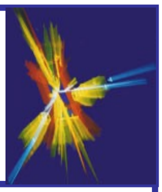
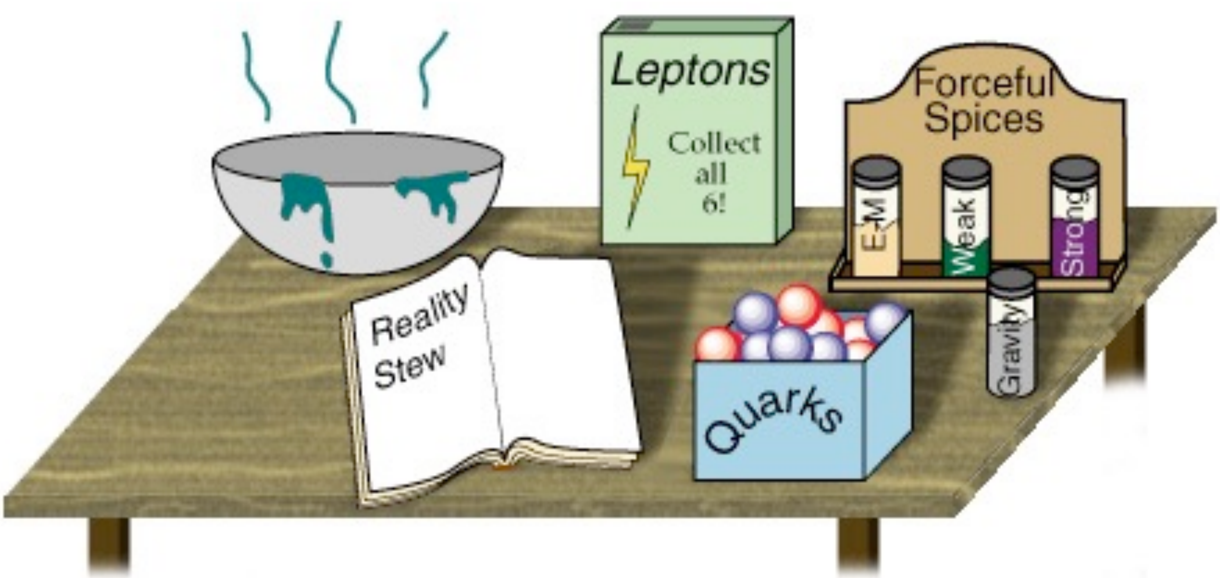


Standard Model

Constituents & Interactions

Gauge Invariance

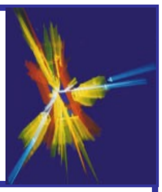


Quantum Chromodynamics

Electroweak Unification

Electroweak Symmetry Breaking

follows A. Pich: The Standard Model of Electroweak Interactions, arXiv:1201.0537



Theoretical Framework

QM ($\hbar$) + special relativity (c) $\Rightarrow$ Quantum Field Theory

STANDARD MODEL:

- Electricity + Magnetism: γ

Quantum Electrodynamics (QED)

- QED + Weak Interaction: γ, Z^0, $W^\pm$

Electroweak Theory $SU(2)_L \otimes U(1)_Y$

- Strong Interaction: **8 gluons**

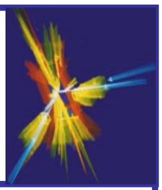
Quantum Chromodynamics (QCD) $SU(3)_C$

Standard Model = $SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$

NB! $Q = T_3 + Y/2$ (Y = weak hypercharge, T_3 = weak isospin)

OPEN QUESTIONS (at least some):

- Grand Unification **electroweak & strong combined ?**
- Supersymmetry **bosons & fermions linked together?**
- Gravitation **quantum theory of gravity? graviton?**



Leptons

- Do not have strong interactions
- Spin ½
- Seen as free particles
- Pointlike (at least for dist ≥ few · 10^{-19} m)

Family Structure:

Why are there 3?

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L , \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

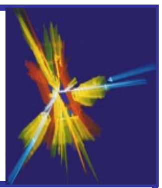
Transformation of fields under SM gauge symmetries:

(2,-1,1)

↕ ↕ ↕

$(SU(2)_L, U(1)_Y, SU(3)_C)$

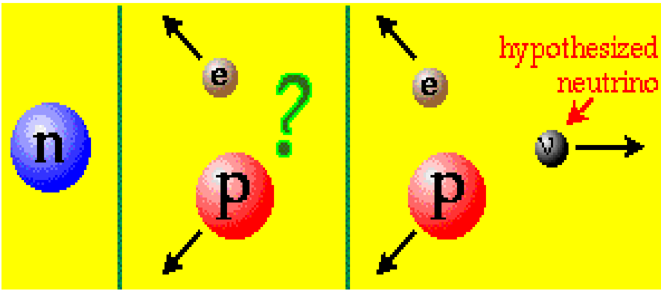
Lepton	Mass (MeV)	τ (s)	q	L	J^P	L_e	L_μ	L_τ	Decay	BR
e (electron)	0.510	stable	-1	1	$½^+$	1	0	0		
ν_e (neutrino)	~0	stable	0	1	$½^+$	1	0	0		
μ (muon)	105.6	$2.2 \cdot 10^{-6}$	-1	1	$½^+$	0	1	0	$\rightarrow e^- \bar{\nu}_e \nu_\mu$	100 %
ν_μ	~0	stable	0	1	$½^+$	0	1	0		
τ (tau)	1777	$2.9 \cdot 10^{-13}$	-1	1	$½^+$	0	0	1	$\rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ $\rightarrow e^- \bar{\nu}_e \nu_\tau$ $\rightarrow$ hadrons ν_τ	17.4% 17.8% 64.8%
ν_τ	~0	stable	0	1	$½^+$	0	0	1		



Neutrinos

- Particles interacting only weakly $\Rightarrow$ difficult to detect
- Abundantly produced in the sun & the atmosphere
- First signs in β decay ("non-energy conversation")

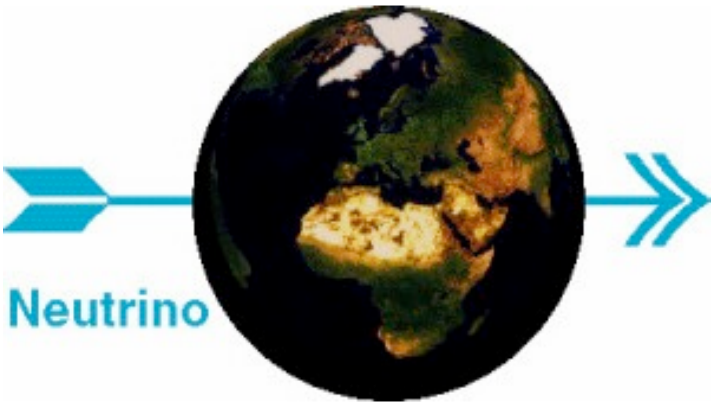
$$ {}^{14}_{6}\mathrm{C} \to {}^{14}_{7}\mathrm{N} + e^- + \bar{\nu}_e \quad (n \to p + e^- + \bar{\nu}_e) $$



$$ q_{\bar{\nu}_e} = q_{\nu_e} = 0 $$
$$ m_{\bar{\nu}_e} \approx m_{\nu_e} \approx 0 $$

$\nu_e \equiv$ neutrino $\qquad \bar{\nu}_e \equiv$ anti-neutrino

- They penetrate earth & humans without interacting

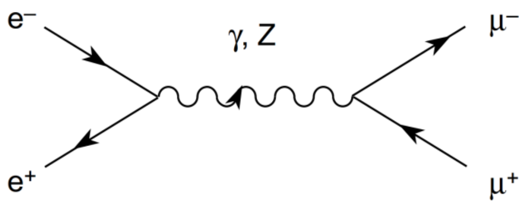


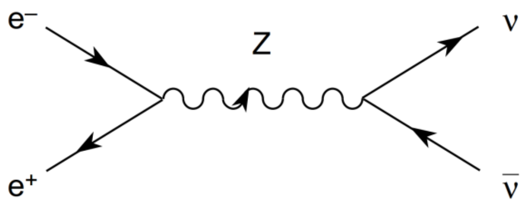
Each second $\sim 10^{14}$ ν_e from the sun pass through your body

($pp \to de^+\nu_e$, ...)

- Artificially produced in nuclear reactors
- Mostly seen in experiments as "missing momentum"
- ν experiments show that different ν flavours turn into each other ("oscillate") and this implies that ν's must have mass, a very small one though ($\lesssim 10^{-2}$ eV). More about this later.

NEUTRAL CURRENTS





- Flavour Conserving

$$\mu \nrightarrow e\gamma \;\&\; Z \nrightarrow e^{\pm}\mu^{\mp}$$

- $g_\gamma \propto q_l \qquad (q_e = q_\mu = q_\tau = 1\,;\, q_\nu = 0)$

- Same γ interaction for both lepton helicities

- NC Universality:

$$g_{Ze+e-} = g_{Z\mu+\mu-} = g_{Z\tau+\tau-} \neq g_{Z\nu\bar{\nu}}$$

- Different Z coupling to l_L and l_R

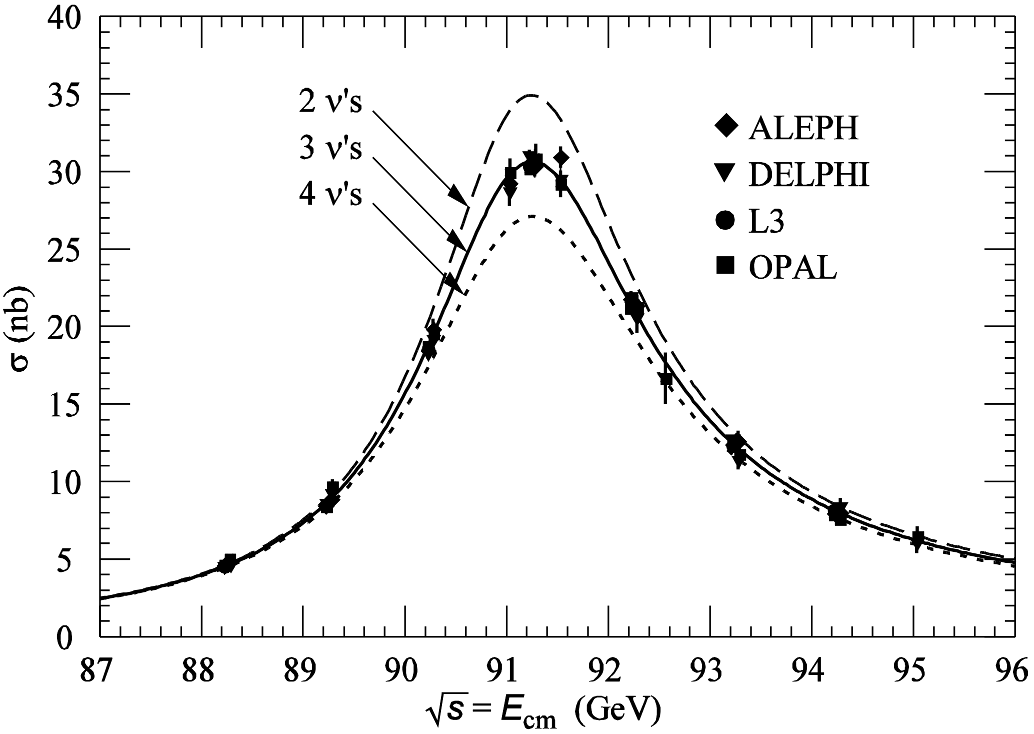
- Left-handed neutrinos only

- 3 Families with light neutrinos

HOW MANY NEUTRINOS ?

