found. The existence of gaugelike invariance for the equation satisfied by (u, ξ) makes it possible to find a self-Bäcklund transformation for q and q'. These Bäcklund transformations can be divided into classes. Equations in the same class have an identical spatial part of their Bäcklund transformations. Their solutions therefore satisfy the same superposition formula. For example the mKdV and sine-Gordon equations have the same superposition formula,

$$\tan\frac{w_3 - w_0}{2} = \frac{k_1 + k_2}{k_1 - k_2}\tan\frac{w_1 - w_2}{2}.$$

This formula renders it possible to construct N-soliton solutions by algebraic manipulations only.

Further generalizations to higher-order inverse problems of what has been done above is possible. I will report some examples in another paper.

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Total Cross Sections of p and $\bar{p}$ on Protons and Deuterons between 50 and 200 GeV/c*

A. S. Carroll, I-H. Chiang, T. F. Kycia, K. K. Li, P. O. Mazur, P. Mockett,† D. C. Rahm, and R. Rubinstein‡
Brookhaven National Laboratory, Upton, New York 11973

and

W. F. Baker, D. P. Eartly, G. Giacomelli,§ P. F. M. Koehler, K. P. Pretzl,‖ and A. A. Wehmann
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

and

R. L. Cool and O. Fackler
Rockerfeller University, New York, New York 10021
(Received 15 July 1974)

Proton and antiproton total cross sections on protons and deuterons have been measured at 50, 100, 150, and 200 GeV/c. The proton cross sections rise with increasing momentum. Antiproton cross sections fall with increasing momentum, but the rate of fall decreases between 50 and 150 GeV/c, and from 150 to 200 GeV/c there is little change in cross section.

We have measured p and $\bar{p}$ total cross sections on protons and deuterons in 50-GeV/c steps between 50 and 200 GeV/c. The experiment, which was carried out in the $M1$ beam[1,2] at the Fermi National Accelerator Laboratory, used a "good geometry" transmission technique.

Incident particles were defined by scintillation counters and identified by two differential gas Cherenkov counters,[3] allowing cross sections of two different particles to be measured simultaneously; in addition, a threshold gas Cherenkov counter[4] could be used in anticoincidence when required. Contamination of unwanted particles in the selected p and $\bar{p}$ beams was always below 0.1%.

The 3-m-long liquid hydrogen and deuterium targets and an identical evacuated target were surrounded by a common outer jacket of liquid hydrogen for temperature stability.[5] By continuously monitoring the vapor pressure in the outer jacket, the target temperature and therefore the hydrogen and deuterium densities were determined[6]; density variations were less than 0.07% throughout the experiment. Target lengths were

TABLE I. Results of this experiment: Cross sections in millibarns.

	Momentum (GeV/c) 50	100	150	200	Momentum-independent scale uncertainty
σ_{pp}	38.14 ± 0.07	38.39 ± 0.06	38.62 ± 0.06	38.90 ± 0.06	$\pm0.5\%$
σ_{pd}	72.98 ± 0.13	73.12 ± 0.11	73.46 ± 0.11	73.84 ± 0.11	$\pm0.6\%$
$\sigma_{\bar{p}p}$	43.86 ± 0.11	42.04 ± 0.09	41.72 ± 0.18	41.54 ± 0.29	$\pm0.5\%$
$\sigma_{\bar{p}d}$	82.21 ± 0.24	79.32 ± 0.19	78.24 ± 0.35	78.77 ± 0.57	$\pm0.6\%$
σ_{pn}	38.86 ± 0.16	38.85 ± 0.14	39.02 ± 0.14	39.18 ± 0.14	$\pm1.5\%$
$\sigma_{\bar{p}n}$	43.69 ± 0.30	42.22 ± 0.23	41.32 ± 0.44	42.09 ± 0.71	$\pm1.5\%$
$\sigma_{\bar{p}p}-\sigma_{pp}$	5.72 ± 0.13	3.65 ± 0.11	3.10 ± 0.19	2.64 ± 0.30	
$\sigma_{\bar{p}d}-\sigma_{pd}$	9.23 ± 0.28	6.20 ± 0.22	4.78 ± 0.37	4.92 ± 0.58	
$\sigma_{\bar{p}n}-\sigma_{pn}$	4.83 ± 0.34	3.37 ± 0.27	2.30 ± 0.46	2.91 ± 0.72	

measured under operating conditions to $\pm0.03\%$.

