# Gluon saturation effects in heavy quark production

JARNO VIERROS
IN COLLABORATION WITH
JANI PENTTALA
CHRISTOPHE ROYON

THE UNIVERSITY OF KANSAS

# CONTENTS

- Introduction
  - Gluon saturation
- Prediction methods
  - Quark antiquark pair production process
  - BK and BFKL dipole amplitudes
  - Inclusive and diffractive cross section formulas
  - Monte Carlo integration
- Comparison with data
  - Both inclusive and diffractive quark production measurements from HERA
- Prediction for new LHC heavy quark production measurement
- Conclusion

# HADRON STRUCTURE

- Protons and neutrons consist of three valence quarks and a sea of gluons and virtual quarks
- At high energy scales gluon density increases due to gluon radiation
- Interactions between hadrons can be facilitated by the gluons
- Therefore, more gluons results in larger cross sections, i.e. greater probability for particles to interact

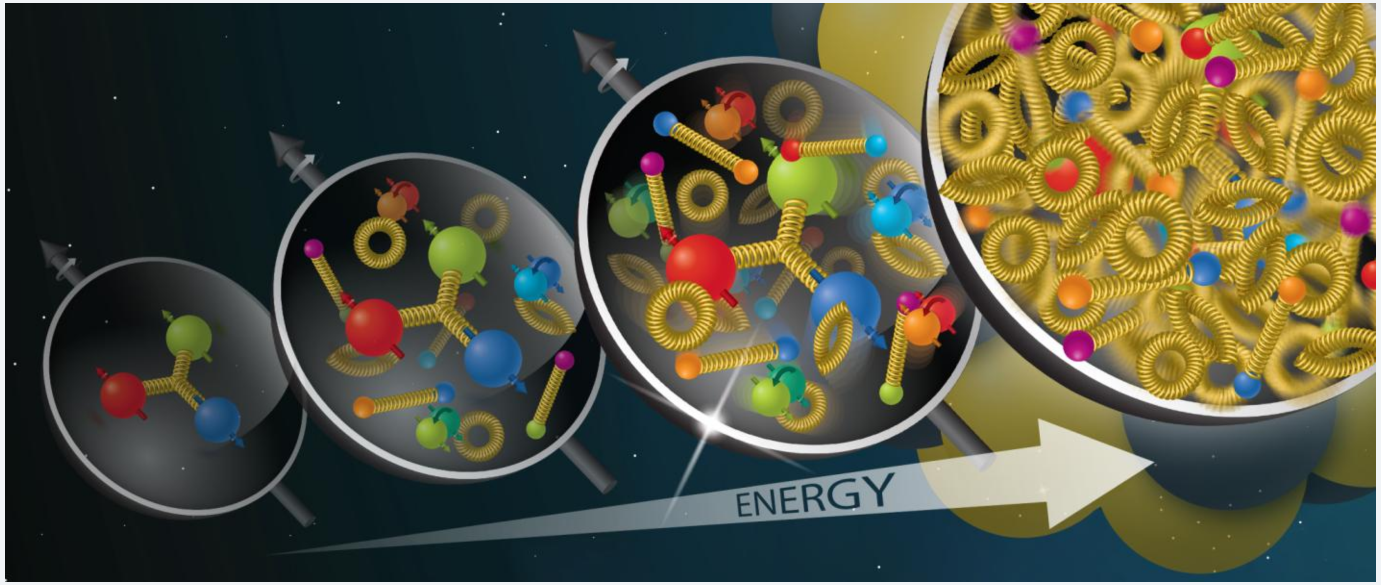
➢ Increase in energy scale

➢ increase in gluon density

➢ increase in scattering cross sections

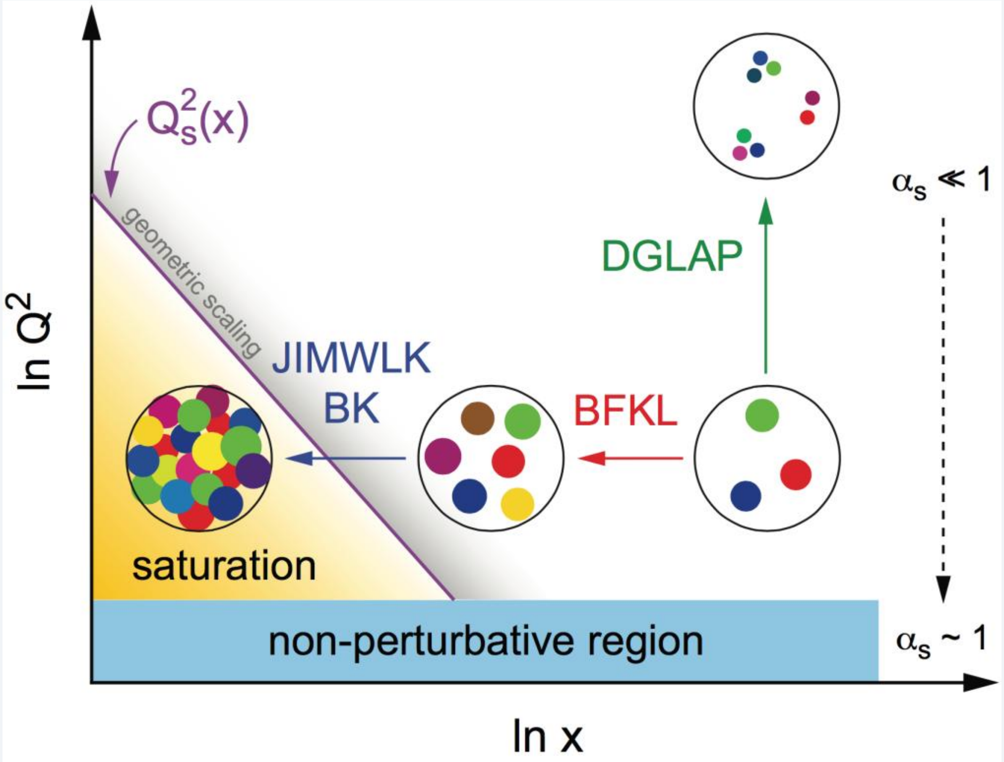
# GLUON SATURATION

- Increase in energy leads to rapid growth in gluon density and cross sections
- Cross sections growing too quickly would break the unitarity of QCD
- To prevent this, gluon absorption must limit the growth of gluon density at some energy scale
- This phenomenon is called gluon saturation



