/2}$$

Tuning: $\Delta \equiv \frac{2\,\delta m_H^2}{m_h^2}$

Christophe Grojean BSM 50 CERN, July 2017

SUSY is Natural
but not plain vanilla

- ~~CMSSM~~
- pMSSM
- NMSSM
- Hide SUSY, e.g. smaller phase space
  - reduce production (eg. split families) Mahbubani et al
  - reduce MET (e.g. ~~R-parity~~, compressed spectrum) Csaki et al
  - dilute MET (decay to invisible particles with more invisible particles)
  - soften MET (stealth susy, stop -top degeneracy) Fan et al

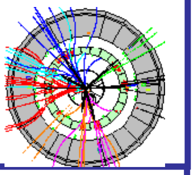
LHC$_{300\text{fb}^{-1}}$ will tell!

Good coverage of

hidden natural susy

- mono-top searches (DM, flavored naturalness - mixing among different squark flavors-, stop-higgsino mixings)
- mono-jet searches with ISR recoil (compressed spectra)
- precise tt inclusive measurement+ spin correlations (stop → top + very soft neutralino)
- multi-hard-jets (RPV, hidden valleys, long decay chains)

Gravity



Gravity and the SM

The existence of gravity is the most solid argument that the SM is not the final theory.

- Gravity interacts with SM fields.

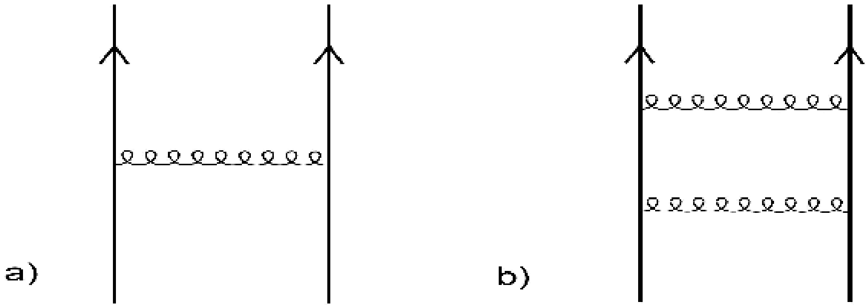
- At some high energy scale, Λ_P gravity will become strong, and quantum effects must be incorporated. This scale could be $M_P \sim 10^{19}$ GeV but (as we will see later) it could also be much lower.

- This fundamental theory, would look like classical gravity plus the SM at energies $E << \Lambda_P$.

- In this sense the SM is an effective theory, valid (at most) up to Λ_P.

- Things look bad, since classical gravity (general relativity) is a non-renormalizable theory.

Gravity at short distances?



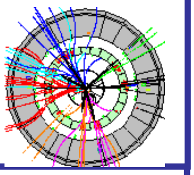
- The classical gravitational theory is non-renormalizable

$$(b) \quad \sim \quad \frac{E^2}{M^4_{\text{Planck}}} \int_0^{\Lambda} dp \; p \quad \sim \quad \frac{\Lambda^2 \; E^2}{M^4_{\text{Planck}}} \; ,$$

- At higher orders it gets worse and worse.

- No clue as to what the short distance theory is.

- This has been an open problem for more than 50 years.



Gravity and string theory

Gravity and String Theory

- String theory is a different framework for describing and unifying all interactions.

- It has become popular because it always includes quantum gravity, without UV problems (divergences)

- Moreover it also includes the other ingredients of the SM: Gauge interactions, chiral matter (fermions) and if needed, supersymmetry.

- It offers some conceptual features that are appealing to physicists:

(a) String theory ALWAYS contains gravity

(b) The existence of fermions implies supersymmetry at high energy.

(c) It has a priori no fundamental parameters but only one dimensionfull scale: the size of the strings. All dimensionless parameters of a given ground state of the theory are "dynamical" (expectation values of scalar fields).

(d) It contains solitonic extended objects (known as branes) that provide an incredible richness to the theory as well as a deep link between gauge theories and gravity.

What is String Theory?

Shift in paradigm: from point particle to a closed string.