The transmission through the targets was measured by twelve scintillation counters of different diameters, with eleven independent channels being formed by coincidences between pairs of adjacent counters to minimize accidental counts and tube noise. These counters were mounted together, smallest upstream, on a movable cart and were positioned such that for each momentum the counters accepted the same range of $|t|$, extending up to 0.008 (GeV/c)^2 for the smallest channel and up to 0.08 (GeV/c)^2 for the largest. The efficiencies of the transmission counters were measured at frequent intervals throughout the experiment using two small counters placed behind them. Such efficiencies were constant and always $>99.8\%$

For each momentum, the beam was tuned to give a final focus at the transmission counters. Two sets of proportional wire chambers were in the incident beam, each set giving two coordinates, and a matrix coincidence was set up between appropriate wires to ensure that each incident particle trajectory would pass through a 2-cm square at the transmission counters.[7] This technique eliminates systematic effects in the extrapolation procedure from beam halo and possible beam instability. In addition, the electronic logic for these chambers required that one and only one particle register in each chamber. This, together with large veto counters around the beam, eliminated possible fluctuations due to accidentals. Cross sections were found to be stable to better than 0.2% for variations in beam flux of a factor of 3 around the value (2×10^5 per pulse) at which data were normally taken.

The data were corrected[8] for single Coulomb scattering ($<0.1\%$) and Coulomb-nuclear interference ($<0.3\%$). For the latter, the ratio ρ of real to imaginary parts of the forward scattering amplitude was obtained from Bartenev *et al.*[9] for pp and the predictions of Cheng *et al.*[10] for $\bar{p}p$. The value of ρ for neutrons was assumed to be the same as for protons.

The extrapolation to $t=0$ of the partial cross sections was carried out using the expression

$$\sigma_i=\sigma_T\exp(At_i+Bt_i^2+Ct_i^3),$$

where σ_i is the partial cross section measured by the ith transmission counter combination subtending a maximum $|t_i|$, and σ_T is the total cross section. For all of the four cross sections measured here, the Bt_i^2 term was necessary, as determined by a substantial reduction in the χ^2 of the fit when it was added. For cross sections on protons, there was no change in the total cross section when the Ct_i^3 term was added, nor was there any change in χ^2, and so C was set equal to zero; for deuteron cross sections, the Ct_i^3 term was found to improve the fit substantially. The extrapolations were carried out using the third through the tenth transmission counter combinations, covering $0.016\leq|t_i|\leq0.062$ (GeV/c)^2. Using fewer counters changed the results by less than 0.1%, indicating negligible multiple Coulomb scattering and beam size effects in the counters used for the extrapolation. This method, of course, cannot take into account a rapid change in slope below 0.016 (GeV/c)^2.

From the reproducibility of our data we quote a momentum-dependent uncertainty in the results for a particular incident particle of $\pm0.15\%$ for those cross sections where the statistical error was smaller than this. The momentum-indepen-

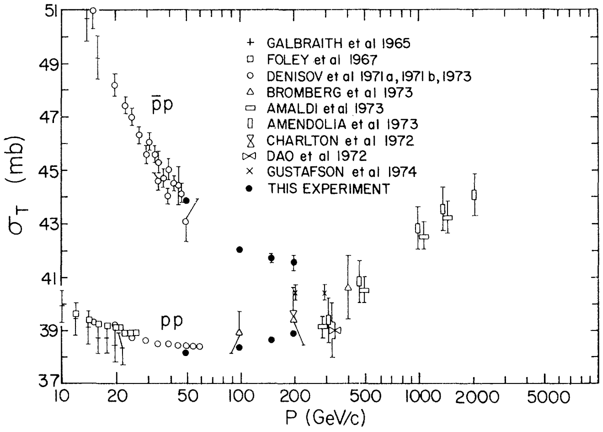

FIG. 1. Total cross sections for pp and $\bar{p}p$. Momentum-dependent errors only are shown. Data of other experiments are from Refs. 11-21.