$\Delta E \cdot \Delta t \leq \hbar \Rightarrow$ small lifetime t leads to large ΔE

$\sigma(Z \to hadrons)$



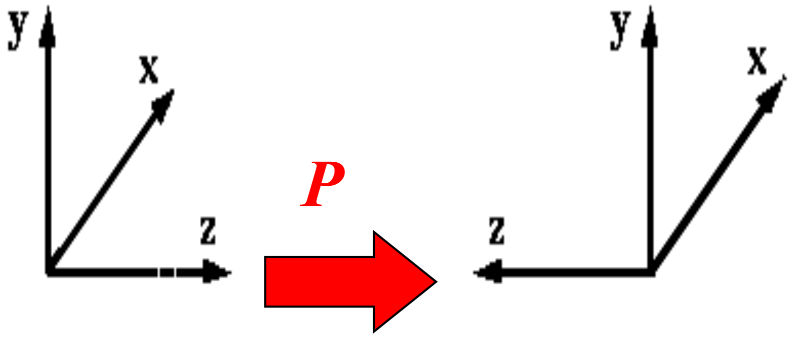
$$\Gamma(Z \to invisible) = \Gamma(Z \to all) - \Gamma(Z \to visible)$$

$$N_\nu = \frac{\Gamma(Z \to invisible)}{\Gamma(Z \to \nu_i \bar{\nu}_i)_{theory}} = 2.9963 \pm 0.0074$$

P. Janot and S. Jadach, PLB 803 (2020) 135319;
G. Vouttsinas et al., PLB 800 (2020) 135068

Bucket analogy: the more “holes” (decay channels) → the “faster” (smaller Δt) the bucket becomes empty → the more “uncertain” (larger ΔE) the particle mass is

Parity (P): reversal of all three axes in a reference frame, P transformation equivalent to a mirror reflection.



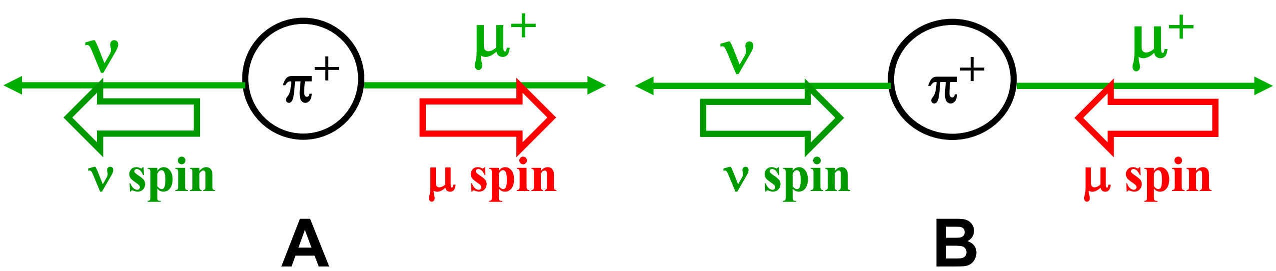
(first, rotate by 180° around z–axis ; then reverse all 3 axes)

most physics laws invariant w.r.t. a P transformation i.e. Nature does not know the difference between right & left.

Angular momentum & spin doesn't change sign under P
$\Rightarrow$ transition prob. dependence on $\bar{s} \cdot \bar{p}$ indicate P violation.

Lee & Yang (1956): weak interaction violates P invariance

Consequence for e.g. $\pi^+ \to \mu^+ \nu$ decay ($S_\pi = 0$, $S_{\mu/\nu} = ½$):
P invariance require that the 2 states

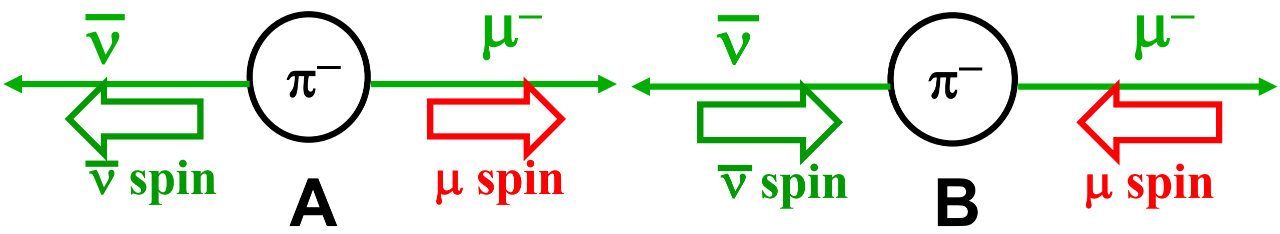


are produced with equal probabilities. Experiments find μ^+ always polarized opposite to the momentum direction (B)

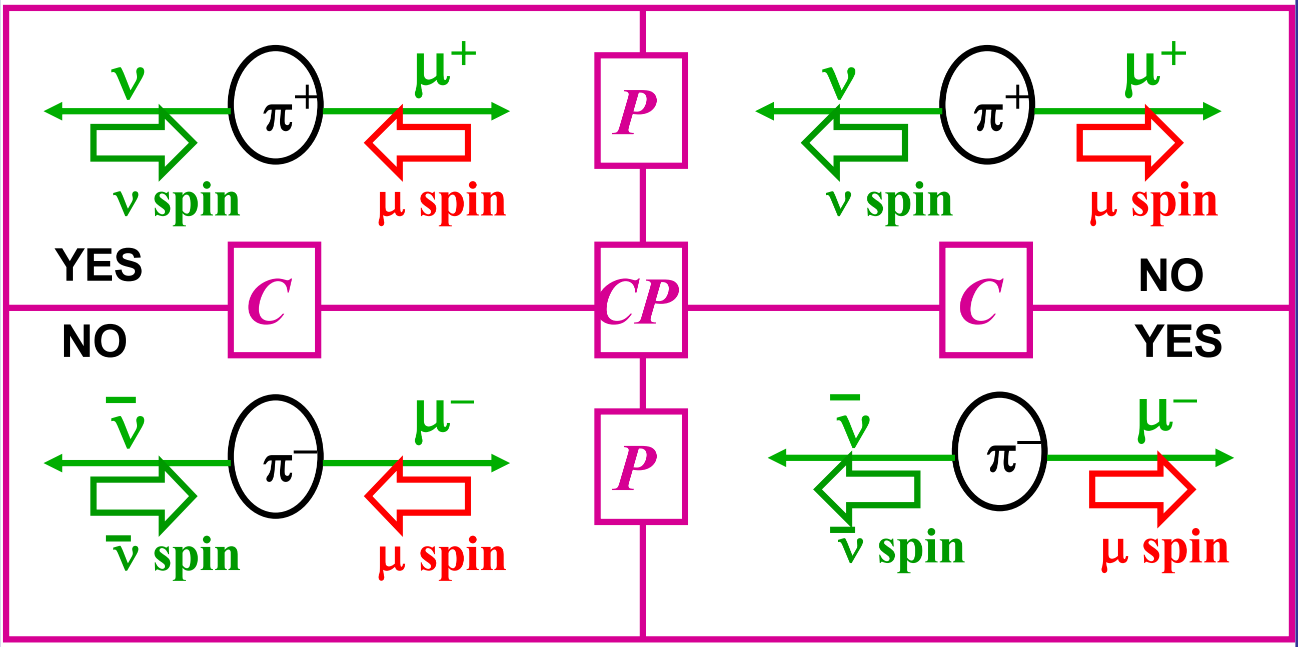
$\Rightarrow$ maximal violation of parity invariance in weak decays !!

Charge conjugation (C): particle ⇔ antiparticle transformation (quantum state defined only for neutrals).

$\pi^- \to \mu^- \bar{\nu}_\mu$ decay: experiments find only decays of type A

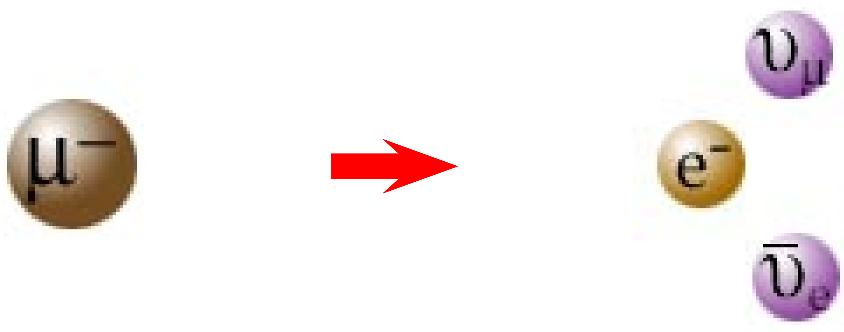


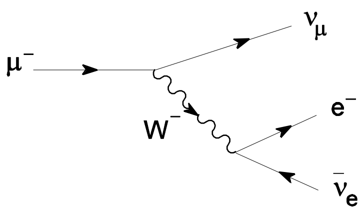
Conclusion for weak (e.g. $\pi^\pm$) decays:

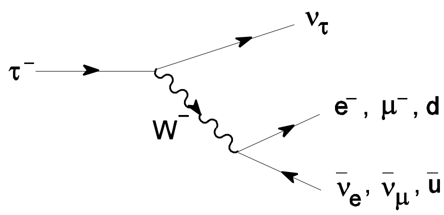


Weak decays violate maximally both P & C invariance but are invariant under CP. In reality CP symmetry is only approximative in weak decays (more later on the slight CP violation in weak decays of neutral K, D & B mesons).