# DETECTING GLUON SATURATION

- Gluon saturation is predicted by QCD
- There are models with and without saturation: BFKL, DGLAP, BK
- An observation of saturation requires:
  - Valid theoretical models
  - Clear difference between predictions given by saturation and non-saturation models
  - A cross section measurement that is precise enough to resolve the difference
- However, past experiments have not led to clear evidence of saturation



# INCLUSIVE QUARK PRODUCTION

- Gluon saturation could be observable in ultraperipheral heavy quark production at the LHC
- One hadron emits a photon, which oscillates into a heavy quark dipole
- The dipole exchanges a gluon with the target hadron, which dissociates
- The interaction between the dipole and the hadron is sensitive to the gluon density of the hadron

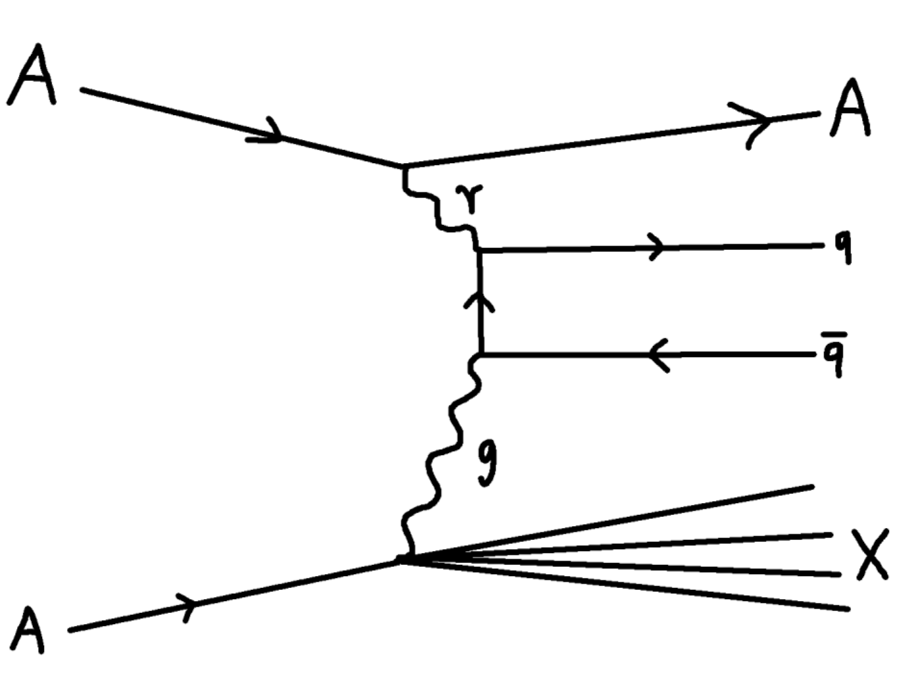
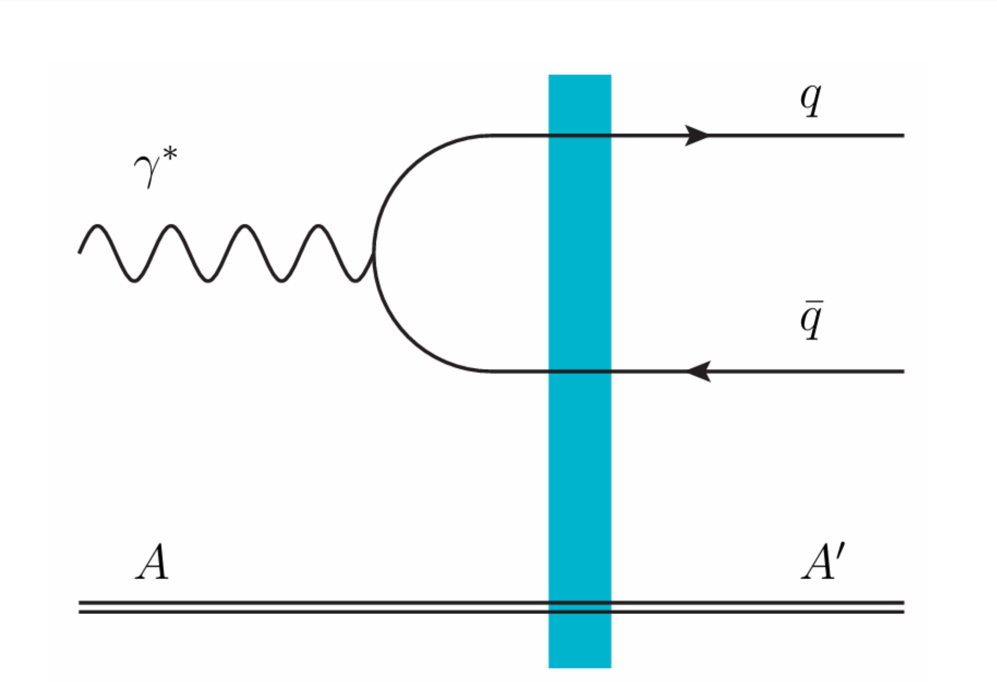



Diagram for quark production in inclusive photon nucleus scattering.

