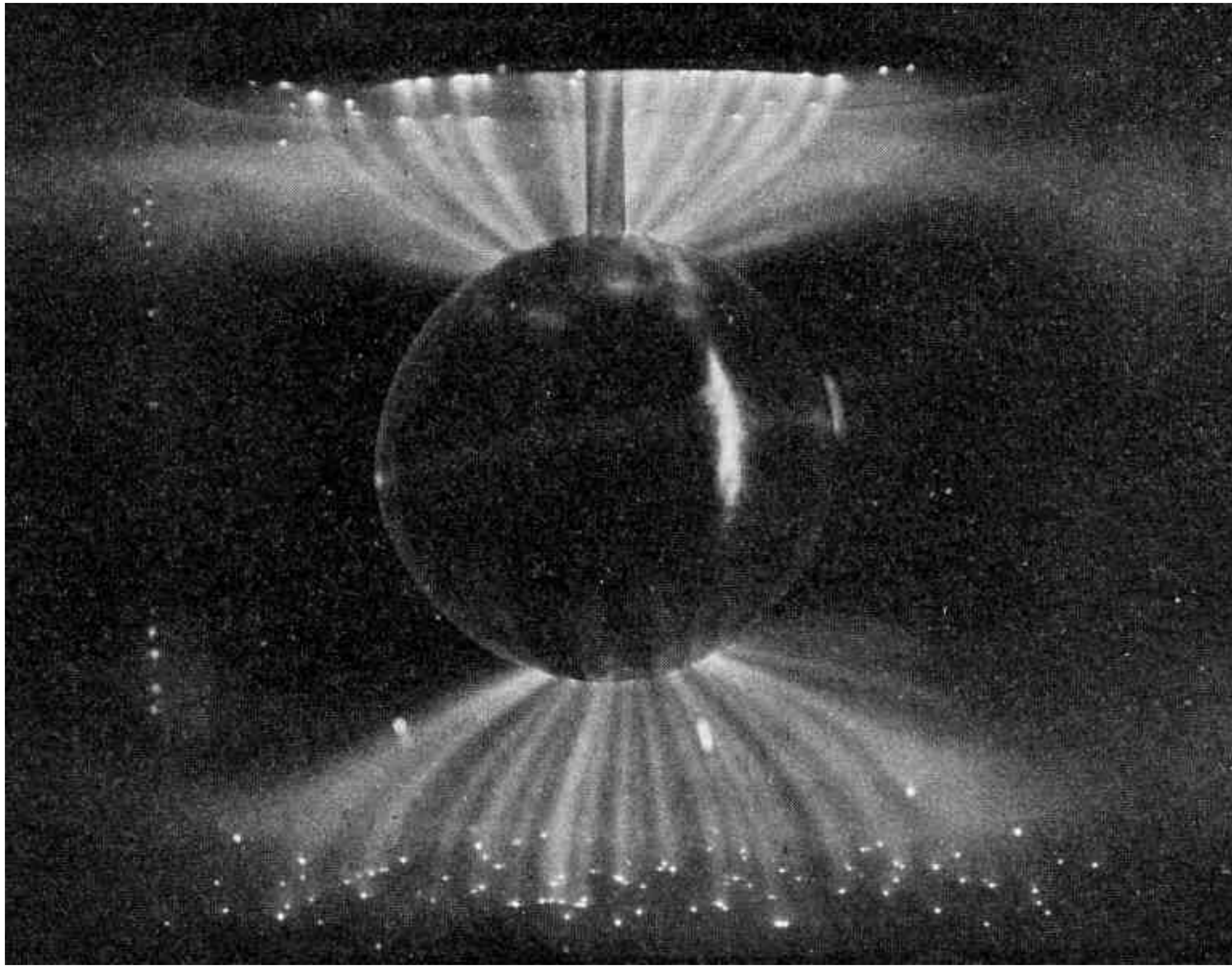
# Fast radial transport of equatorially trapped electrons in the Earth's radiation belts