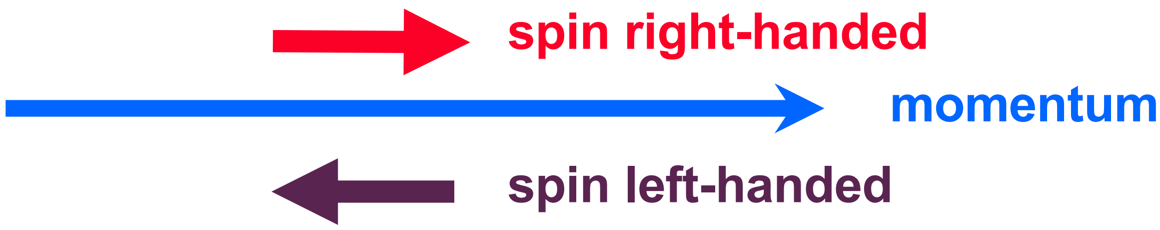
The heavier leptons μ and τ are unstable

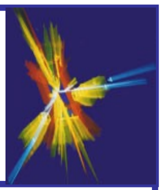




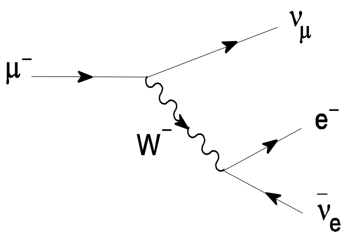


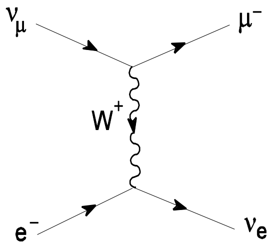
- same process as in the case of radioactivity. Note that the W boson virtual ($Q^2 \ll m_W^2$) in the process.
- only left-handed leptons (right-handed antileptons) (determined from direction of spin) take part in the reaction.





CHARGED CURRENTS





- Left-handed leptons (Right-handed antileptons)

$$l^-, \nu_l \qquad\qquad l^+, \overline{\nu_l}$$

$$\Leftarrow \qquad \vec{J} \qquad \Rightarrow$$

$$\rightarrow \qquad \vec{p} \qquad \rightarrow$$

- Doublet partners: $l^- \Leftrightarrow \nu_l$

$$\nu_\mu X \rightarrow \mu^- X' \qquad \nu_\mu X \not\rightarrow e^- X'$$

- Universal Strength

$$M(l \rightarrow \nu_l l' \overline{\nu_{l'}}) \sim \frac{g_W^2}{M_W^2 - Q^2} \xrightarrow{Q^2 << M_W^2} \frac{g_W^2}{M_W^2} \sim G_F$$

$$\Rightarrow \quad \Gamma(l \rightarrow \nu_l l' \overline{\nu_{l'}}) \sim G_F^2 m_l^5$$

"lepton universality"

$$\Gamma(\tau \rightarrow \nu_\tau e \overline{\nu_e}) \big/ \Gamma(\mu \rightarrow \nu_\mu e \overline{\nu_e}) = (m_\tau / m_\mu)^5$$

Quarks

- Strong interaction bind them into hadrons
- Spin ½ ; pointlike (at least for dist ≥ few · 10^{-19} m)
- Not observed as free particles ⇒ **confinement**
- $q_u = 2/3$; $q_d = -1/3$

Family Structure:

Why are there 3?

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L , \begin{pmatrix} c \\ s' \end{pmatrix}_L , \begin{pmatrix} t \\ b' \end{pmatrix}_L \qquad (2,1/3,3) \leftrightarrow (SU(2)_L, U(1)_Y, SU(3)_C)$$

Mass eigenstates ≠ Weak eigenstates

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$V_{CKM} \cdot V^{\dagger}_{CKM} = \mathbf{1}$

More later !

"Wolfenstein" parametrisation of the CKM matrix. Decay rates depend on the CKM elements e.g. $\Gamma(t \to bW^+) \propto |V_{tb}|^2 \sim 1$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$\lambda = 0.22650 \pm 0.00048$

$A = 0.790^{+0.017}_{-0.012}$, $\bar{\rho} = 0.141^{+0.016}_{-0.017}$

$\bar{\eta} = 0.357 \pm 0.011$

$$\approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

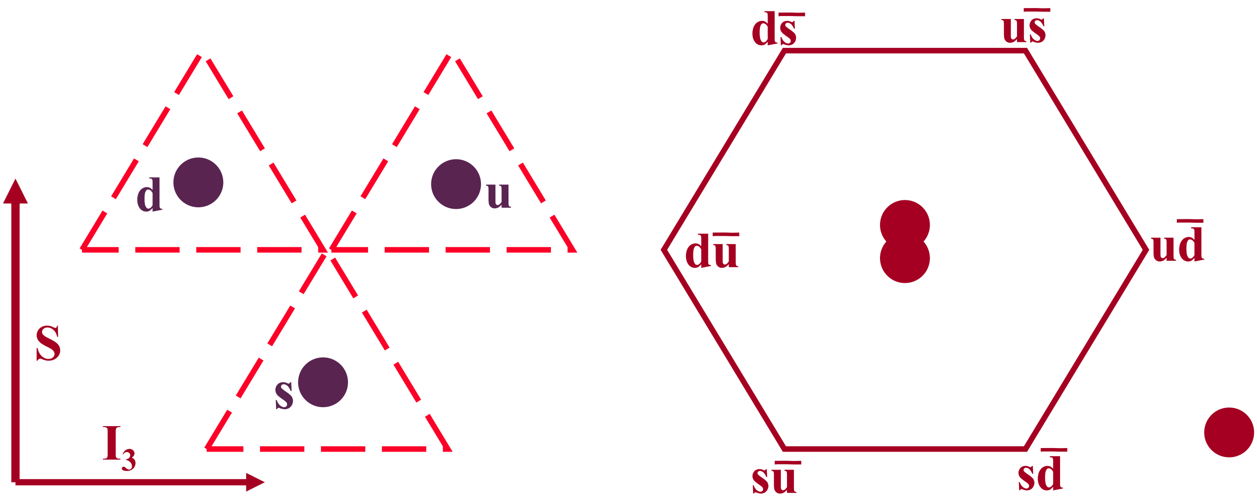
$\bar{\rho} = \rho\,(1 - \lambda^2/2 + \ldots)$

$\bar{\eta} = \eta\,(1 - \lambda^2/2 + \ldots)$

Quark model

(Nearly?) all observed **hadrons** can be explained within the **quark model** by constituting **colourless** combinations, each (anti)quark having a (anti)colour:

Mesons ≡ $q_1\bar{q}_2$ (colour & anticolour combination)



$$3 \otimes \bar{3} = 8 \oplus 1$$

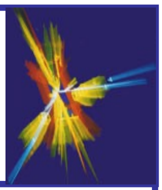
Baryons ≡ $q_1q_2q_3$ (combination of R B G)

Antibaryons ≡ $\bar{q}_1\bar{q}_2\bar{q}_3$ (combination of $\bar{R}$ $\bar{B}$ $\bar{G}$)

Most hadrons very short-lived ("resonances"). Heisenberg uncertainty principle implies that their mass is not precisely defined (~ħ/Δt). Their mass M_R is described by a Breit-Wigner.

$$f(M) \propto \frac{(\Gamma_R^{tot})^2}{(M-M_R)^2+(\Gamma_R^{tot}/2)^2},$$

Γ_R^{tot} is the total decay width = 1/τ, the lifetime.



Hadronization scale $\Lambda_{had} \sim 1$ GeV

Isospin: u,d: $m_{u,d} \ll \Lambda_{had}$

M_p = 938.3 MeV M_n = 939.6 MeV (p = uud, n = udd)

accidental but gives rather good predictions (~ 1 %) !!

Quark mass not well defined since quarks confined in hadrons:

- ”**constituent**” mass. The mass which a quark appears to have in a hadron in terms of properties, such as magnetic moments. For u and d quarks, ~1/3 of the nucleon mass.

- ”**current**” mass. The ”bare” mass, the one inserted into a fundamental theory, i.e. difference between constituent & current masses due to strong interactions effects.Theoretical arguments applied to observed hadron masses, suggest the u and d current masses to be very small compared to proton mass: $m_u \approx 2$ MeV & $m_d \approx 5$ MeV. Thats why isospin works !! NB! In current mass picture, masses ”runs” with energy scale μ.

NB2! small u-d mass difference results in stable p & unstable n.

Special case: top since $M_t \approx 173$ GeV

$$\tau_t \equiv \quad 1/\Gamma(t \to \text{anything}) \approx 1/\Gamma(t \to bW^+) << 1/\Lambda_{\text{had}}$$

Top quark has no time to hadronize !!!

Summary of Quarks

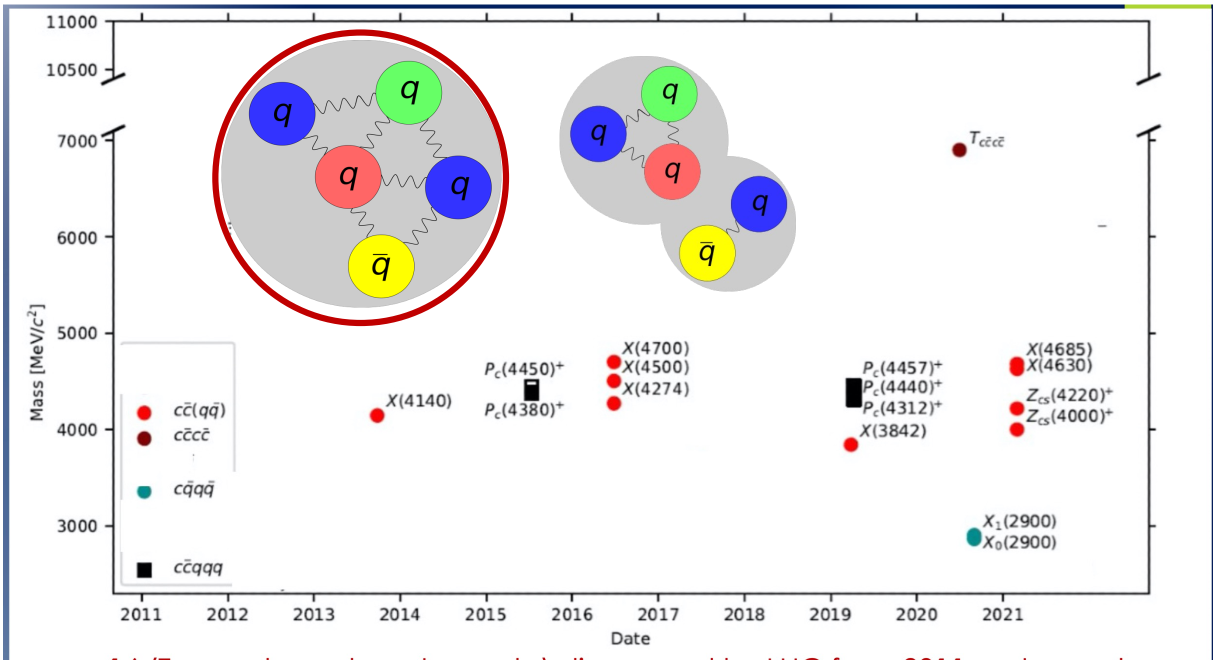
Masses definied below as current masses at a certain Q^2