- In QFT fields are "point-like". In string theory, they depend not on a point of space-time but a loop in space-time (the position of a closed string).

What is the difference between a closed "fundamental" string and a loop of wire?

(A) The fundamental string is much smaller: its size is definitely smaller than 10^{-18} m. This would explain why we have not seen one so far.

(B) Apart from the usual degrees of freedom (their coordinates in space-time), fundamental strings have also fermionic degrees of freedom. There a kind of supersymmetry relating the coordinates to such fermionic degrees of freedom.

Since the smallest length we can see today (with accelerators) is approximately 10^{-18} m strings would appear in experiments so far as point-like objects.

String theory

String Theory, Vol II

- Fundamental strings, like the analogous classical objects, can vibrate in an infinite possible number of harmonics.

- Upon quantization, these harmonics behave like different particles in space-time.

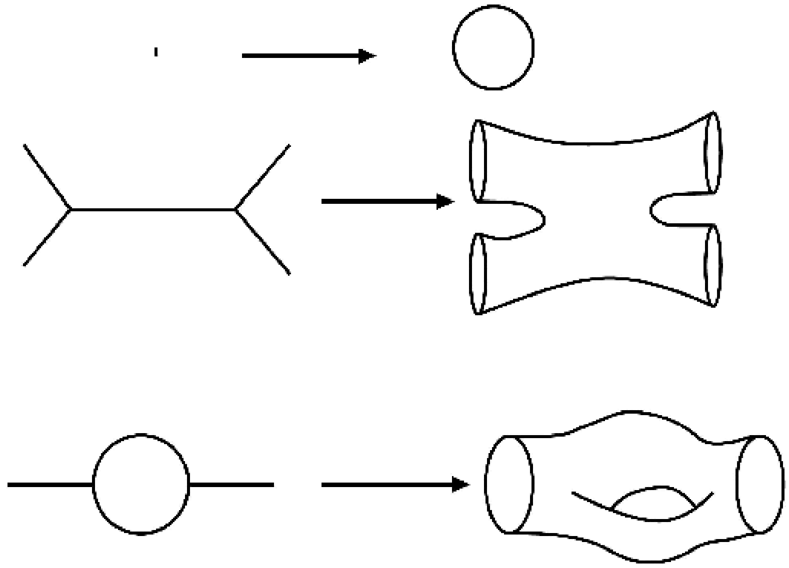
A single string upon quantization $\Longrightarrow$ an infinite number of particles with ever increasing mass.

- Infinity of particles is responsible for the unusual properties of string theory (and its complicated structure).

- Strings live in diverse dimensions. Lorentz invariances $\Leftrightarrow$ 9+1 dimensions. Although this seems to contradict common experience it can be compatible under certain circumstances. How do we see the extra dimensions? More on this later .....

String Theory, Vol III

- In perturbation theory, standard QFT Feynman diagrams are replaced with string diagrams (two-dimensional surfaces)

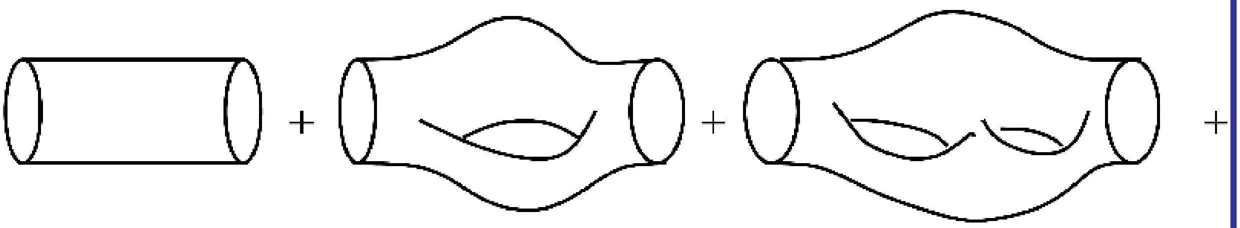


String theory

String perturbation theory

♣ In QFT perturbation theory is formulated using Feynman diagrams.