## PAP301 Seminar

Saara Takala (saara.takala@helsinki.fi) 28.11.2023

# Outline

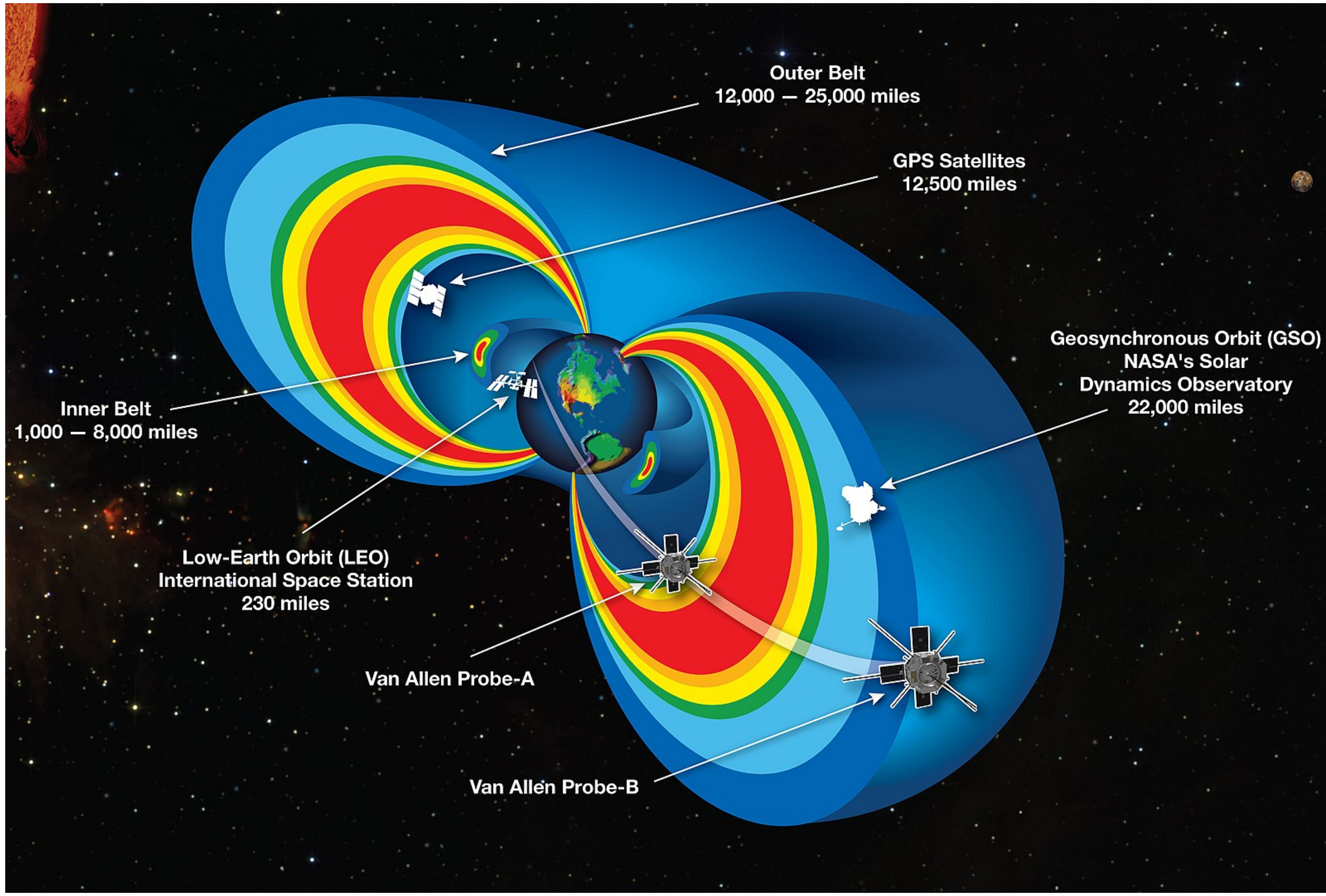
- **Motivation**: radiation belt dynamics, ULF wave acceleration, drift-periodic signatures
- **Theoretical framework** for fast transport: the drift kinetic description
- **Numerical framework**: spectral methods
- **Results**: simulations in three different systems, comparison to analytical results