$m_q = m_q$ (2 GeV^2) ; $M_q = m_q (m_q^2)$

Light (MeV)	$m_u \sim 2$	$m_d \sim 5$	$m_s \sim 93$
Heavy (GeV)	$M_c \sim 1.3$	$M_b \sim 4.2$	$M_t \sim 173$

Tetraquarks and Pentaquarks

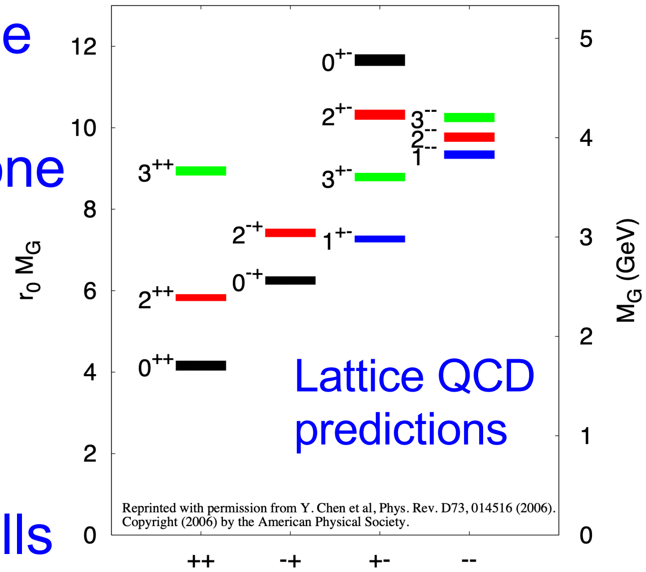
- Currently one of the unresolved questions about tetraquarks concerns the arrangement of their structure. We know that they are made up of 4 quarks but we do not know how tight the bond of these components is. According to some physicists, the tetraquark can be thought of as a compact object, like the proton or the neutron. Another hypothesis represents them as molecular states, such as structure composed of 2 meson substructures. In a similar way for pentaquarks we can think of them as compact 5 quarks or as baryon –meson molecular states. Data seems to favour the first option.



16 (Traquarks and pentaquarks) discovered by LHC from 2011 up to now!

Glueballs (particles made up of only gluons): only candidates upto now, none definitely yet confirmed

However Pomeron (~ 2 gluons) & Odderon (~ 3 gluons) existence imply also existence of glueballs



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QUARKS HAVE COLOUR

$\Delta^{++} \sim u^{\uparrow} u^{\uparrow} u^{\uparrow}$ $\quad (J = \frac{3}{2}\,,\ J_3 = \frac{3}{2})$

Pauli (Fermi statistics) ➡ $\Delta^{++} \sim \varepsilon^{\alpha\beta\gamma}\, u^{\uparrow}_{\alpha}\, u^{\uparrow}_{\beta}\, u^{\uparrow}_{\gamma}$

NEW QUANTUM NUMBER: **COLOUR**

baryons $B \sim \varepsilon^{\alpha\beta\gamma}\, q^i_\alpha\, q^j_\beta\, q^k_\gamma \quad ; \quad M \sim \delta^{\alpha\beta}\, q^i_\alpha\, \bar{q}^j_\beta$ mesons

$(i, j, k = u, d, s, \ldots \quad ; \quad \alpha, \beta, \gamma = 1, \ldots, N_C)$

$N_C = 3$ ➡ $q^i\, q^i\, q^i$

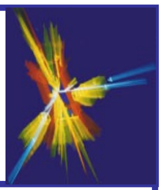
We don't see Colour Multiplets

➡ Hadrons are Colour Singlets

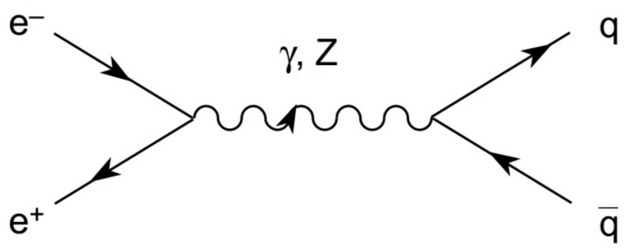
$(q\,q\,q,\ \ \bar{q}\,\bar{q}\,\bar{q}$ and $q\,\bar{q}$; BUT NOT $q\,q$ and $q\,q\,q\,q)$

We don't see Quarks

➡
- Don't exist ?
- CONFINEMENT



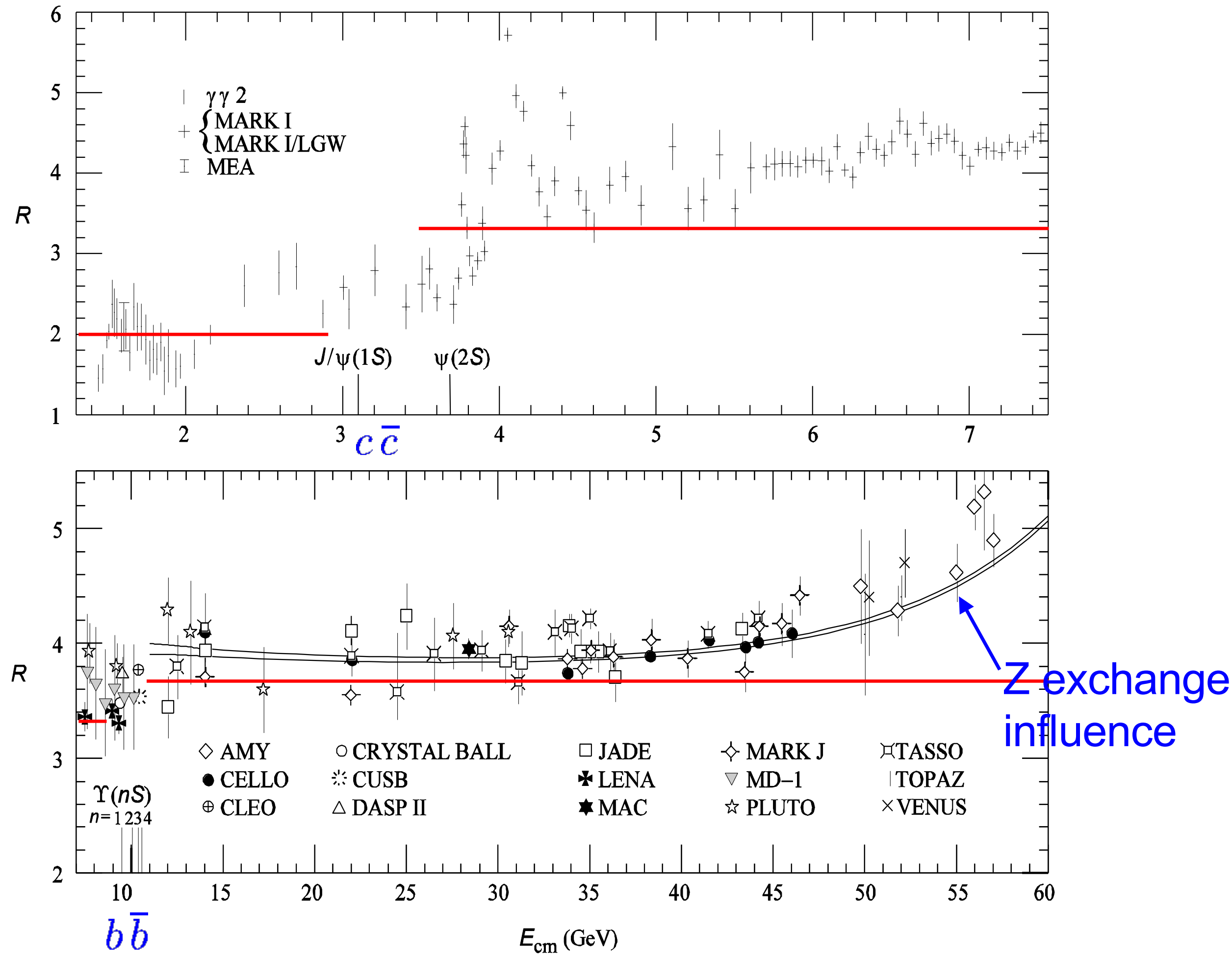
R_{ee}

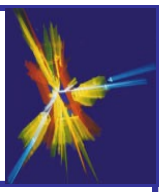


$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \approx \frac{\sum_q \sigma(e^+e^- \to q\,\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} \approx N_C \sum_q Q_q^2$$

$$= \begin{cases} \frac{2}{3} N_C & , \quad (u,d,s) \\ \frac{10}{9} N_C & , \quad (u,d,s,c) \\ \frac{11}{9} N_C & , \quad (u,d,s,c,b) \end{cases}$$