♠ In string theory we have Riemann surfaces. For closed strings, each order contains a single diagram. At low energy, they reduce to the (many) QFT Feynman diagrams.



- String theory diagrams, when appropriately defined, give finite amplitudes in the UV. Quantum gravity, which is part of string theory is essentially finite.

Extra space dimensions

- The idea that space has extra, hitherto unobservable dimensions goes back to the beginning of the twentieth century, with Nordström (1914), Kaluza (1925) and Klein (1926).

- It comes naturally in string theory.

How come they are not visible today?

(A) Because they compact and sufficiently small.

(B) Because we are “stuck” on the four-dimensional world.

(C) Because they are of a more bizarre kind (for example, they are discretized appropriately)

Extra dimensional models

Extra dimensional models

- Problematic aspect of SM:

$$v_{EW} \ll M_{GUT}, M_{Pl}$$

Electroweak scale: $v_{EW} \approx 246\,\text{GeV}\ (2\cdot10^{-18}\,\text{m})$

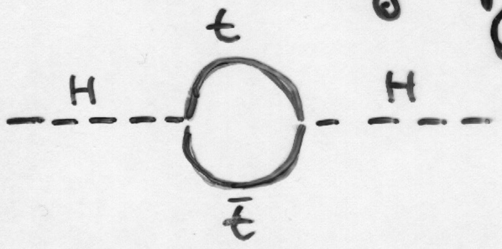
Mass scale of (quantum) gravity:

$$M_{Pl} = \left(\frac{\hbar c}{G_N}\right)^{-\frac{1}{2}} = 1.2\cdot10^{19}\,\text{GeV}\ (2\cdot10^{-32}\,\text{m})$$

This large separation leads to:

Hierarchy problem:

(how keep $m_H \sim v_{EW}$)



$$\delta m_H^2 \sim \Lambda^2$$

$$\Lambda \sim M_{GUT}$$

possible solutions:

(a) connect bosons & fermions by symmetry ⇒ SUPERSYMMETRY

boson + fermion loops cancel !

fundamental scale M_{Pl}

Large extra dimensions

(b) exploit geometry of space time

& fundamental scale v_{EW}

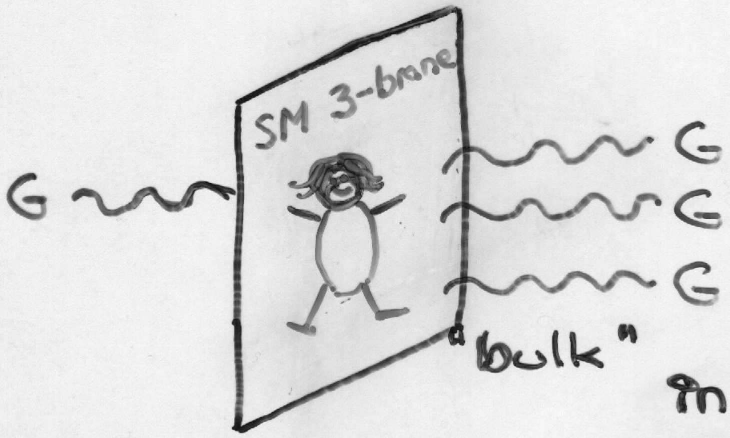
$$\Rightarrow \Lambda \sim v_{EW}$$

NB! here M_{Pl} only derived effect

$$\Longrightarrow$$

- (large) extra spatial dimensions
- confinement of matter on subspace

Natural setting in string theory!



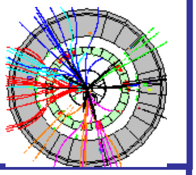
SM confined to 3+1 dimensions ("3-brane")

Gravity propagates in D-dimensional space-time ("bulk")

$D \equiv (3 + \delta + 1)$ ← extra dimensions!