[Birkeland, 1913]

# Radiation belts

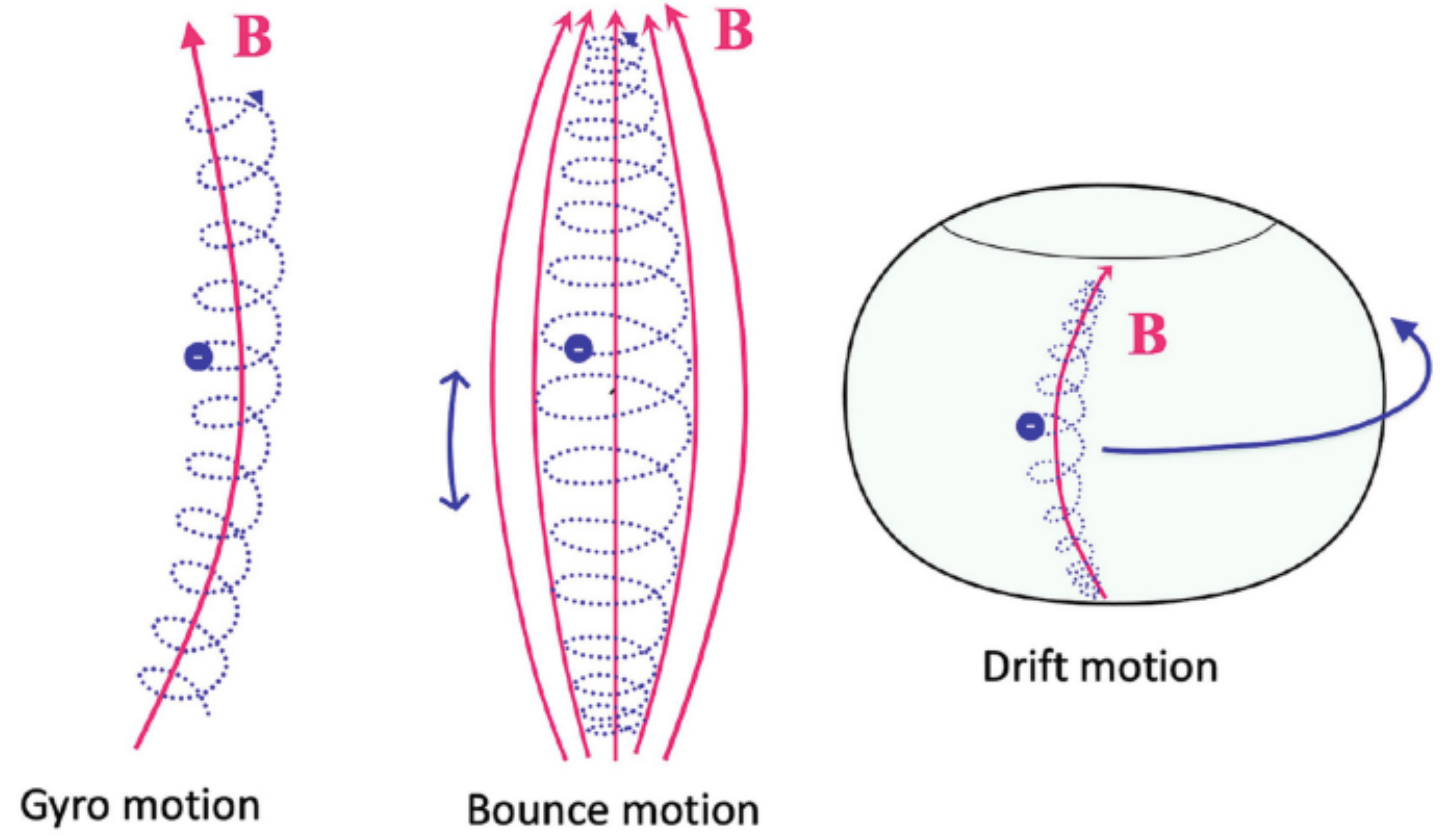
- Radiation belts are trapped populations of high-energy particles around the planet
- Earth's radiation belts: stable proton belt at $1.5 - 3R_E$, highly varying electron belt at $4 - 10R_E$
- Particles are continuously added to and lost from the belts due to particle precipitation and wave-particle interactions
- The variability of the radiation belts is a major space weather concern for satellite operators