# PROCESS FACTORIZATION

- The process can be factorized in two parts
  1. The photon fluctuates into a quark-antiquark pair
  2. The quark-antiquark pair interacts with the target hadron
- The first part is described by the light-cone wave function of the photon
- It can be calculated using perturbative QCD




Factorization of the process in the dipole picture. The blue rectangle depicts the nonperturbative interaction between the dipole and the target.

# DIPOLE AMPLITUDE

- The second part of the process is given by the dipole scattering amplitude
- The dipole amplitude is a nonperturbative object
- However, its energy dependence can be described using perturbative evolution equations
- The dipole amplitude must be fitted to measurement data at some initial energy scale
- The evolution equations can then be used to calculate the dipole amplitude at other energy scales
  - ➢ The measurement at the initial scale affects predictions at all other scales

Evolution equations

- Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation
  - Linear
  - Does not include saturation
- Balitsky-Kovchegov (BK) equation
  - Nonlinear
  - Does include saturation

# INCLUSIVE FORMULA

- The inclusive cross section is a function of two variables:
  - The Bjorken x
  - Photon virtuality $Q^2$
- The three integrations are done over:
  - The size and orientation of the dipole **r**
  - The impact parameter **b**
  - The fraction of photon momentum carried by the quark z
- The dipole amplitude depends on **r**, **b** and the shifted Bjorken x:

$$\tilde{x} = x\left(1 + \frac{4m_f^2}{Q^2}\right)$$

Dipole amplitude

$$\sigma^{\gamma_\lambda^* \to q_f \bar{q}_f \to X} = \frac{4\alpha_{\mathrm{em}} N_c e_f^2}{(2\pi)^2} \int d^2\mathbf{r} d^2\mathbf{b} \int_0^1 dz K_\lambda(\mathbf{r}, z) N_{\tilde{x}}(\mathbf{r}, \mathbf{b})$$

$$K_\lambda(\mathbf{r}, z) = \begin{cases} 4Q^2 z^2 (1-z)^2 K_0(\epsilon|\mathbf{r}|)^2 & \lambda = L \\ m_f^2 K_0(\epsilon|\mathbf{r}|)^2 + \epsilon^2 \left[z^2 + (1-z)^2\right] K_1(\epsilon|\mathbf{r}|)^2 & \lambda = T \end{cases}$$

$$\mathbf{r} = \mathbf{x} - \mathbf{y}$$

$$\mathbf{b} = \frac{1}{2}(\mathbf{x} + \mathbf{y})$$

$$\epsilon^2 = m_f^2 + z(1-z)Q^2$$

Modified Bessel function of the second kind

Locations of the quark and antiquark relative to the nucleus

Different cases for longitudinally and transversely polarized photons

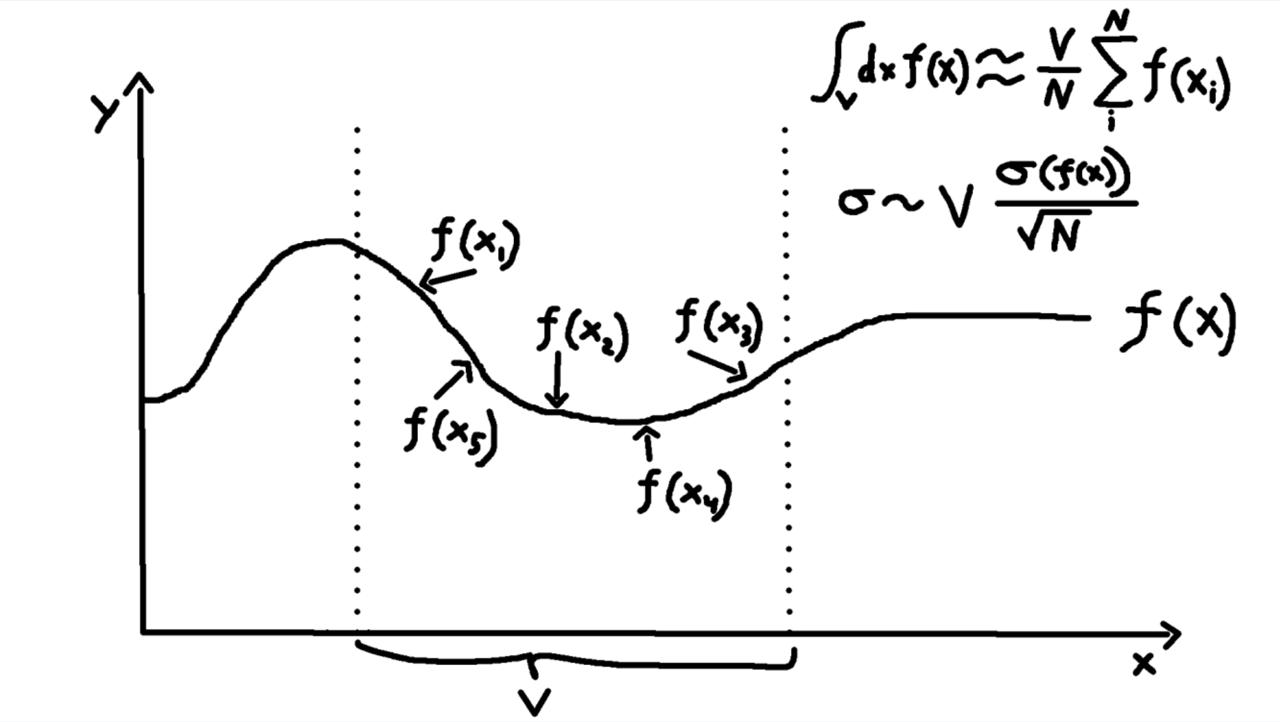
# COMPUTATION

- The dipole amplitudes have been computed from fits to vector meson data and stored in tables by Jani Penttala
- Different protons and lead ions have their own tables
- Due to the difference in their evolutions, the BK and BFKL models also have separate tables
- The integrals are computed using GSL VEGAS  Monte Carlo integration

GNU Scientific Library (GSL)