(c) mere coincidence, corrections cancel!

$$\Lambda_a^2 - \Lambda_b^2 \sim v_{EW}^2 ; \Lambda_a, \Lambda_b \sim M_{GUT}$$



Large extra dimensions

Compute Newtons Constant

- Einstein action in D-dimensions:

$$S_E^D = \frac{1}{16\pi \hat{G}_N} \int d^Dx \sqrt{-\hat{g}}\, R(\hat{g})$$

("^" D-dimensional equivalents)

Assume factorizable geometry:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + h_{ij}(y) dy^i dy^j$$

$\mu, \nu = (0 \ldots 3)$; $i, j = (1 \ldots \delta)$

and that the extra dimensions, δ, are compactified on circles with radius R_c → $V_c = 2\pi R_c$ →

- Effective action in 4D:

$$S_E = \frac{V_c^\delta}{16\pi \hat{G}_N} \int d^4x \sqrt{-g}\, R(g)$$

$$\rightarrow \frac{1}{G_N} = \frac{V_c^\delta}{\hat{G}_N} ; \quad \hat{G}_N = \frac{1}{M_D^{D-2}}$$

$$\rightarrow M_{Pl} = M_D^{\frac{\delta+2}{2}} V_c^{\delta/2}$$

Large extra dimensions

M_D fundamental scale!

Possible for $M_D \sim$ TeV? →

M_{Pl} large if $V_c(R_c)$ large

Arkani Hamed-Dimopoulos-Dvali 98

NB! electroweak & strong tested to $\sim 10^{-18}$ m, gravity to $\sim 10^{-4}$ m

- R_c (radius of compactification):

if $M_D \sim 1$ TeV

$$\left\{ \begin{array}{llll} 10^{12}\,\text{m} & \sim (10^{-15}\,\text{eV})^{-1} & \delta=1 & \text{excluded} \\ 5\cdot 10^{-4}\,\text{m} & \sim (10^{-4}\,\text{eV})^{-1} & \delta=2 & \\ 10^{-8}\,\text{m} & \sim (10^{4}\,\text{eV}) & \delta=3 & \text{not apriori excluded} \\ 3\cdot 10^{-10}\,\text{m} & \sim (10^{7}\,\text{eV}) & \delta=4 & \end{array} \right.$$

Gravity weak because it is diluted by a large space!

NB! No convincing explanation why $R_c M_D \gg 1$ exist

Large extra dimensions

- Gravitational interactions modified at small distances:

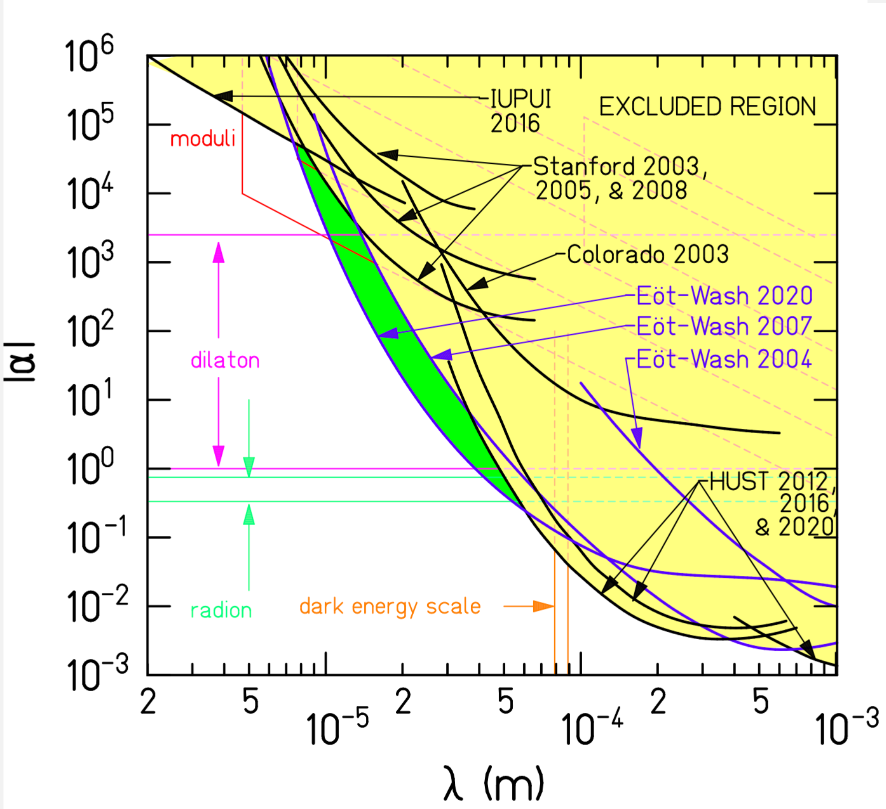
$$V(r) = G_N \frac{m_1 m_2}{r} \quad \text{at } r \gg R_c$$