[NASA, 2013]

# Particle motion in the magnetosphere

- Three types of motion:
    - Gyro motion around the magnetic field line ($10^{-3} - 1$ s)
    - Bounce motion between magnetic mirror points ($10^{-1} - 10^{1}$ s)
    - Drift motion around the Earth ($10$ min)




[Kilpua and Koskinen, 2022]

- Bounce motion results in the formation of the radiation belts
- Drift rates $\sim L = \dfrac{r}{R_E}$ & $\sim E$

# Radiation belts are particle accelerators

- Ultra-low frequency (ULF) waves in the 2-25 mHz (Pc4-Pc5) range accelerate radiation belt particles to near-relativistic energies

- Acceleration leads to particle losses and injections in and out of the belts

- Observational studies show wave-particle phenomena occurring on timescales comparable to particle drift periods: **drift-periodic signatures**

# Drift echoes

- Result from injections of energetic particles from the magnetotail and energization due to interplanetary shocks
- Show up as peaks in particle fluxes
- Pc5 event observed in 1968 shows drift echoes in association with the ULF wave

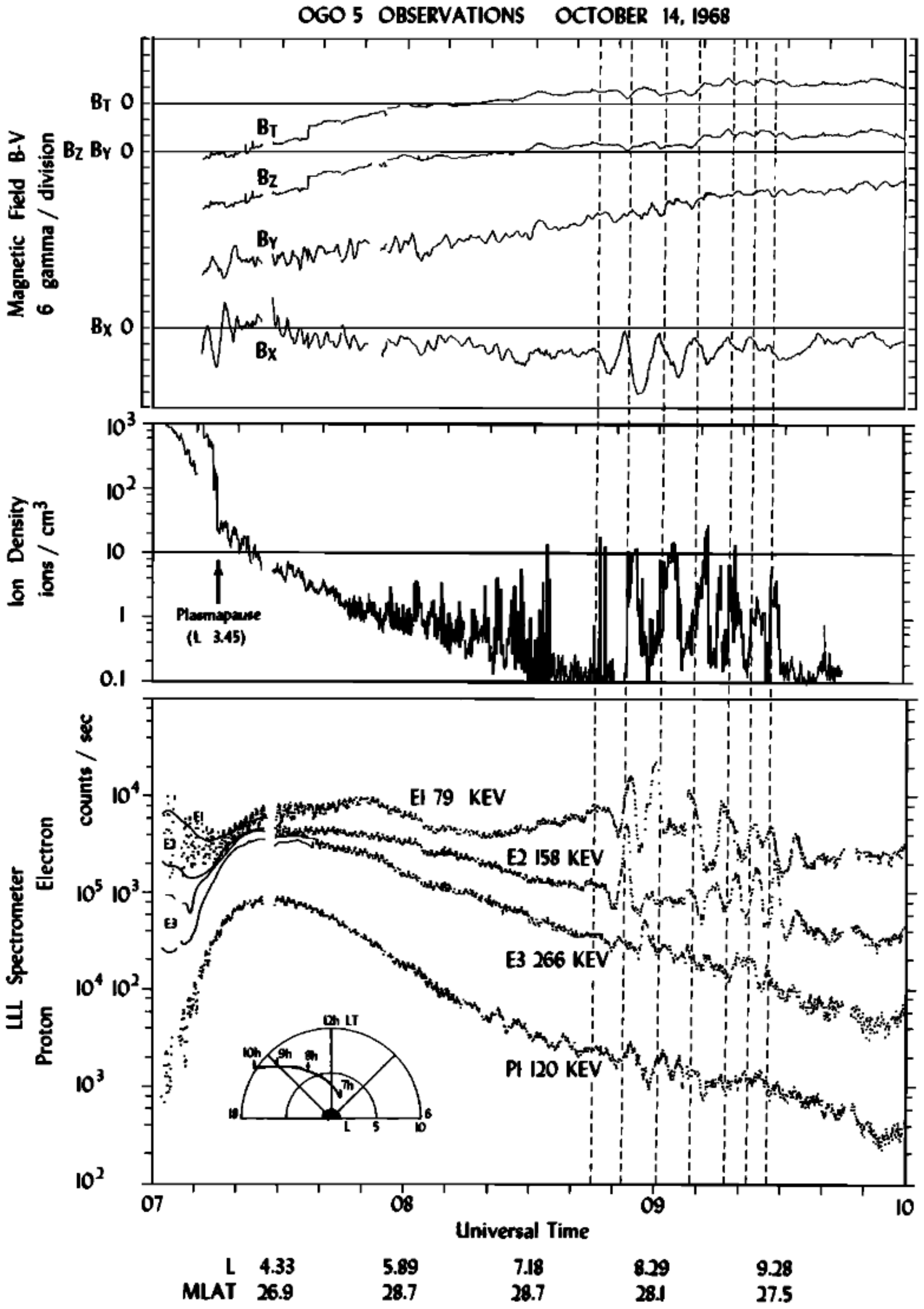


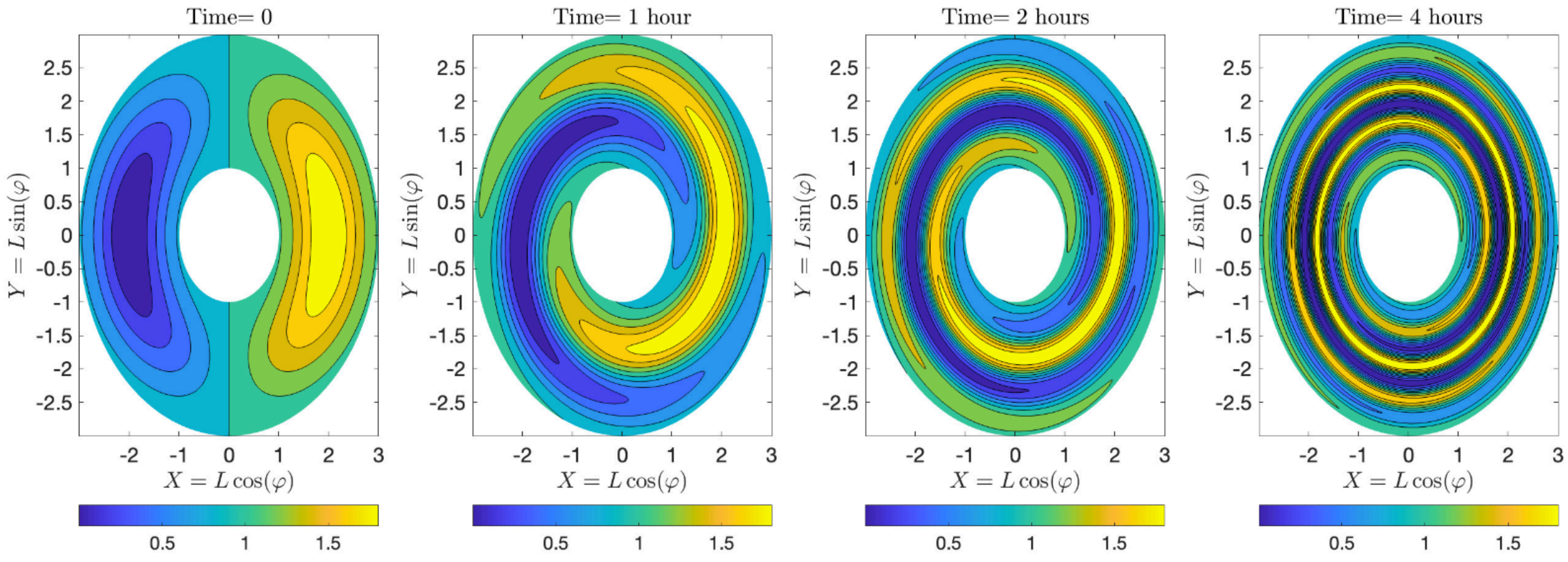

Fig. 6. Plots of magnetic field (deviation from model field), ion density, and electron and proton fluxes during the Pc 5 wave event observed on the outbound pass of Ogo 5 in the afternoon sector on October 14, 1968. The dashed lines indicate the times of peaks in the $B_X$ component of the magnetic field. Large modulations in ion density and electron fluxes occurred simultaneously in association with the Pc 5 wave starting at 0845 UT. The peaks in $B_X$ fall on portions in the ion flux density that are increasing in intensity.