dent scale uncertainty for the absolute magnitude of the cross section, caused by uncertainties in the form of the extrapolation and in the hydrogen and deuterium densities and contaminations, is estimated to be $\pm 0.5\%$ for protons and $\pm 0.6\%$ for deuterons.

The results are listed in Table I and shown in Figs. 1 and 2, together with previous data.[11-21] Agreement with other experiments in the same momentum range is within the quoted scale errors, except for that of Gustafson *et al.*[21]

As the incident momentum increases from 50 to 200 GeV/c the pp total cross section rises by 2%. The rise is consistent with the rise observed at the CERN intersecting storage rings.[17,18] The $\bar{p}p$ cross sections continue to fall with increasing momentum, but the rate decreases markedly, and above 150 GeV/c there is very little variation.

Cross sections of p and $\bar{p}$ on deuterons show a momentum dependence similar to those on protons. The $\bar{p}d$ cross section is also nearly constant above 150 GeV/c.

The antiparticle-particle differences $\sigma_{\bar{p}p} - \sigma_{pp}$ and $\sigma_{\bar{p}d} - \sigma_{pd}$ are shown in Fig. 3. These differences are becoming smaller with increasing momentum, and can be fitted by the form $As^{\alpha-1}$. Using only data from this experiment gives $\alpha = 0.39 \pm 0.04$ for $\sigma_{\bar{p}p} - \sigma_{pp}$ and $\alpha = 0.43 \pm 0.05$ for $\sigma_{\bar{p}d} - \sigma_{pd}$. If this form for $\sigma_{\bar{p}p} - \sigma_{pp}$ is extrapolated to higher momentum, together with the known σ_{pp}, an estimate of $\sigma_{\bar{p}p}$ at higher momenta can be obtained. This procedure predicts a minimum in the antiproton-proton cross section at about 200 GeV/c.

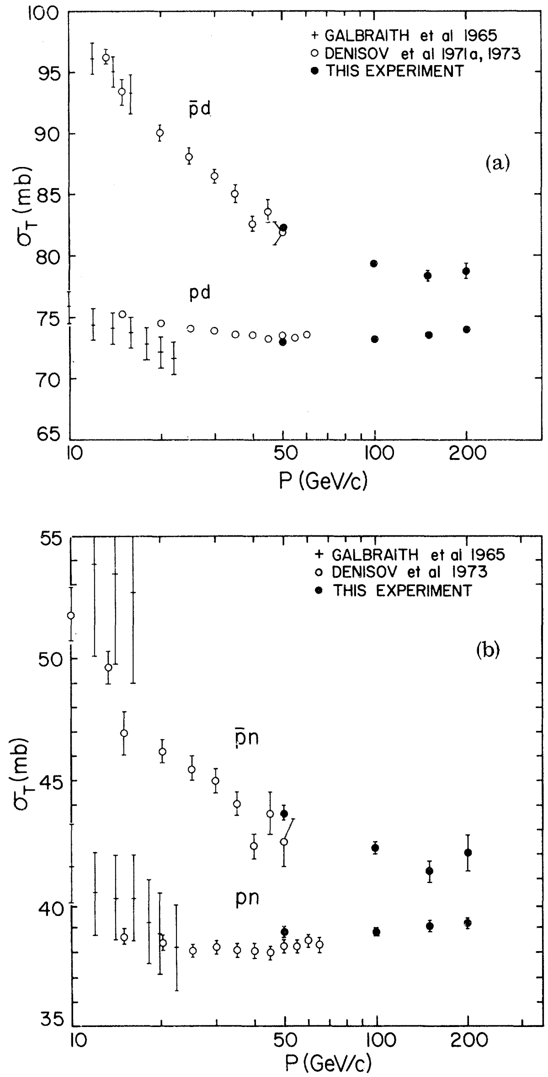

FIG. 2. Total cross sections for (a) pd and $\bar{p}d$, and (b) pn and $\bar{p}n$. Momentum-dependent errors only are shown. References as for Fig. 1.