$$V(r) = G_N^{(4+\delta)} \frac{m_1 m_2}{r^{1+\delta}} = G_N R_c^{\delta} \frac{m_1 m_2}{r^{1+\delta}}$$

at $r \ll R_c$

- Experimental tests of Newtons Law: (torsion experiments)

$$V(r) = -G_N \frac{m_1 m_2}{r} \left(1 + \alpha e^{-r/\lambda}\right)$$



$R_C < 30\ \mu$m for $\delta = 2$ leading to $M_D \geq 4.0$ TeV. Bounds from astrophysics tighter but these limits from torsion experiments more general.

First ideas for extra dimensions

Nordström-Kaluza-Klein unification

1914 - Gunnar Nordström: Unification of scalar gravity with electromagn. in 5D:

$$A_M (M=0,1,2,3,5) \Rightarrow A_\mu (\mu=0,1,2,3) + \overset{\phi}{\overset{\equiv}{A_5}}$$

where scalar field ϕ is a gravitational field coupled to T^μ_μ

1915 - Albert Einstein: General relativity gravitational field is a tensor $g_{\mu\nu}(x^\mu) T^{\mu\nu}$

1921 - Theodor Kaluza: General relativity in 5D as an unified theory of gravity and electromagnetism

1926 - Oscar Klein: rediscovered Kaluza's theory and gave a geometrical interpretation of extra dimension (compactness)

4D gravity and electromagnetism unified in 5D! Charge & masses quantized! $R_c = \sqrt{\frac{4G_N}{\alpha}} \approx 10^{-31}$ cm

Doesn't describe the real world!

charge states of $m \sim M_{Pl}$, $F_{\mu\nu}F^{\mu\nu} = 0$ ...

Manifestations in 4D

- example: massless free scalar $\varphi(x^\mu, y)$ field in 5D; $x^5 \equiv y$ is a circle of radius R_c, i.e. $y + 2\pi R_c \approx y$

Klein-Gordon equation:

$$\left.\begin{aligned} &(\partial_\mu \partial^\mu - \partial_y \partial^y)\, \varphi(x^\mu, y) = 0 \\ &\varphi(x^\mu, y + 2\pi R_c) = \varphi(x^\mu, y) \end{aligned}\right\}$$

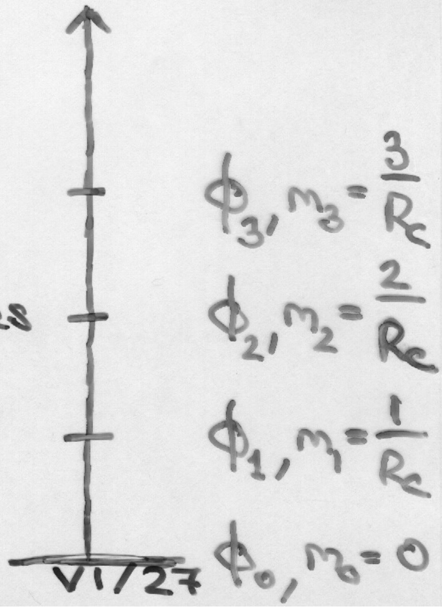
$$\begin{cases} \varphi(x^\mu, y) = \displaystyle\sum_{n=-\infty}^{\infty} \phi_n(x^\mu)\, f_n(y) \\ f_n(y) = e^{iny/R_c}\,; \quad n = 0, \pm 1, \pm 2 \dots \end{cases}$$

solution same as QMs "particle in a box"

$$(\partial_\mu \partial^\mu + m_n^2)\, \phi_n(x^\mu) = 0\,;$$

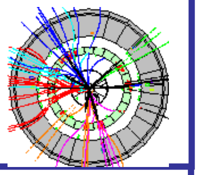
$$m_n^2 = \frac{n^2}{R_c^2}$$

Get in 4D a tower of "Kaluza-Klein" (KK)-states with equal quantum numbers!