[Kokubun et al., 1977]

# Phase-mixing leads to zebra stripes

- Zebra stripes are signatures of drift echoes that appear as peaks and valleys in the distribution function

- Given a localized equilibrium distribution, patches of different densities will produce spiraling structures as they rotate with different velocities → **phase-mixing**




[Osmane et al., 2023]

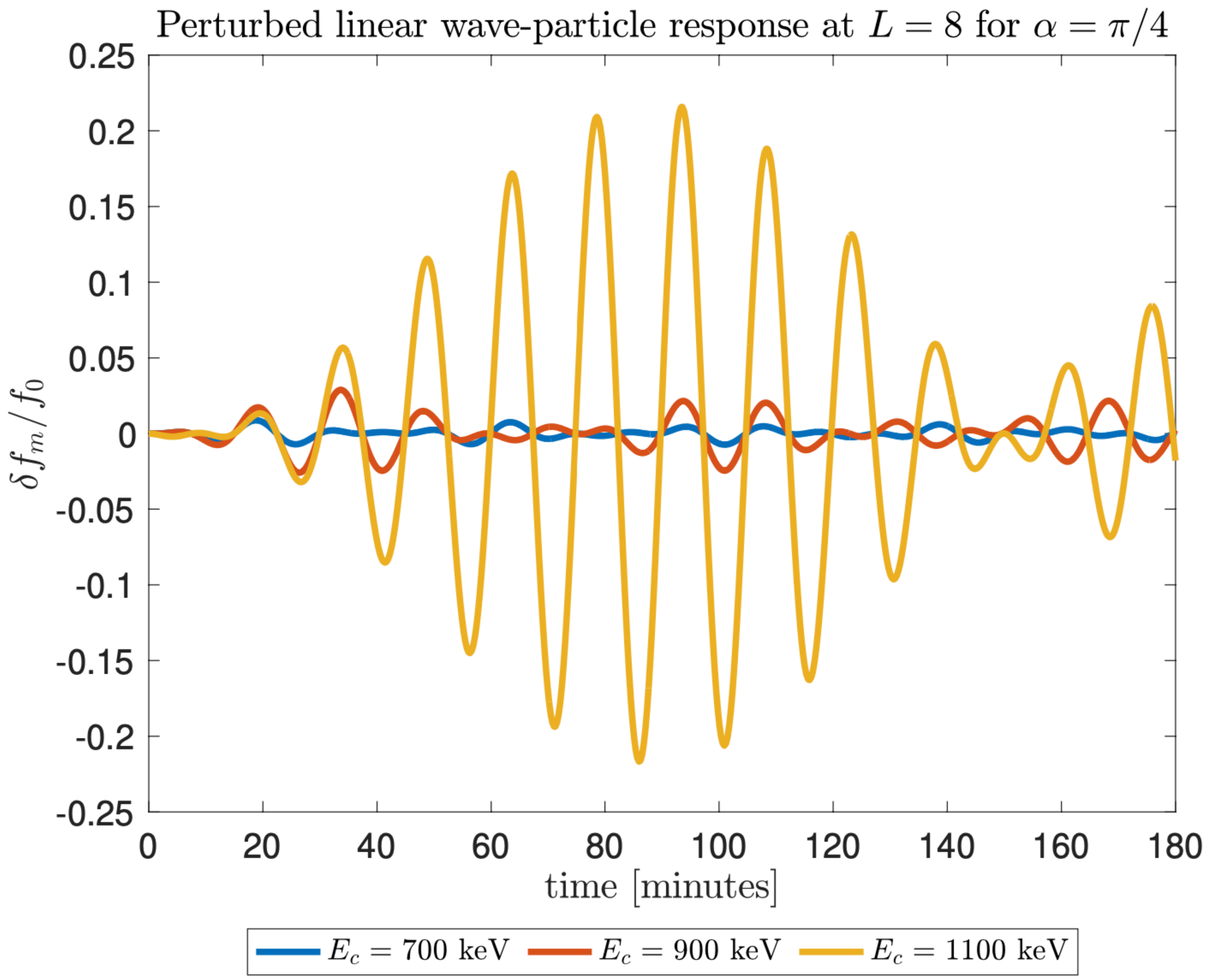
# Drift resonance

- Occurs when ULF wave frequency ($\omega_m$) matches with integer multiples of the particle's drift frequency ($\omega_d$):

$$\omega_m - m\omega_d = 0$$

- Leads to spreading of the distribution function towards higher and lower $L$

- Transient event → amplifies the ULF wave signal