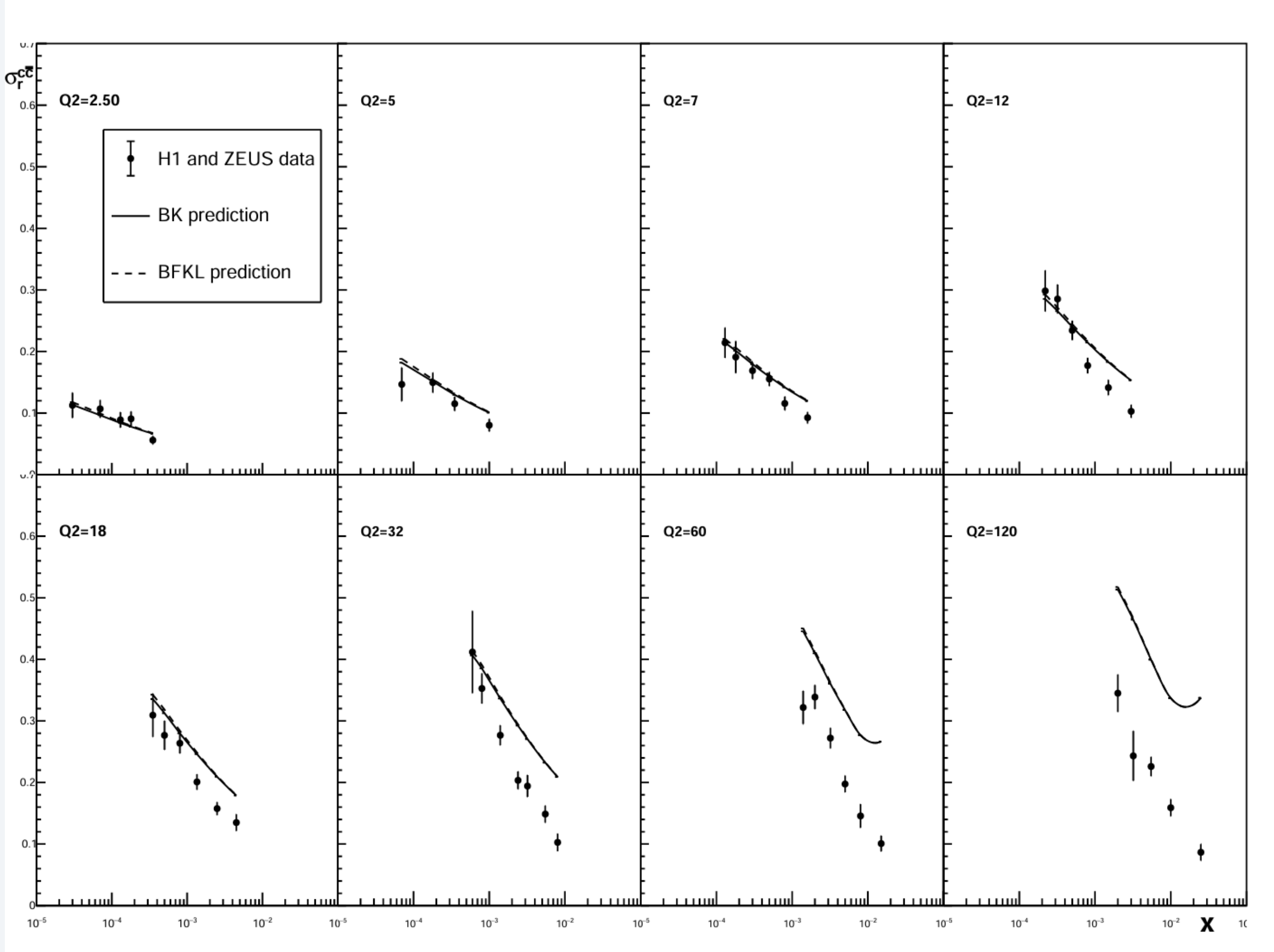
# MONTE CARLO INTEGRATION

- Monte Carlo integration is a numerical method for calculating multidimensional definite integrals
- The function is evaluated at random points in the integration volume
- Integral is estimated by the mean of the results multiplied with the integrated volume
- Advanced algorithms sample more points from areas where function changes rapidly
- Integration time doesn't depend exponentially on number of dimensions like in most deterministic methods



# INCLUSIVE $c\bar{c}$ DATA COMPARISON

- The formula can be tested by comparing it with measurements of the reduced cross section
  - Reduced cross section is a linear combination of longitudinal and transverse polarization cross sections
- The data on the right is from photon proton collisions at H1 and ZEUS
- The prediction is valid at small $Q^2$
- However, the $Q^2$ dependence is unexpected and requires further study
- The difference between BK and BFKL is negligible because no saturation is expected for protons at this low energies




Reduced $c\bar{c}$ production cross section measurements in inclusive photon proton scattering at H1 and ZEUS with varying $Q^2$.

# DIFFRACTIVE QUARK PRODUCTION

- In a diffractive process the target nucleus emits two gluons (a pomeron) and stays intact
- The cross section is much smaller than for the inclusive process
- However, sensitivity to gluon density is stronger
- Diffractive calculation uses a different dipole amplitude compared to inclusive