Beyond SM KÖ VI/27

Predictions of extra dimensional models



- Mass splitting between gravitons

$$\Delta m \sim \frac{1}{R_c} = M_D\left(\frac{M_D}{M_{Pl}}\right)^{2/\delta}$$

$$\begin{cases} 5\cdot 10^{-4}\,\mathrm{eV} & \delta=2 \\ 20\ \mathrm{keV} & \delta=4 \end{cases}$$

a continum of states!

- Probability for producing a KK graviton

$\alpha_{grav} \sim E^2/M_{Pl}^2$; impossible to see!

\# of KK states with $m_n < E$

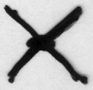
$\sim E^{\delta} M_{Pl}^2 / M_D^{2+\delta}$ → LHC@13 TeV: $M_D >$ 5.9 – 11.2 TeV

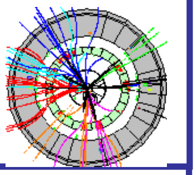
e.g. $\sum_n \sigma(pp \to G_n\,\mathrm{jet}) \simeq \frac{\alpha_s}{\pi}\frac{E^{\delta}}{M_D^{2+\delta}}$

$\sigma(pp \to G_n\gamma) \Rightarrow$ LHC@13 TeV: $M_D >$ 2.9 TeV

accessible for colliders if $M_D \sim$ O (TeV)

- will give supersymmetry-like missing energy signal when gravitons escape into the bulk
- also 2 → 2 scattering processes modified through virtual graviton exchange ("contact" interaction type)

Beyond SM  $\sim 4/M_{TT}^4$ LHC @13 TeV: $M_{TT} >$ 6-9 TeV



Mini black hole - production?

- Schwarzschild radius, R_s, (i.e. within which nothing escapes gravitation)

4D : $R_s \sim \frac{2M_{BH}}{M_{Pl}^2}$

DD : $R_s \sim \frac{1}{M_D}\left(\frac{M_{BH}}{M_D}\right)^{1/\delta+1}$

LHC@13 TeV: $M_{BH,\ semi\text{-}classical}$ > 9.0-10.1 TeV

Quantum BH ($M_{BH} \sim M_D$): 2-particle decay

M_{QBH} > 2.3-9.4 TeV (model detail dependence)

- If $M_D \sim 1$ TeV and $\sqrt{s} > M_D > R_s^{-1}$, tiny black holes ($M_{BH} \sim$ TeV) can be produced if two partons pass at a distance, b, < R_s. Cross sections large! $\sim \pi R_s^2$

$\sigma(pp \to BH) \sim 100$ fb ($M_D \sim 3$ TeV, $\delta = 4$)

$\Rightarrow$ 1000 events/year at low $\mathcal{L}$ LHC

- Mini black holes decay immediately ($\tau \sim 10^{-26}$ s) by evaporation to q, ℓ ...

expected signature: ~ spherical events with many high energy jets, leptons, γ's

NB! BH's should be produced also in cosmic ray experiments

Constraints on extra dimensions

Most significant constraints come from astrophysics!