circumstancial evidence for $N_C = 3$



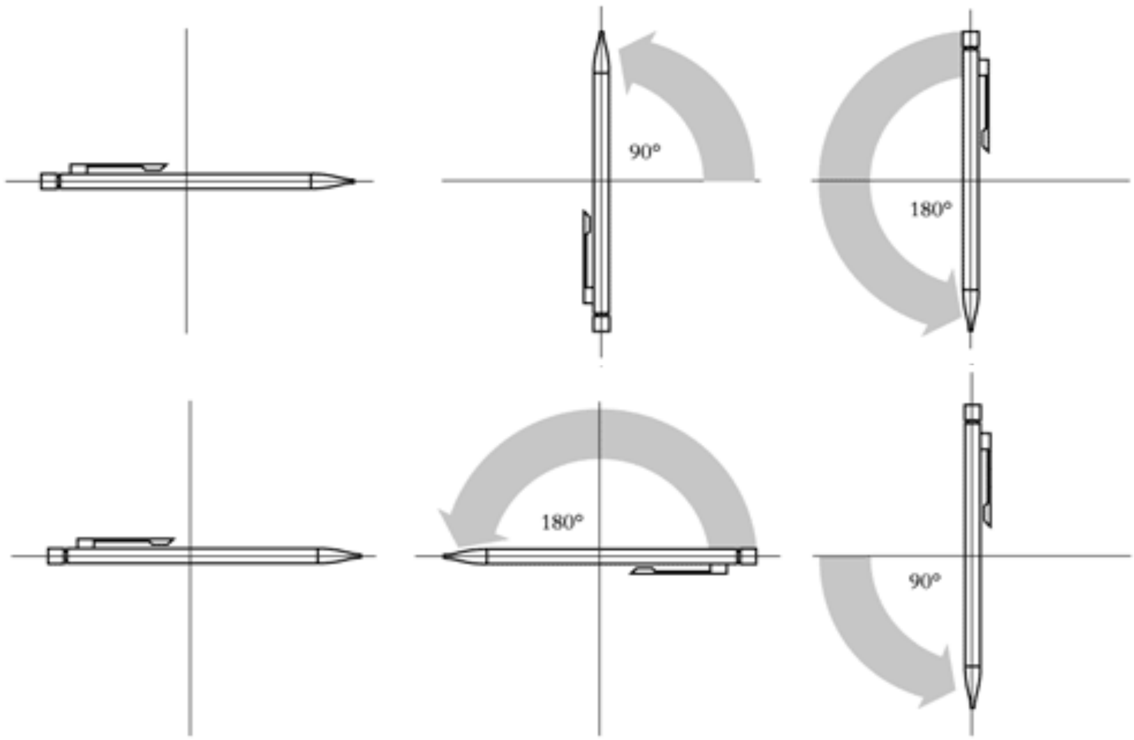


Group Theory

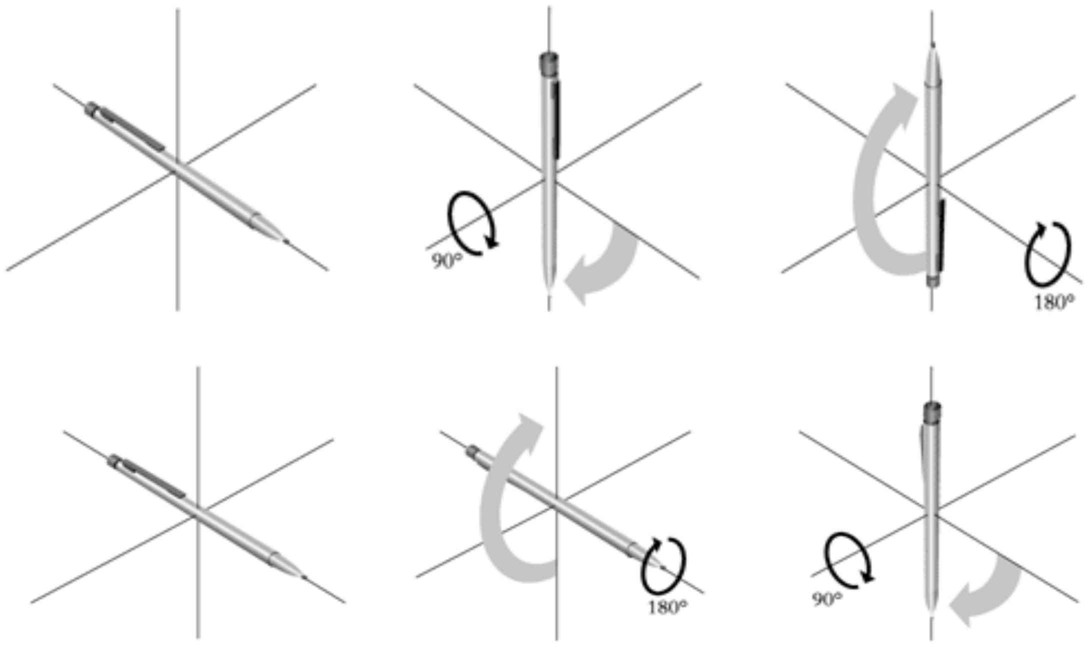
- An example of a group is the set of rotations of a system (2 successive rotations described by 1, inverse rotations & identity elements (no rotation) exist). The rotation group is a Lie group i.e. each rotation can be expressed as a set of successive infinitesimal rotations $1-i\varepsilon J_{k.}$
- The result of the experiment should not depend on the specific laboratory orientation hence the rotation group a symmetry group & the physics invariant under rotations.
- Assume $|\Psi\rangle \rightarrow |\Psi'\rangle = U\,|\Psi\rangle$ under a rotation; probability that system $|\Psi\rangle$ is in state $|\phi\rangle$ must be unchanged $|\langle\phi'\,|\,\Psi'\rangle|^2 = |\langle\phi\,|\,\Psi\rangle|^2 \Rightarrow U^\dagger U = 1$ U is a unitary operator
- Hamiltonian unchanged: $\langle\phi'\,|\,H\,|\,\Psi'\rangle = \langle\phi\,|\,H\,|\,\Psi\rangle \Rightarrow$ $[U,H] = 0$ & a constant of motion i.e. there is a conserved quantity. Rotations $\rightarrow$ angular momentum conservation.
- For rotations the J_k's are called <u>generators</u>.
Since $U^\dagger U = 1$, they are hermitian $J_k^\dagger = J_k$.
They are related by $[J_{i.}, J_{j.}] = i\varepsilon_{ijk}\, J_k$.
Due to this the rotational group is a non-abelian group.
- The lowest-dimensional non-trivial generators of the rotational group are the Pauli matrices. The basis they describe is a spin ½ particle (spin up & down).
- The Pauli matrices are also traceless hence called special unitary matrices and form the group SU(2). They also constitute its <u>fundamental representation</u>, basis for the building of all other representations.

Abelian vs non-abelian

Abelian group: rotation on a plane



Non-abelian group: rotation in space



LAGRANGIAN FORMALISM

$$S = \int d^4x \; \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\delta S = 0 \quad \rightarrow \quad \frac{\partial \mathcal{L}}{\partial \phi} - \partial^\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi)} \right) = 0$$

Eq. Motion

Klein–Gordon: (spin 0)

$$\mathcal{L} = \partial^\mu \phi^* \, \partial_\mu \phi - m^2 \, \phi^* \, \phi \quad \rightarrow \quad \left(\Box + m^2 \right) \phi = 0$$

Dirac: (spin $\frac{1}{2}$) $\bar{\psi} \equiv \psi^\dagger \gamma^0$

$$\mathcal{L} = \bar{\psi} \, (i \, \gamma^\mu \, \partial_\mu - m) \, \psi \quad \rightarrow \quad (i \, \gamma^\mu \, \partial_\mu - m) \, \psi = 0$$

Gauge Invariance

Quantum Mechanics

- A QM state is described by a complex wave function $\Psi(x)$
- Observables: $< O >= \int dx \Psi^*(x) O \Psi(x)$

$O = O^\dagger$

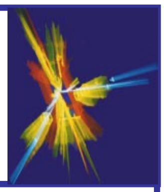
$\Psi \to e^{i\theta}\,\Psi,\ \Psi^* \to \Psi^* e^{-i\theta}$

- The "global" gauge θ is abritrary & non-measurable
- Make a position dependent change of phase ("local" gauge): $\Psi \to e^{i\alpha(x)}\,\Psi$
- Schrödinger equation involves derivates,

$(-\nabla^2\Psi / 2m = i\ \partial\Psi/\partial t)$

$$\partial_\mu\Psi \to \partial_\mu[e^{i\alpha(x)}\,\Psi(x)] = e^{i\alpha(x)}\,[\partial_\mu\Psi(x) + i\,\Psi(x)\partial_\mu\alpha(x)]$$

$\Rightarrow$ Local phase invariance is spoiled



How to proceed ?

- Add a four vector to remove the extra stuff :

  $\partial_\mu \rightarrow D_\mu \equiv \partial_\mu - ieA_\mu$ (e = e.m. charge)
- D_μ transformation properties require

  $D_\mu\psi \rightarrow e^{i\alpha(x)} D_\mu\psi(x)$

- Thus $(\partial_\mu - ieA'_\mu)(e^{i\alpha(x)}\psi(x)) = D'_\mu\psi'(x)$

  $= e^{i\alpha(x)} [\partial_\mu + i(\partial_\mu\alpha(x)) - ieA'_\mu]\psi(x)$

  $= e^{i\alpha(x)} [\partial_\mu - ieA_\mu]\psi(x)$

- This is true IF $A_\mu \rightarrow A'_\mu = A_\mu + 1/e\, \partial_\mu\alpha(x)$

  $\Rightarrow \psi^* D_\mu\psi$ is locally phase invariant

  Momentum operator $p_\mu = i\hbar\, \partial_\mu \rightarrow i\hbar\, D_\mu$

  (= covariant derivative)

Mass of the photon ?

A mass term $L = \frac{1}{2} m_\gamma^2 A_\mu A^\mu$ violates local phase invariance:

$A'_\mu A'^\mu = [A_\mu + 1/e\, \partial_\mu\alpha]\,[A^\mu + 1/e\, \partial^\mu\alpha] \neq A_\mu A^\mu$

- Local phase invariance $\Rightarrow m_\gamma = 0$

Gauge principle: Phase invariance should hold locally !!

OK for photon but what about $W^\pm$, Z^0? More later !!

Gauge Principle

- Symmetries has always played an important role in physics especially in particle physics
- Noethers theorem: If an action is invariant under some group transformations, there exists one or more conserved quantities. Example: rotational invariance ⇒ angular momentum conservation
- Salam & Ward (1961): Possible to generate interaction terms for forces by requiring local gauge transformation invariance of the free Lagrangian

What does that mean e.g. for a U(1) group?

- Start from the Dirac free Lagrangian:

$$L_\psi = \bar{\psi}(i\not\partial - m)\psi \;;\; \not\partial = \partial_\mu \gamma^\mu$$

- In local gauge transformation:

$$\psi \to e^{i\alpha(x)}\psi \;; \qquad L_\psi \to L_\psi + \bar{\psi}\gamma_\mu \psi \partial^\mu \alpha$$

- Introducing a gauge field A_μ through minimal coupling: $D_\mu = \partial_\mu - ieA_\mu$ and require:

$$A_\mu \to A_\mu + 1/e\, \partial_\mu \alpha(x)$$

- Then $L_\psi \to L_\psi + e\bar{\psi}\gamma_\mu \psi A^\mu$

- The Lagrangian for the free gauge field is:

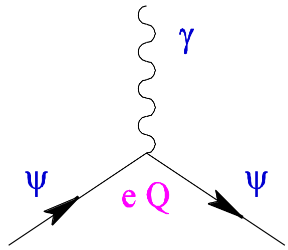
$$L_A = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \;;\; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

QUANTUM ELECTRODYNAMICS

based on U(1) gauge invariance

$$\mathcal{L} = \bar{\psi}\,(i\,\gamma^{\mu}\,D_{\mu} - m)\,\psi = \bar{\psi}\,(i\,\gamma^{\mu}\,\partial_{\mu} - m)\,\psi + \mathcal{L}_I$$

$$\mathcal{L}_I = e\,Q\,A_{\mu}\,\left(\bar{\psi}\,\gamma^{\mu}\,\psi\right)$$



conservation of electric charge

Kinetic term: **Maxwell**

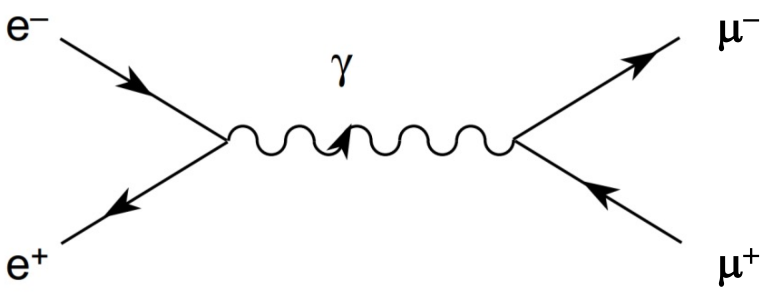
$$\mathcal{L}_K = -\tfrac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu} \;\Rightarrow\; \partial_{\nu}F^{\nu\mu} = -e\,Q\,\left(\bar{\psi}\,\gamma^{\mu}\,\psi\right)$$

$$\mathcal{L}_m = \tfrac{1}{2}\,m_{\gamma}^{2}\,A^{\mu}\,A_{\mu}$$ **Not Gauge Invariant**