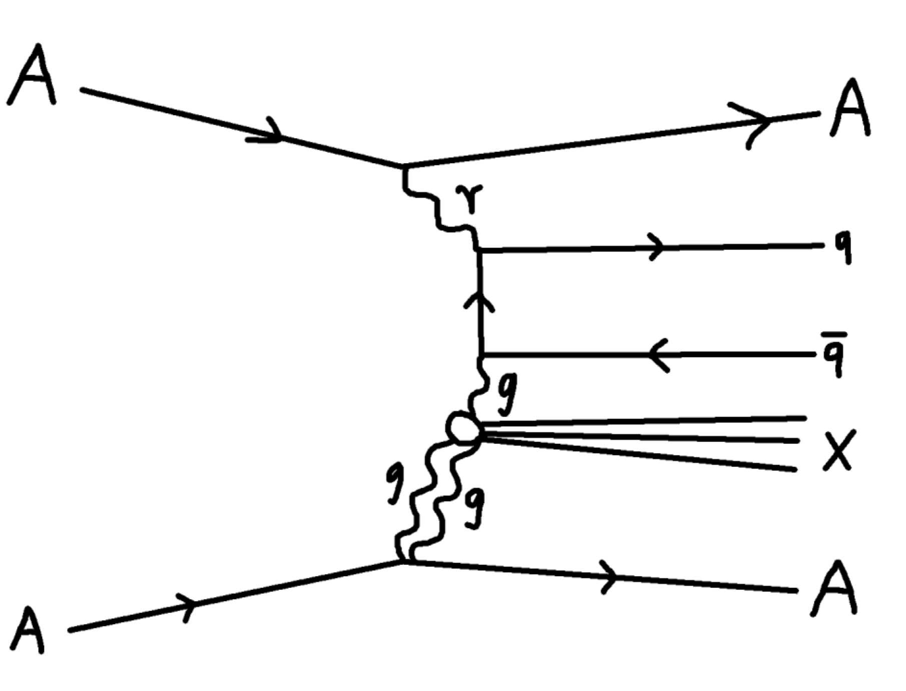



Diagram for quark production in diffractive photon nucleus scattering.

# DIFFRACTIVE FORMULA

- Here the complex conjugate coordinates $\bar{r}$ and $\bar{b}$ must be integrated separately
- Due to the pomeron, the cross section depends on two new independent variables:
  - Invariant mass of the diffractive system $M_X$
  - Momentum transfer $t$
- In the diffractive case instead of the shifted Bjorken x, the $x_{pom}$ variable is used:

$$\tilde{x}' = x_{\text{pom}} = \frac{x}{\beta} = \left(Q^2 + M_X^2 - t\right)\frac{x}{Q^2}$$

Dipole amplitude appears twice

Bessel function of the first kind

Depends on two new variables

$$\frac{d\sigma^D}{dt dM_X^2} = \frac{1}{2(2\pi)^4}\alpha_{\text{em}} N_c e_f^2 \int_0^1 dz \int d^2\mathbf{r} \int d^2\bar{\mathbf{r}} \int d^2\mathbf{b}$$

$$\int d^2\bar{\mathbf{b}} J_0\left(\sqrt{|t|}\left|\mathbf{b}-\bar{\mathbf{b}}\right|\right) J_0\left(\sqrt{z(1-z)M_X^2 - m_f^2}\,|\mathbf{r}-\bar{\mathbf{r}}|\right)\theta\left(\sqrt{z(1-z)M_X^2 - m_f^2}\right) z(1-z) H(\mathbf{r}, \bar{\mathbf{r}}, z) N_{\tilde{x}'}(\mathbf{r}, \mathbf{b}) N_{\tilde{x}'}\left(\bar{\mathbf{r}}, \bar{\mathbf{b}}\right)$$

$$\mathbf{r} = \mathbf{x} - \mathbf{y}$$

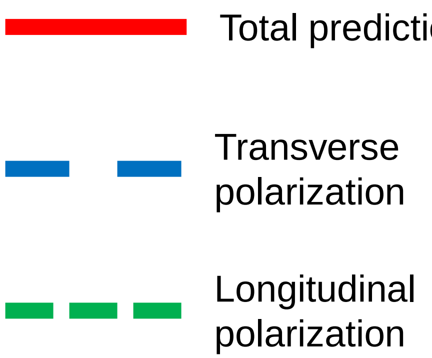
$$\mathbf{b} = z\mathbf{x} + (1-z)\,\mathbf{y}$$