- Copious emission of KK gravitons and ν compete in Supernova cooling

neutrinos $\sim G_F^2 T^2$; gravitons $\sim \frac{T^{\delta}}{M_D^{2+\delta}}$

SN1987A: $M_D > 27$ TeV for $\delta = 2$

$M_D > 2.4$ TeV for $\delta = 3$

- $G \to \gamma\gamma$ decay distorts cosmic γ-ray background and leads to anomalous heating of neutron stars

$\tau_G \sim 3 \cdot 10^9 \left(\frac{100\text{ MeV}}{m_G}\right)^3$ years

$$\left.\begin{array}{l} M_D > 1700 \text{ TeV for } \delta = 2 \\ M_D > 76 \text{ TeV for } \delta = 3 \end{array}\right\}$$

reduced if KK gravitons decay mainly to non-SM particles

- Cosmology: Relic KK gravitons contribution to cosmic gamma radiation: $M_D > 100$ TeV for $\delta = 2$
- NB! Very weak limits from astrophysics & cosmology if $\delta \geq 4$

Non-factorizable geometry

Non-factorizable geometry

Randall-Sundrum 99

- A classical mechanism to make quanta softer

- Time independent metrics $g_{0\mu} = 0$

$$\Rightarrow \quad E\sqrt{|g_{00}|} \text{ conserved } \left(\begin{array}{l}\text{proper time} \\ d\tau^2 = g_{00}\,dt^2\end{array}\right)$$

Schwarzschild metric $\quad g_{00} = 1 - \frac{2G_N M}{d}$

$$\frac{E_{obs} - E_{em}}{E_{em}} = \sqrt{|g_{00}|} - 1 \approx -\frac{G_N M}{d}$$

- In non-trivial metrics, we see far-away objects as red-shifted

("curved" or "warped" space time)

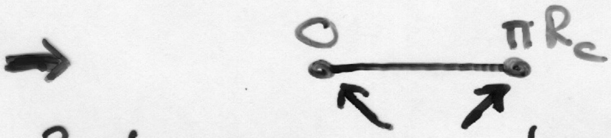
Non-factorizable geometry

- Non-factorizable geometry in 5D:

$$ds^2 = e^{-2ky} g_{\mu\nu} dx^\mu dx^\nu - dy^2$$

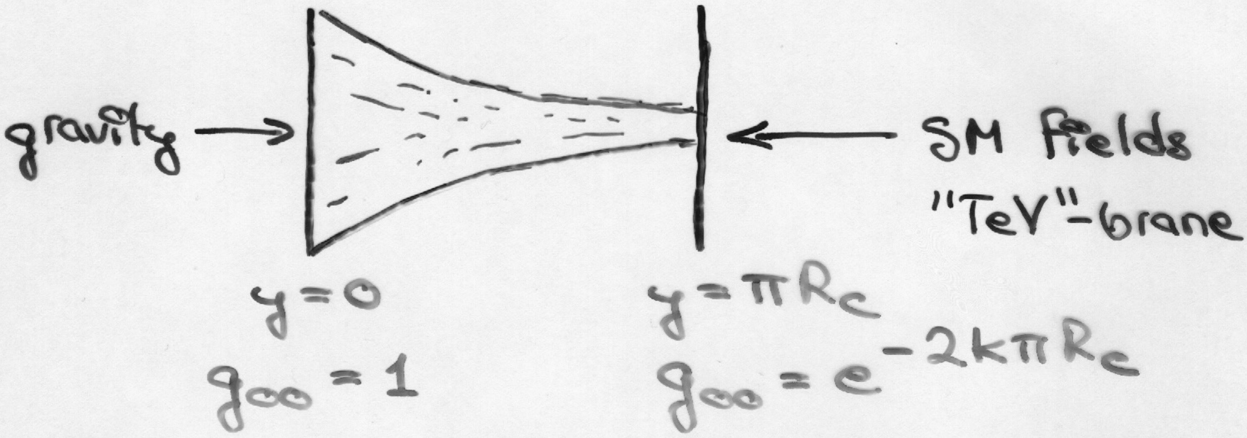
- 5th dimension $S_1/\mathbb{Z}_2$

identify $y \to y + 2\pi R_c$ & $y \to -y$



2 3-branes <u>on</u> boundaries!