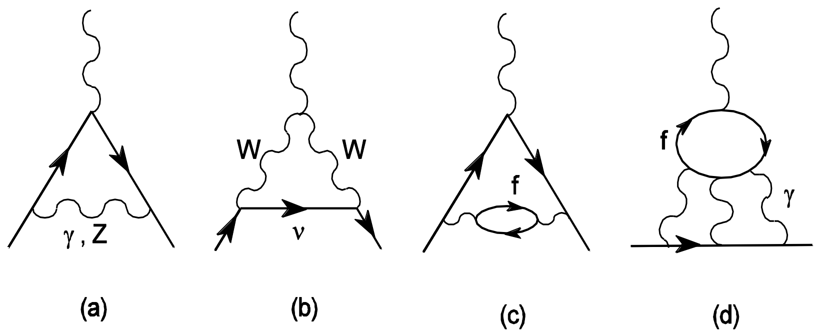
$\Rightarrow$ $m_{\gamma} = 0$ [exp: $m_{\gamma} < 1 \cdot 10^{-18}$ eV]

Gauge Symmetry $\Rightarrow$ **QED Dynamics**

Successful Theory



Anomalous Magnetic Moment



$\mu_l \equiv g_l \frac{e}{2m_l}$; $a_l \equiv \frac{1}{2}(g_l - 2)$

$a_e = (115\,965\,218.073 \pm 0.028) \cdot 10^{-11}$ ⇒
$1/\alpha_{em} = 137.035\,999\,150 \pm 0.000\,000\,033$ ⇒
$a_\mu^{th} = (116\,591\,834 \pm 44) \cdot 10^{-11}$

[Exp: $(116\,592\,055 \pm 24) \cdot 10^{-11}$]

Lattice calculations: a_μ^{th} closer to exp, value

precision up to 8th significant digit, but ~4.4σ discrepancy

SU(N) ALGEBRA

$N \times N$ matrices: $U U^\dagger = U^\dagger U = 1$; $\det U = 1$

$$\rightarrow \quad U = \exp\{i T^a \theta_a\}$$

$T^a = T^{a\dagger}$; $\mathrm{Tr}(T^a) = 0$; $a = 1, \ldots, N^2 - 1$

Commutation Relation:

$$[T^a, T^b] = i f^{abc} T^c$$

Structure Constants f^{abc} real , antisymmetric

Fundamental Representation: $T_F^a = \frac{\lambda^a}{2}$ $N \times N$

The local gauge transformation invariance of the Lagrangian for the U(1) group can be generalized to SU(N) groups:

$D_\mu = \partial_\mu + igT^a A^a_\mu$

then $F^{a\mu\nu} = \partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} - g f^{abc} A^{b\nu} A^{c\mu}$

and $L_A = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$

SU(2)

2×2 **matrices:** $U\,U^\dagger = U^\dagger\,U = 1 \quad ; \quad \det U = 1$

$$\Rightarrow \quad U = \exp\{i\,T^a\,\theta_a\}$$

$$T^a = T^{a\dagger} \quad ; \quad \mathrm{Tr}\,(T^a) = 0 \quad ; \quad a = 1, \ldots, 3$$

Commutation Relation:

$$\left[T^a, T^b\right] = i\,\epsilon^{abc}\,T^c$$

Fundamental Representation: $T_F^a = \frac{1}{2}\sigma^a$ **Pauli**

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; ; \; \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \; ; \; \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\left\{\sigma^a, \sigma^b\right\} = 2\,\delta_{ab}$$

SU(3)

$$\left[T^a, T^b\right] = i\, f^{abc}\, T^c$$

Fundamental Rep.: $T_F^a = \frac{1}{2}\lambda^a$ **Gell-Mann**

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; ; \; \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; ; \; \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \; ; \; \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \; ; \; \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

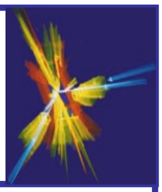
$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\frac{1}{2} f^{123} = f^{147} = -f^{156} = f^{246} = f^{257} = f^{345}$$

$$= f^{345} = -f^{367} = \frac{1}{\sqrt{3}} f^{458} = \frac{1}{\sqrt{3}} f^{678} = \frac{1}{2}$$

Quantum chromodynamics

- Basic structure
- QCD Lagrangian
- Quantum corrections
- Running couplings
- α_S measurements



QUANTUM CHROMODYNAMICS

$$N_C = 3 \quad \rightarrow \quad \mathbf{q} \equiv \begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

FREE QUARKS: $\mathcal{L} = \bar{\mathbf{q}}\,[i\,\gamma^\mu \partial_\mu - m]\,\mathbf{q}$

SU(3) Colour Symmetry:

$$\mathbf{q} \rightarrow \mathbf{U}\,\mathbf{q} = \exp\left\{-i\,g_s \frac{\lambda^a}{2}\,\theta_a\right\}\mathbf{q}$$

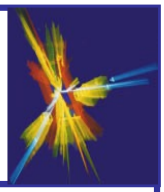
Gauge Principle: Local Symmetry $\theta_a = \theta_a(x)$

$$\mathbf{D}^\mu \mathbf{q} \equiv (\mathbf{I}_3\,\partial^\mu - i\,g_s\,\mathbf{G}^\mu)\,\mathbf{q} \rightarrow \mathbf{U}\;\mathbf{D}^\mu \mathbf{q}$$

$$\mathbf{D}^\mu \rightarrow \mathbf{U}\,\mathbf{D}^\mu\,\mathbf{U}^\dagger \quad ; \quad \mathbf{G}^\mu \rightarrow \mathbf{U}\,\mathbf{G}^\mu\,\mathbf{U}^\dagger - \frac{i}{g_s}\,(\partial^\mu \mathbf{U})\,\mathbf{U}^\dagger$$

$$[\mathbf{G}^\mu]_{\alpha\beta} \equiv \frac{(\lambda^a)_{\alpha\beta}}{2}\,G^\mu_a(x)$$

8 Gluon Fields



Infinitesimal SU(3) Transformation:

$$\mathbf{q}^{\alpha} \rightarrow \mathbf{q}^{\alpha} - i\, g_s\, \delta\theta_a \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} \mathbf{q}^{\beta}$$

$$G_a^{\mu} \rightarrow G_a^{\mu} - \partial^{\mu}(\delta\theta_a) + g_s\, f^{abc}\, \delta\theta_b\, G_c^{\mu}$$

Non Abelian Group: $f^{abc} \neq 0$

- δG_a^{μ} depends on G_a^{μ}
- Universal g_s **No Colour Charges**

Kinetic Term:

$$\mathbf{G}^{\mu\nu} \equiv \frac{i}{g_s}[\mathbf{D}^{\mu}, \mathbf{D}^{\nu}] = \partial^{\mu}\mathbf{G}^{\nu} - \partial^{\nu}\mathbf{G}^{\mu} - i\, g_s\, [\mathbf{G}^{\mu}, \mathbf{G}^{\nu}]$$

$$\mathbf{G}^{\mu\nu} \rightarrow \mathbf{U}\, \mathbf{G}^{\mu\nu}\, \mathbf{U}^{\dagger} \quad ; \quad \mathbf{G}^{\mu\nu} \equiv \frac{\lambda^a}{2} G_a^{\mu\nu}$$

$$G_a^{\mu\nu} = \partial^{\mu} G_a^{\nu} - \partial^{\nu} G_a^{\mu} + g_s\, f^{abc}\, G_b^{\mu}\, G_c^{\nu}$$

$$\mathcal{L}_K = -\frac{1}{2}\,\mathrm{Tr}\,(\mathbf{G}^{\mu\nu}\,\mathbf{G}_{\mu\nu}) = -\frac{1}{4}\, G_a^{\mu\nu}\, G^a_{\mu\nu}$$

Mass term for gluons? Not gauge invariant ⇒ $\mathrm{m_G} = 0$

QCD LAGRANGIAN

$$\mathcal{L}_{QCD} = -\tfrac{1}{2}\,\mathrm{Tr}\,(\mathrm{G}^{\mu\nu}\,\mathrm{G}_{\mu\nu}) + \bar{\mathrm{q}}\,[i\,\gamma^{\mu}\,\mathrm{D}_{\mu} - m_q]\,\mathrm{q}$$

$$= -\frac{1}{4}\left(\partial^{\mu}G_a^{\nu} - \partial^{\nu}G_a^{\mu}\right)\left(\partial_{\mu}G_{\nu}^{a} - \partial_{\nu}G_{\mu}^{a}\right)$$

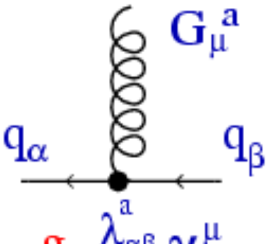
Kinetic

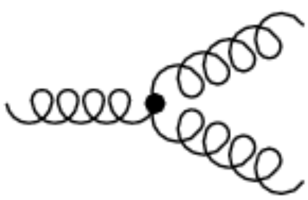
$$+ \sum_q \bar{q_\alpha}\,[i\,\gamma^{\mu}\,\partial_{\mu} - m_q]\,q_\alpha$$

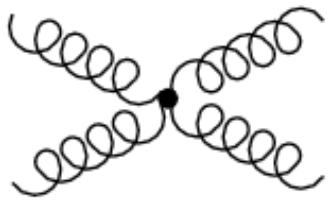
$$+ \frac{1}{2}\sum_q g_s\,\left[\bar{q_\alpha}\,(\lambda^a)_{\alpha\beta}\,\gamma^{\mu}\,q_\beta\right]\,G_{\mu}^{a}$$

$$- \frac{1}{2}\,g_s\,f_{abc}\,\left(\partial_{\mu}G_{\nu}^{a} - \partial_{\nu}G_{\mu}^{a}\right)\,G_b^{\mu}\,G_c^{\nu}$$

$$- \frac{1}{4}\,g_s^2\,f_{abc}\,f_{ade}\,G_b^{\mu}\,G_c^{\nu}\,G_{\mu}^{d}\,G_{\nu}^{e}$$