The purpose of measuring deuteron cross sections is to extract cross sections on neutrons. We have done this to within the accuracy of the Glauber-Wilkin formula,[22,23] which takes into account the shadowing in the deuteron. A parameter $\langle r^{-2}\rangle$ is used in the formula, and we have derived it from our pion data[24]; it is consistent with being momentum independent and averages 0.039 mb^{-1}. There has been recent discussion as to whether this parameter is dependent upon the incident particle, or whether a more complex formula should be used.[25-28] Using the work of Gorin *et al.*[28] our measured value of $\langle r^{-2}\rangle$ would

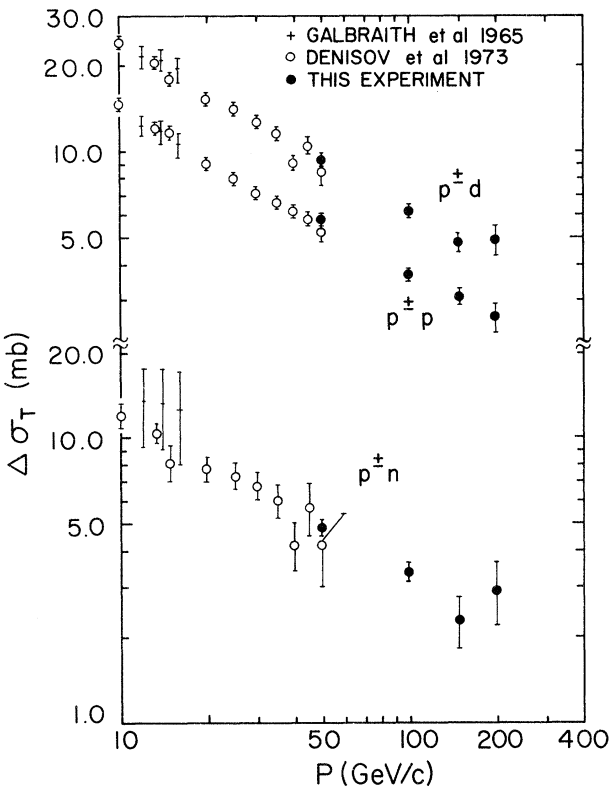

FIG. 3. Values of total-cross-section differences $\sigma_{\bar{p}d}-\sigma_{pd}$, $\sigma_{\bar{p}p}-\sigma_{pp}$, and $\sigma_{\bar{p}n}-\sigma_{pn}$. References as for Fig. 1.

be scaled to 0.031 mb^{-1} for incident protons. In view of the uncertainties in the method, we have used a value of 0.035 mb^{-1} with a systematic uncertainty of ± 0.004 mb^{-1} in extracting pn and $\bar{p}n$ cross sections. We note that this systematic uncertainty causes a ±1.5% scale uncertainty in the neutron cross sections, but does not affect the momentum dependence. We note further that the cross sections on neutrons shown in Fig. 1(b) have a behavior with momentum similar to those on protons. The difference $\sigma_{\bar{p}n}-\sigma_{pn}$, which is only slightly affected by the values of $\langle r^{-2}\rangle$, is shown in Fig. 3, and also shows the same behavior as on protons.

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†Present address: Physics Department, University of Washington, Seattle, Wash. 98195.

‡Present address: Fermi National Accelerator Laboratory, P. O. Box 500, Batavia, Ill. 60510.

§Visitor from Istituto di Fisica, University of Padova, and Istituto Nazionale di Fisica Nucleare, Sezione di Padova, Padova, Italy.

¶Present address: Max Planck Institute for Physics and Astrophysics, Munich, Germany.

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