- Gravitational redshift!



masses related by $\frac{m_\pi}{m_0} = e^{-k\pi R_c}$

$$R_C \approx 11/k \Rightarrow m_\pi/m_0 \approx O(1\,\mathrm{TeV})/M_{Pl}$$

Non-factorizable geometry

$$\Rightarrow \quad M_{Pl}^2 = \frac{M_D^3}{k}\left(1-e^{-2k\pi R_c}\right)$$

$$\Rightarrow \quad M_D \sim k \sim \frac{11}{R_c} \sim M_{Pl}$$

(not a model with large extra dimensions)

- Effective theory on TeV-brane:

$$\Lambda_\pi = M_{Pl}\, e^{-k\pi R_c} \sim 1 \text{ TeV}$$

- Masses of bulk graviton KK towers:

$$m_n = x_n k e^{-k\pi R_c} = x_n k \frac{\Lambda_\pi}{M_{Pl}}$$

x_n roots of first order Bessel functions
$$\begin{cases} x_1 = 3.8 \\ x_2 = 7.0 \\ x_n \approx (n+\frac{1}{4})\pi \end{cases}$$

- KK states not evenly spaced
- characteristic mass $k\frac{\Lambda_\pi}{M_{Pl}} \sim \text{TeV}$
- couplings ("strong" for excitations)

$$\mathcal{L} = -\frac{h^{(0)}_{\mu\nu}}{M_{Pl}} T^{\mu\nu} - \frac{T^{\mu\nu}}{\Lambda_\pi}\sum_{n=1}^{\infty} h^{(n)}_{\mu\nu}$$

Predictions of non-factorizable models

Physical interpretation:

- Gravity concentrated at $y=0$, our world confined at $y=\pi R_c$ small overlap ⇒ gravity seems weak
- Graviton KK states are not equally spaced and couples strongly to matter; can be observed as resonances
- Lightest new state in this scenario radion (or graviscalar), r,

$$O(10\text{ GeV}) \lesssim m_r \lesssim \Lambda_\pi$$

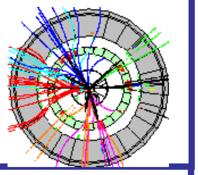
Dominant decay mode $r \to gg$ if $m_r \lesssim 2M_W$, otherwise $r \to W^+W^-, ZZ, hh$

couplings $\approx v_{EW} / (\sqrt{24}\Lambda_\pi) \cdot$ SM couplings

radion orginates from quantum excitations of the distance between branes

radion can mix with Higgs and alter g_{Hff} and g_{HVV} !

Extra dimensional models



Constraints on non-factorizable models

- most astrophysical limits weak
- best limits from LHC

$G \to \gamma\gamma/ee/\mu\mu$ $m_G > 2.3 – 4.8$ TeV
(no SM fields in the bulk)

$G \to WW/ZZ/tt$ $m_G > 2.3$ -3.7 (VV) – 4.55 (tt) TeV
(SM fields also in the bulk)

Radion → WW/ZZ $m_{radion} > 3.2$ TeV

Open problems in extradimensional models

- no large mass scales to suppress violation of approximate symmetries (proton decay, flavour changing neutral currents, neutrino mass)
exception warped space time & no SM fields in bulk
- unification of gauge couplings
- cosmology, baryogenesis ...
- no theory at $\sqrt{s} \sim M_D$!
quantum gravity? string theory?

For more details see PDG review on Extra dimensions searches
https://pdg.lbl.gov/2023/reviews/rpp2023-rev-extra-dimensions.pdf

Beyond SM conclusion

What is physics beyond the Standard Model?

?

I don't know. Nobody knows

If it were known, it would be part of the SM!

You ~~won't learn~~ haven't learned during these lectures what BSM is.

(maybe) ~~You'll learn~~ You learned what BSM could be.

" Looking and not finding is different than not looking"

We'll study the limitations/defaults of the SM as a guide towards BSM.

We want to learn from our failures

The hierarchy problem made easy

only a few electrons are enough to lift your hair (~ 10^{25} mass of e^-)

the electric force between 2 e^- is 10^{43} times larger than their gravitational interaction

we don't know why gravity is so weak?

we don't know why the masses of particles are so small?

Several theoretical hypothesis

new dynamics? new symmetries? new space-time structure?

modification of special relativity? of quantum mechanics?