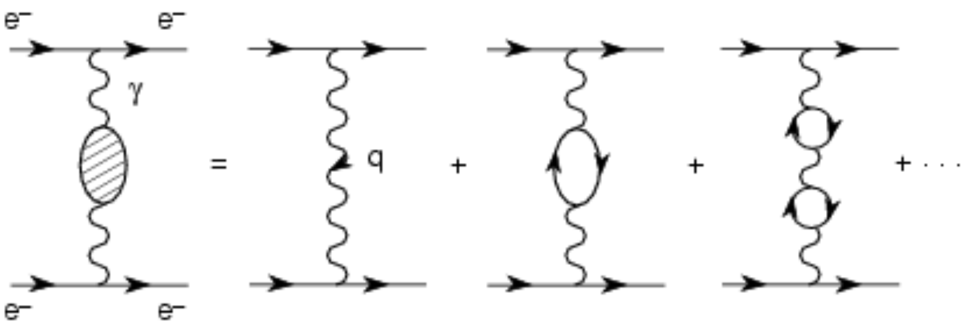






- **Gluon Self-interactions** G^3, G^4
- **Universal Coupling** g_s
- **No Colour Charges**

QUANTUM CORRECTIONS



$$T(Q^2) \sim \frac{\alpha}{Q^2} \left\{1 + \Pi(Q^2) + \Pi(Q^2)^2 + \cdots\right\} \sim \frac{\alpha(Q^2)}{Q^2}$$

Effective *(Running)* **Coupling:**

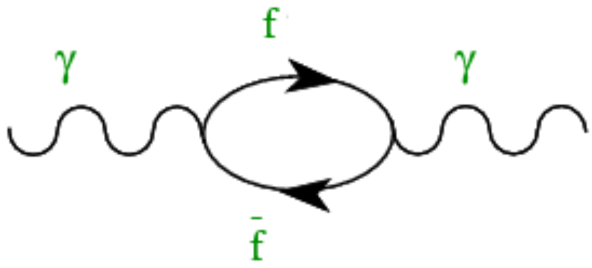
$$\alpha(Q^2) = \frac{\alpha}{1 - \Pi(Q^2)} \approx \frac{\alpha}{1 - \frac{\alpha}{3\pi} \ln\left(\frac{Q^2}{m^2}\right)}$$

$\alpha(Q^2)$ Increases with $Q^2 \equiv -q^2$

$\Longrightarrow$ increases at shorter distances
$\Longrightarrow$ decreases at larger distances

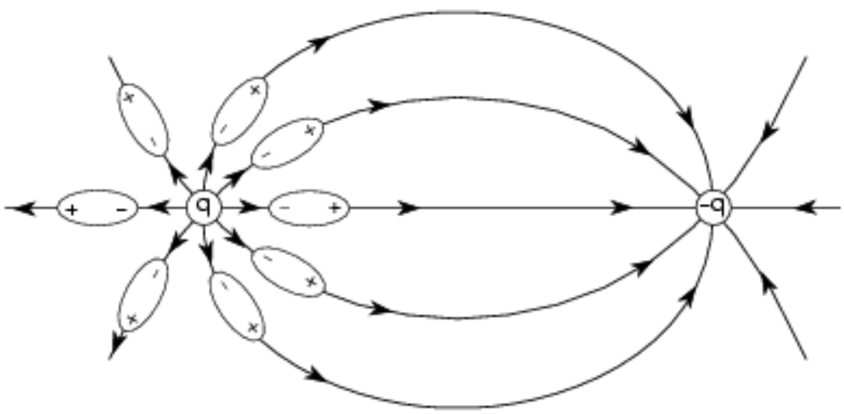
SCREENING

VACUUM POLARIZATION



The Photon Couples to *Virtual* $f\,\bar{f}$ *Pairs*

Vacuum ⟷ Polarized Dielectric Medium

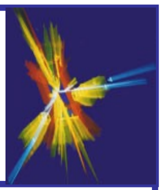


$$1/\alpha = 1/\alpha\,(m_e^2) = 137.035999180\,(10)$$

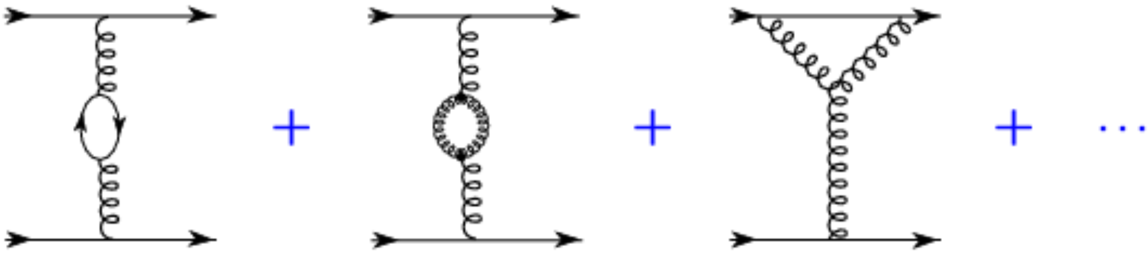
$$1/\alpha\,(m_Z^2) = 127.951 \pm 0.009$$

($l^- l^+$ and $q\,\bar{q}$ contributions included)

source: PDG review on electro-weak model



QCD RUNNING COUPLING



$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 - \beta_1 \frac{\alpha_s(Q_0^2)}{2\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)}$$

$$\beta_1 = \frac{1}{3} N_F - \frac{11}{6} N_C$$

quarks gluons

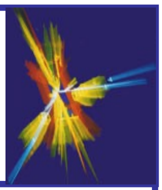
$N_C = 3$, $N_F = 6$ ➡ $\beta_1 < 0$

$Q^2 > Q_0^2$ ➡ $\alpha_s(Q^2) < \alpha_s(Q_0^2)$

$\alpha_s(Q^2)$ **Decreases at Short Distances**

ANTI–SCREENING

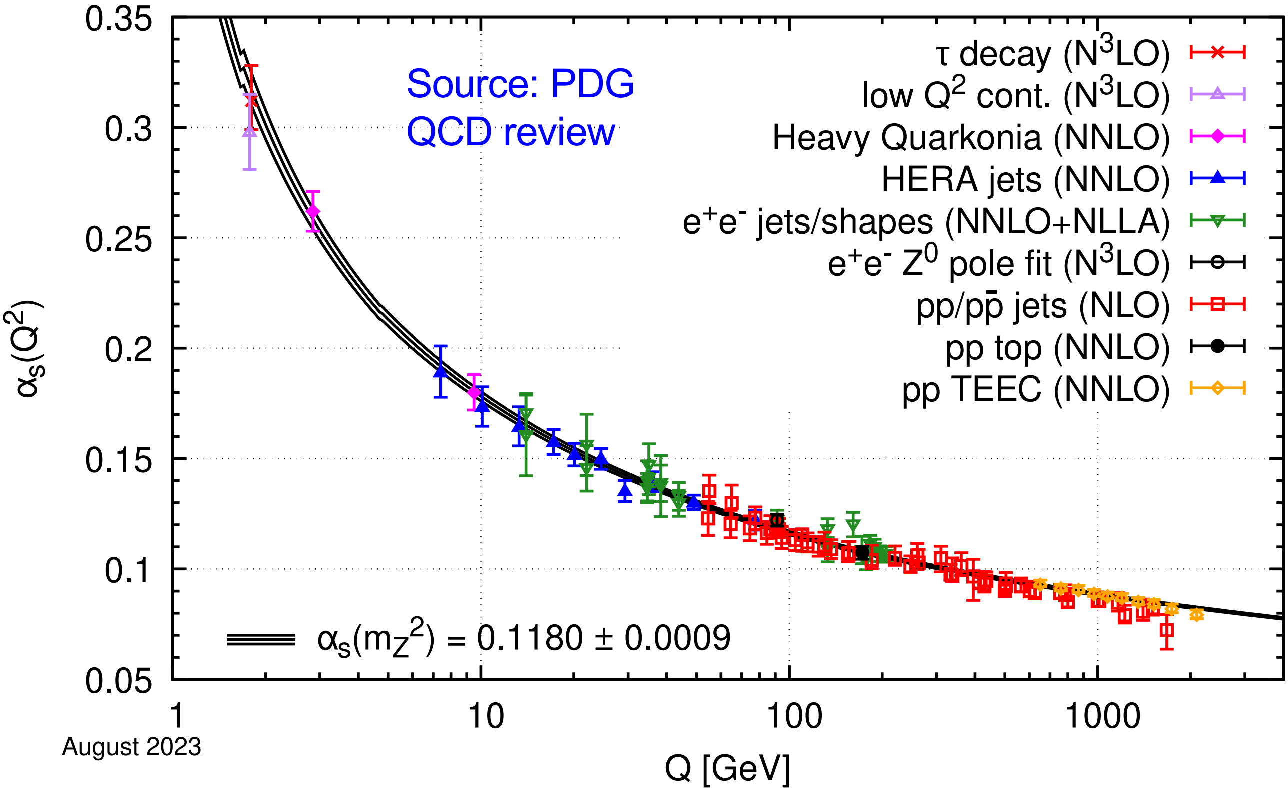
Asymptotic freedom & confinement



$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 - \beta_1 \frac{\alpha_s(Q_0^2)}{2\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)}$$

$\beta_1 < 0 \quad \rightarrow \quad \lim_{Q^2 \to \infty} \alpha_s(Q^2) = 0$

ASYMPTOTIC FREEDOM

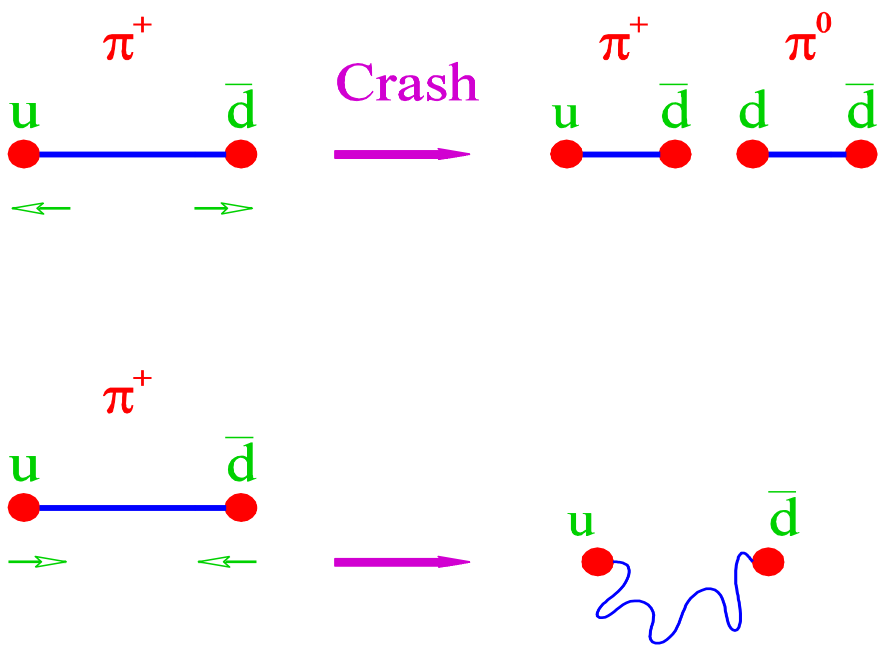


$\alpha_s(Q^2)$ **increases at low energies**

$\alpha_s \sim O(1)$ at 1 GeV   Non-Perturbative Region

$\rightarrow$ **CONFINEMENT ?**

CONFINEMENT



ASYMPTOTIC FREEDOM

Asymptotic freedom:

$\alpha_s \rightarrow 0$ at large E (short distances)

Confinement:

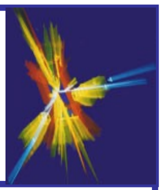
large α_s at small E (large distances)

Quark flavour (u,d,s,c,b,t):

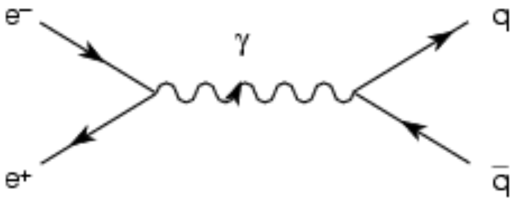
strong interactions — flavour independent & conserving

weak interactions — change quark flavour

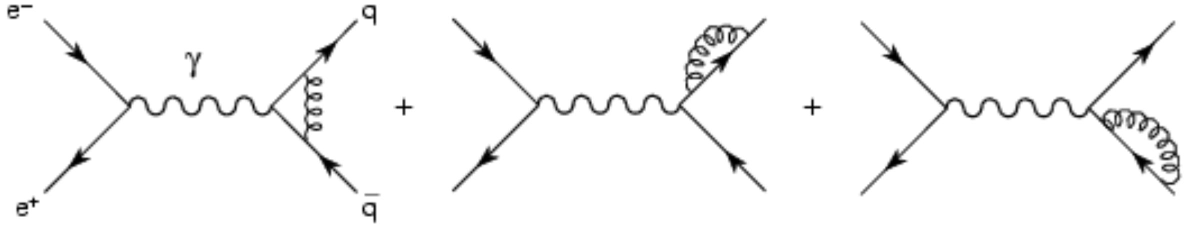
e^+e^- annihilation



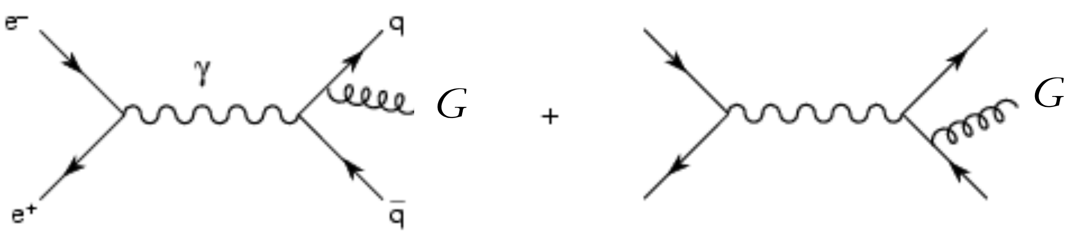
$$T\left(e^+e^- \rightarrow q\,\bar{q}\right) =$$



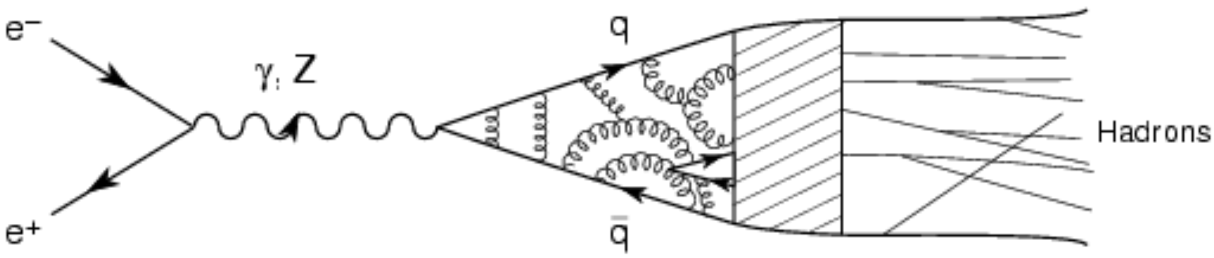
$$+$$



$$T\left(e^+e^- \rightarrow q\,\bar{q}\,G\right) =$$



Confinement ⟷ Prob. Hadronization = 1



$$\sigma(e^+e^- \to \text{hadrons}) =$$

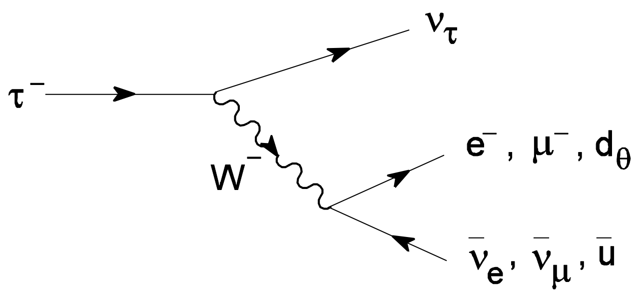
$$\sigma\left(e^+e^- \to q\bar{q} + q\bar{q}G + q\bar{q}GG + q\bar{q}q\bar{q} + \cdots\right)$$

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_q Q_q^2 \, N_C \quad \left\{1 + \frac{\alpha_s(s)}{\pi} + \cdots\right\}$$

$$R_Z \equiv \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to e^+e^-)} = R_Z^{EW} \, N_C \quad \left\{1 + \frac{\alpha_s(s)}{\pi} + \cdots\right\}$$

$\tau^- \rightarrow \nu_\tau$ + Hadrons

$$d_\theta = \cos\theta_C d + \sin\theta_C s$$



$$B_l \equiv Br(\tau^- \rightarrow \nu_\tau l^- \bar{\nu}_l) \approx 1/(2+N_C) = 1/5 = 20\%$$

difference mainly due to QCD

$$B_e = (17.82 \pm 0.04)\ \% \qquad B_\mu = (17.39 \pm 0.04)\ \%$$

QCD :

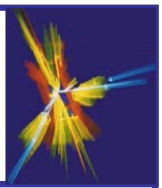
$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \nu_\tau)} = N_C(1+S_{ew})\left\{1 + \frac{\alpha_s(m_\tau^2)}{\pi} + 5.2\frac{\alpha_s(m_\tau^2)^2}{\pi^2} + \ldots\right\}$$

$$R_\tau = 3.636 \pm 0.010 \quad \Rightarrow \quad \alpha_s(m_\tau^2) = 0.314 \pm 0.014$$

$$\alpha_s(m_\tau^2) = 0.314 \pm 0.014 \quad \gg \quad \alpha_s(m_Z^2) = 0.1180 \pm 0.0009$$

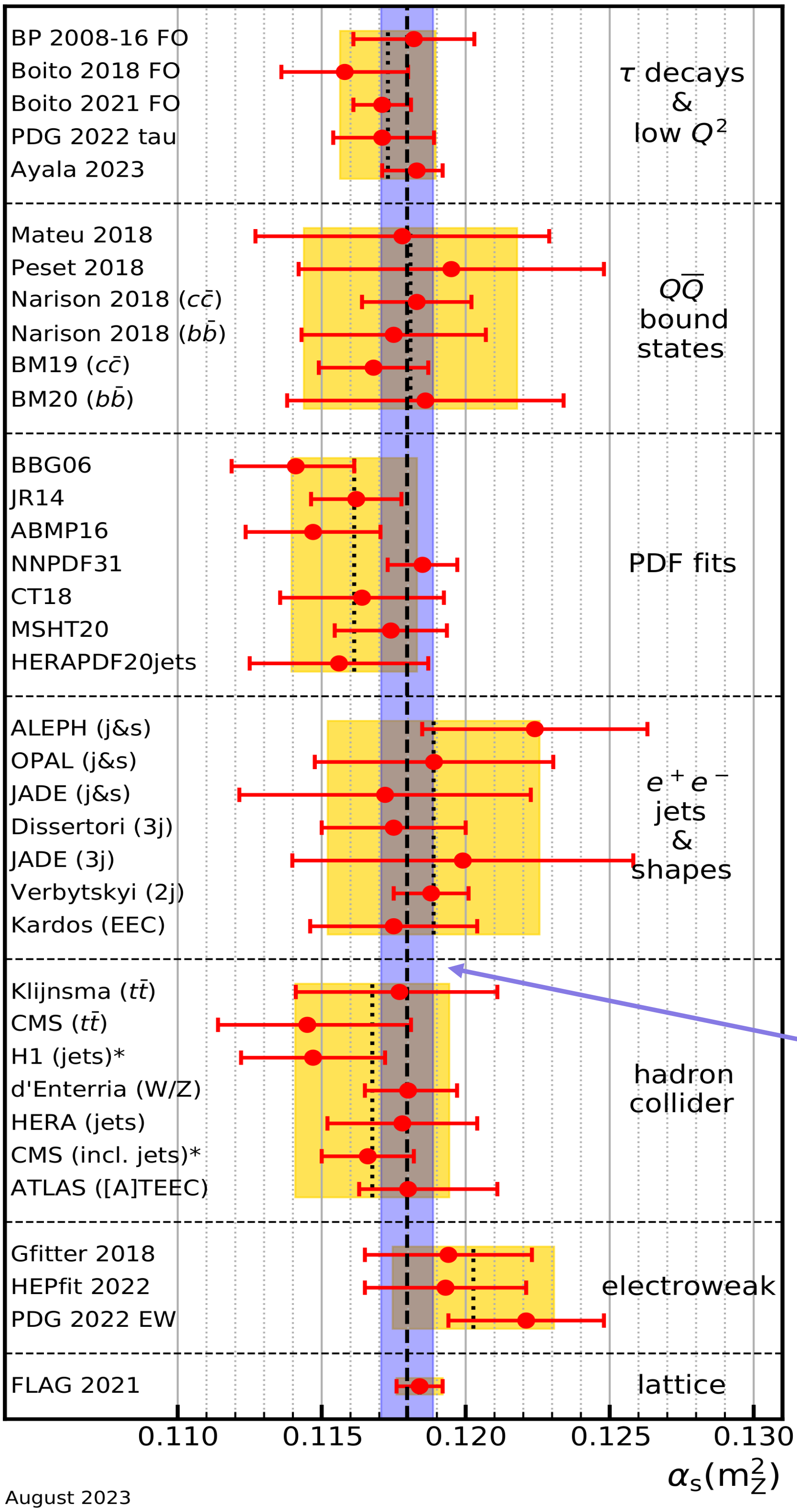
NB! electroweak correction $S_{ew} \approx 0.019$

Sources: PDG QCD & electro-weak reviews



Summary of different α_s measurements:

(for details see Quantum chromodynamics review in PDG)



PDG 2023: $\alpha_s(m_Z^2) =$ 0.1180 ± 0.0009