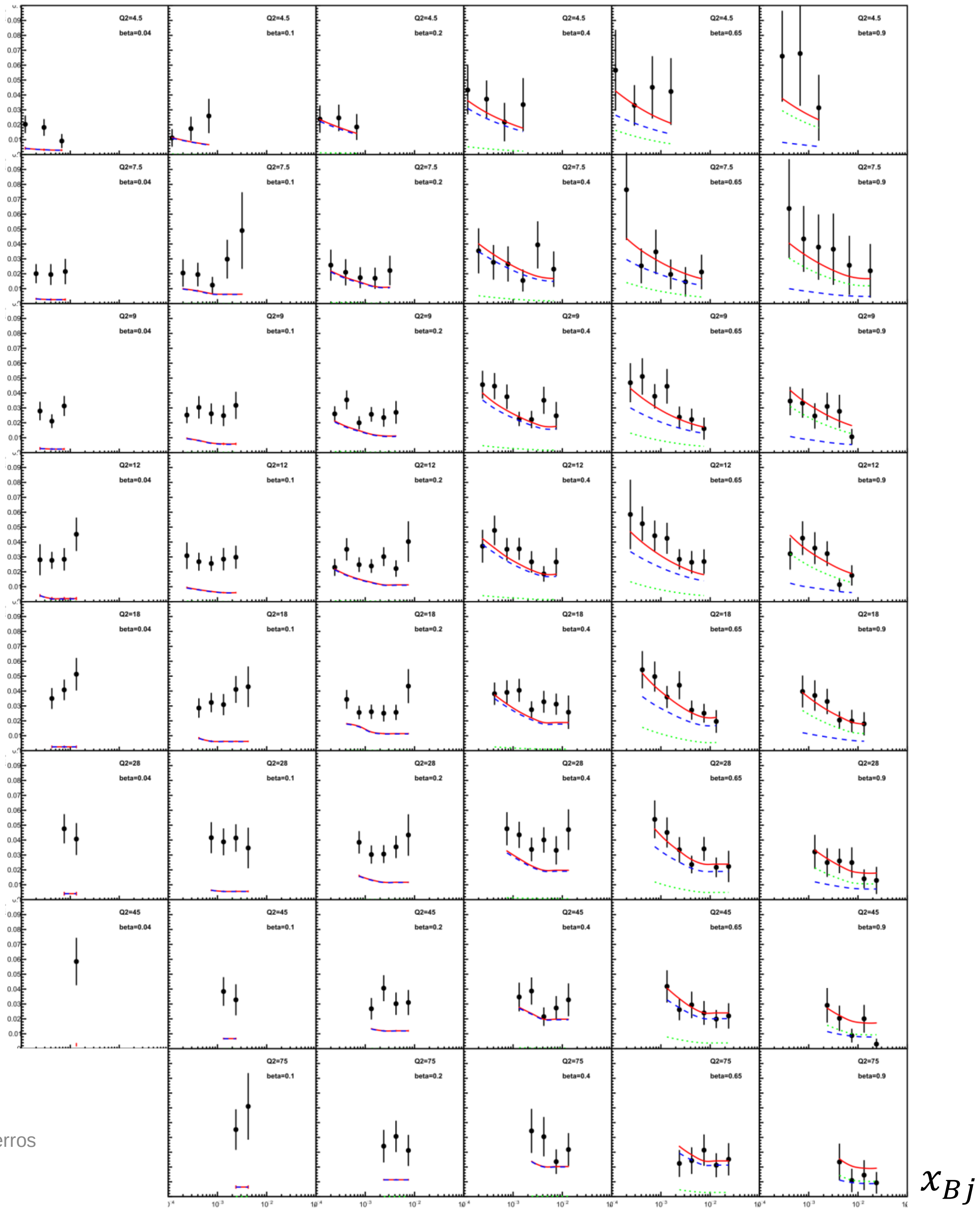
Different b definition

$$H_\lambda(\mathbf{r}, \bar{\mathbf{r}}, z) = \begin{cases} 4Q^2 z^2 (1-z)^2 K_0(\epsilon|\mathbf{r}|) K_0(\epsilon|\bar{\mathbf{r}}|) & \lambda = L \\ m_f^2 K_0(\epsilon|\mathbf{r}|) K_0(\epsilon|\bar{\mathbf{r}}|) + \epsilon^2 \left[z^2 + (1-z)^2\right] \frac{\mathbf{r}\cdot\bar{\mathbf{r}}}{|\mathbf{r}||\bar{\mathbf{r}}|} K_1(\epsilon|\mathbf{r}|) K_1(\epsilon|\bar{\mathbf{r}}|) & \lambda = T \end{cases}$$

$$\epsilon^2 = z_0 z_1 Q^2 + m_f^2$$

$x_{pom}F_2^{D(3)}$




Measurements from H1 of the differential structure function $F_2^{D(3)}(x_{pom}, \beta, Q^2)$ multiplied with $x_{pom}$ in diffractive photon proton scattering at varying β and $Q^2$.

# Diffractive data comparison

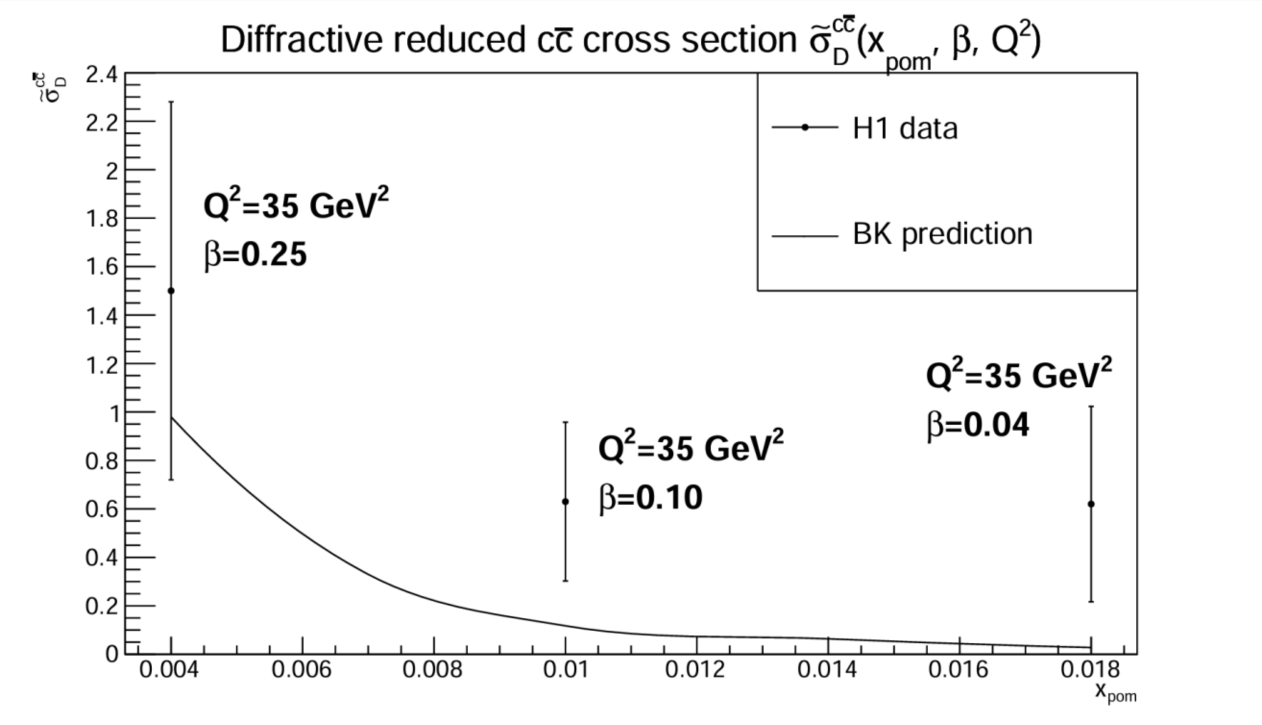
- Comparison between diffractive prediction and $x_{pom}F_2^{D(3)}$ measurements from H1
- $F_2^{D(3)}$ is another linear combination of the cross sections
- The diffractive cross section is integrated over t
- $M_X^2$ dependence is replaced with β:

$$\beta \approx \frac{Q^2}{Q^2 + M_X^2}$$

- The prediction contains lightest quarks up to charm
- The prediction at low β is too small because of a large next to leading order contribution
- The agreement is good at high β

# DIFFRACTIVE CHARM DATA COMPARISON

- The diffractive formula can be compared with specifically charm production cross section measurements from H1
- Here again t is integrated and $M_X^2$ is replaced with β
- The precision of the measurements is not sufficient to tell much about the validity of the prediction



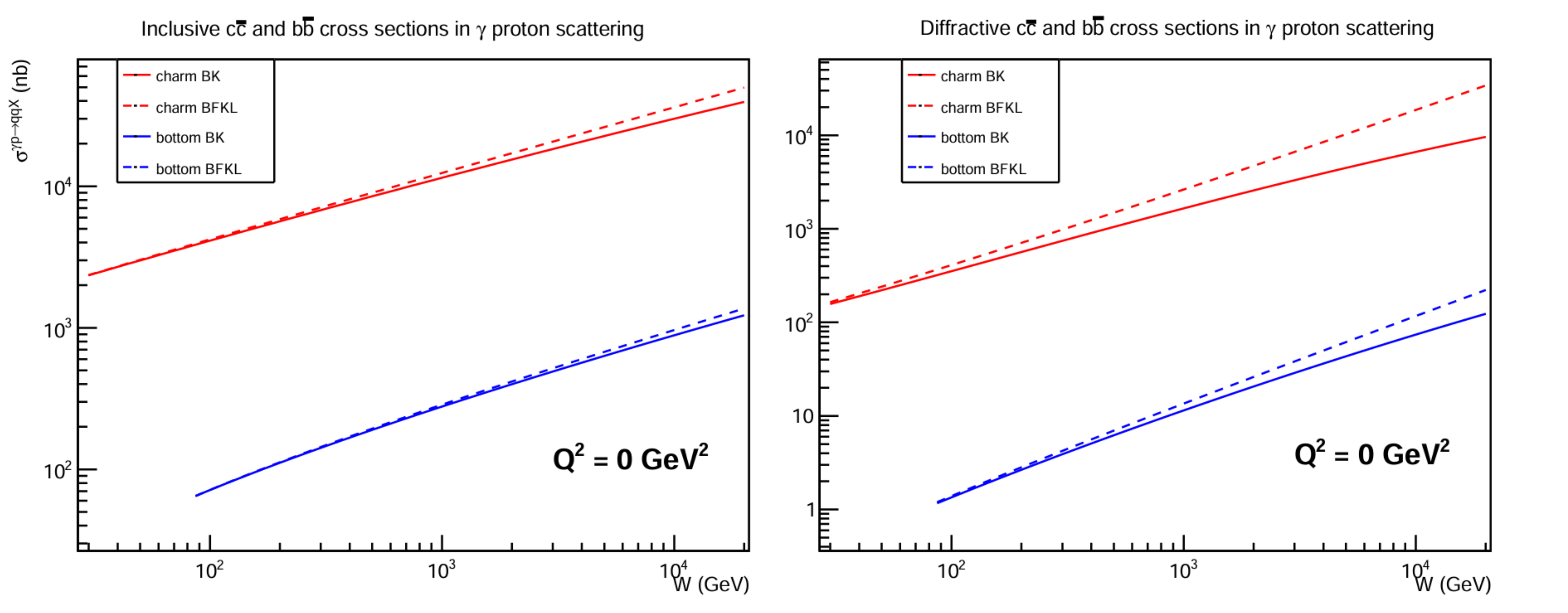

Reduced $c\bar{c}$ production cross section in diffractive photon proton scattering measurements from H1

# LHC MEASUREMENT

- MIT is analyzing charm quark production cross section measurements in photon lead scattering in the CMS detector
- Due to the large center of mass energy at the LHC and the use of lead ions there is a chance of detecting saturation effects
- The LHC measurement is ultraperipheral, so the photon virtuality is effectively zero: $Q^2 = 0$
- For the diffractive process both t and $M_X^2$ are integrated
- Bottom quark measurements may also be analyzed later

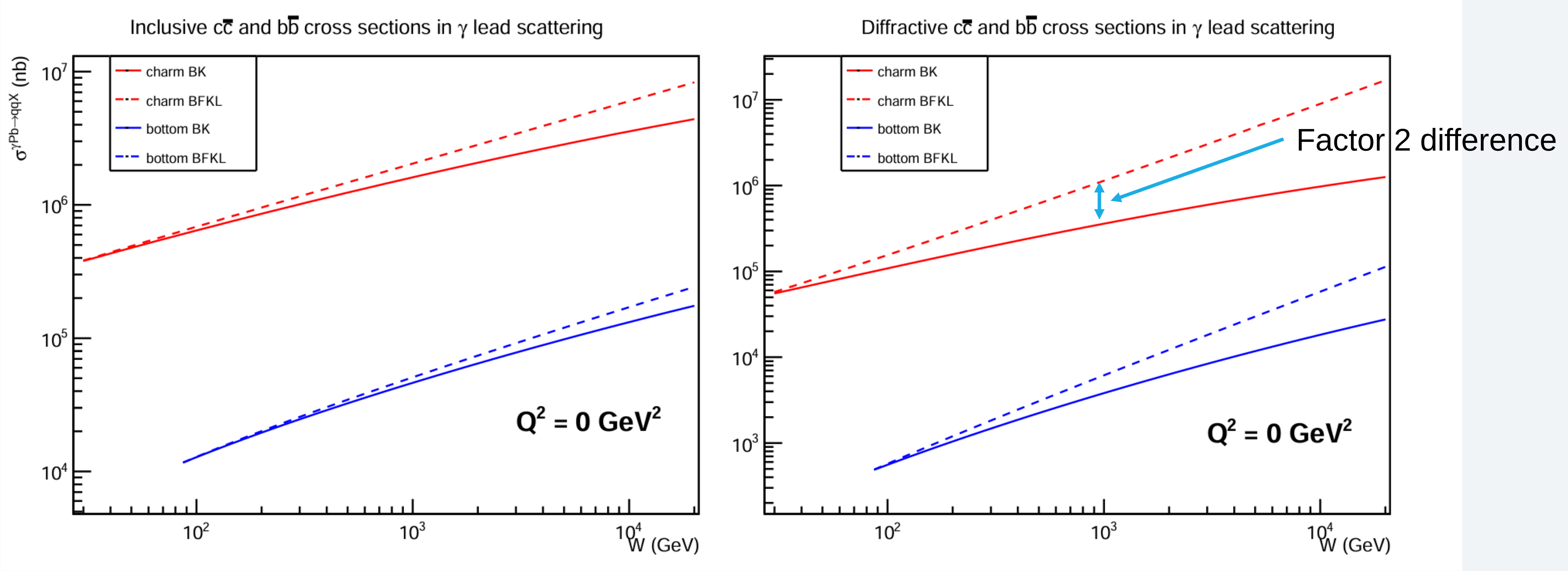
Massachusetts Institute of Technology

# LHC PROTON SCATTERING PREDICTION




Predictions for heavy quark production cross sections in photon proton scattering with BK evolution (saturation) and BFKL evolution (no saturation).

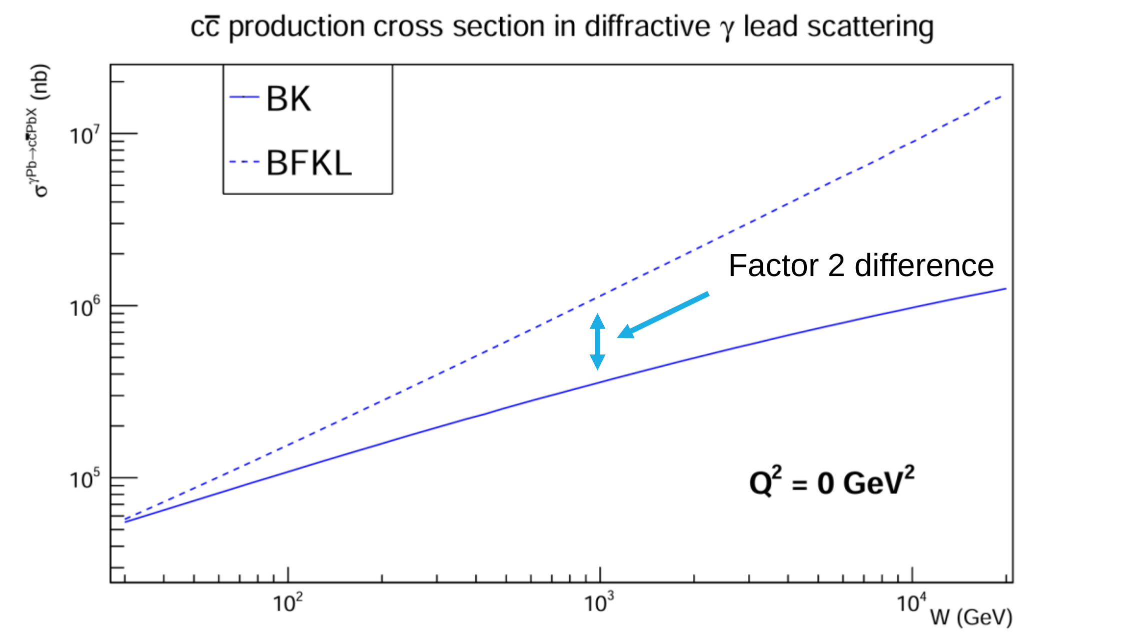
# LHC LEAD SCATTERING PREDICTION




Predictions for heavy quark production cross sections in photon lead scattering with BK evolution (saturation) and BFKL evolution (no saturation).

# CONCLUSION

- A clear observation of gluon saturation would be a major achievement
- A significant difference is predicted between BK and BFKL models in diffractive charm production cross section in photon lead scattering
- However, it will depend on measurement precision if the difference is sufficient
- The prediction can be improved further:
  - Adding contribution from $q\bar{q}g$ final state to improve prediction at small β and large $Q^2$
  - Better dipole amplitude fits
  - Full next to leading order calculation is in progress




Predictions for charm quark production cross sections in photon lead scattering with BK evolution (saturation) and BFKL evolution (no saturation).

# SOURCES


- Inclusive $c\bar{c}$ cross section measurement
  - Abramowicz, H., Abt, I., Adamczyk, L. et al. Combination and QCD analysis of charm and beauty production cross-section measurements in deep inelastic ep scattering at HERA. Eur. Phys. J. C 78, 473 (2018).
  - https://doi.org/10.1140/epjc/s10052-018-5848-3
- Differential structure function $F_2^{D(3)}(x_{pom}, \beta, Q^2)$ measurement
  - H1 Collaboration. Inclusive measurement of diffractive deep-inelastic ep scattering. Z Phys C - Particles and Fields 76, 613–629 (1997).
  - https://doi.org/10.1007/s002880050584
- Diffractive $c\bar{c}$ cross section measurement
  - Aktas, A., Andreev, V., Anthonis, T. et al. Diffractive open charm production in deep-inelastic scattering and photoproduction at HERA. Eur. Phys. J. C 50, 1–20 (2007).
  - https://doi.org/10.1140/epjc/s10052-006-0206-2