[Osmane et al., 2023]

# What has been done so far?

- Current models of radiation belts are diffusion models written as Fokker-Planck equations → can't describe particle phenomena on fast timescales
- Many models use electromagnetic fields that violate Liouville's theorem
- Recent approaches at modeling fast transport rely on F-P theory and assume stochasticity of field fluctuations

**→ aim to construct a physically sensible and computationally inexpensive model for fast transport**

**→ look for drift-periodic signatures**

# Description for fast timescales: kinetic theory

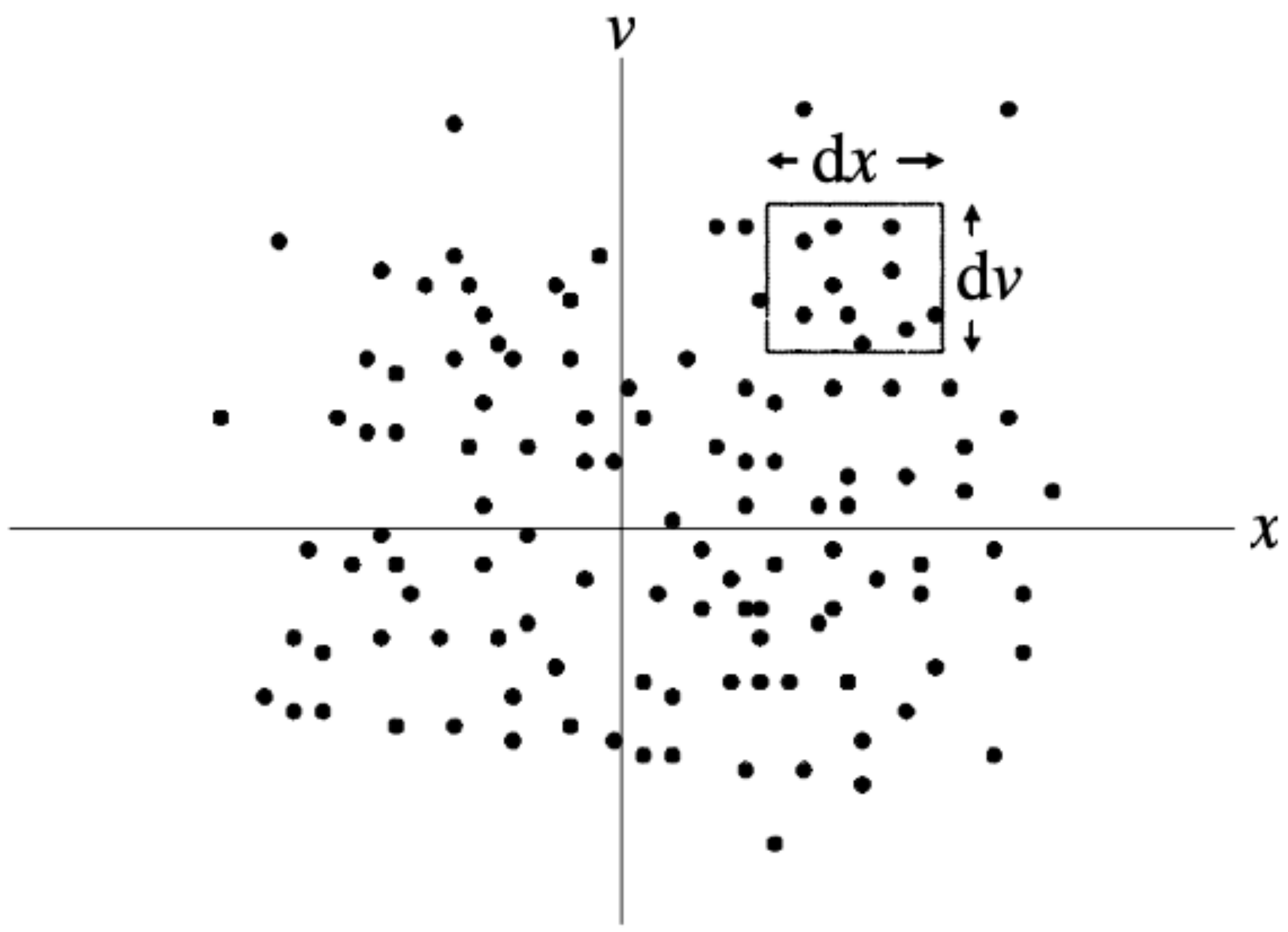
- Kinetic theory is a statistical approach that describes particle motion using the 6D distribution function:

$$f(\vec{r}, \vec{v}, t) = \underbrace{f_0(\vec{v}, t)}_{\text{equilibrium distribution}} + \underbrace{\delta f(\vec{r}, \vec{v}, t)}_{\text{perturbation}}$$

- Look for particles within the same phase-space volume element at a given time

- The time evolution of the distribution function is described by the Vlasov equation:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f - \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f}{\partial \vec{v}} = 0$$




[Bellan, 2006]

# Mead field

- Magnetic field model consisting of a background dipole field, and a symmetric and antisymmetric perturbation

- For equatorial particles:

$$B = B_0 + \delta B = \left(\frac{B_E R_E^3}{r^3} - S(t) - \Sigma A_m(t) r e^{im\varphi}\right)\hat{z}$$

- Conserves phase-space density → consistent with Liouville's theorem

$R_E, B_E$: Earth radius & magnetic field

$S(t)$: symmetric fluctuation

$A_m(t)$: asymmetric fluctuation

$\varphi$: azimuthal angle

# Analytical theory of fast transport

- Fourier decomposition of the perturbed distribution function:

$$\delta f = \sum_m e^{im\varphi}\delta f_m(r,t)$$

- Time evolution for a wave mode $m$:

$$\frac{\partial \delta f_m}{\partial t} + \underbrace{im\Omega_d \delta f_m}_{\text{particle streaming}} = \underbrace{-(\gamma_m - i\omega_m)\frac{\delta B}{B_0} e^{-i\omega_m t + \gamma_m t} r \frac{\partial f_0}{\partial r}}_{\text{wave term}}$$

- No ULF wave: RHS = 0
- Field perturbations are not assumed to be stochastic

$\Omega_d$: drift frequency

$\gamma_m$: ULF wave damping rate

$\omega_m$: ULF wave frequency

$\delta B / B_0$: magnetic field amplitude

$\partial f_0 / \partial r$: background distribution gradient

# Numerical framework: spectral methods

- Global methods for solving partial differential equations numerically
- Use weighted residuals to approximate the solutions of the given problem over the domain
- **Tau method** approximates the function by a polynomial and seeks an exact solution by adding perturbation terms $\tau$ to the equation
- The boundary conditions are enforced by the $\tau$ term

# Dedalus

- Spectral PDE framework utilizing the tau method
- Written in Python
- Solves linear and nonlinear PDEs, eigenvalue problems, initial and boundary value problems on different bases and coordinate systems
- Using Dedalus to solve an initial value problem on a disk basis (polar coordinates)
- Minimal computational requirements: programs can be run on a laptop

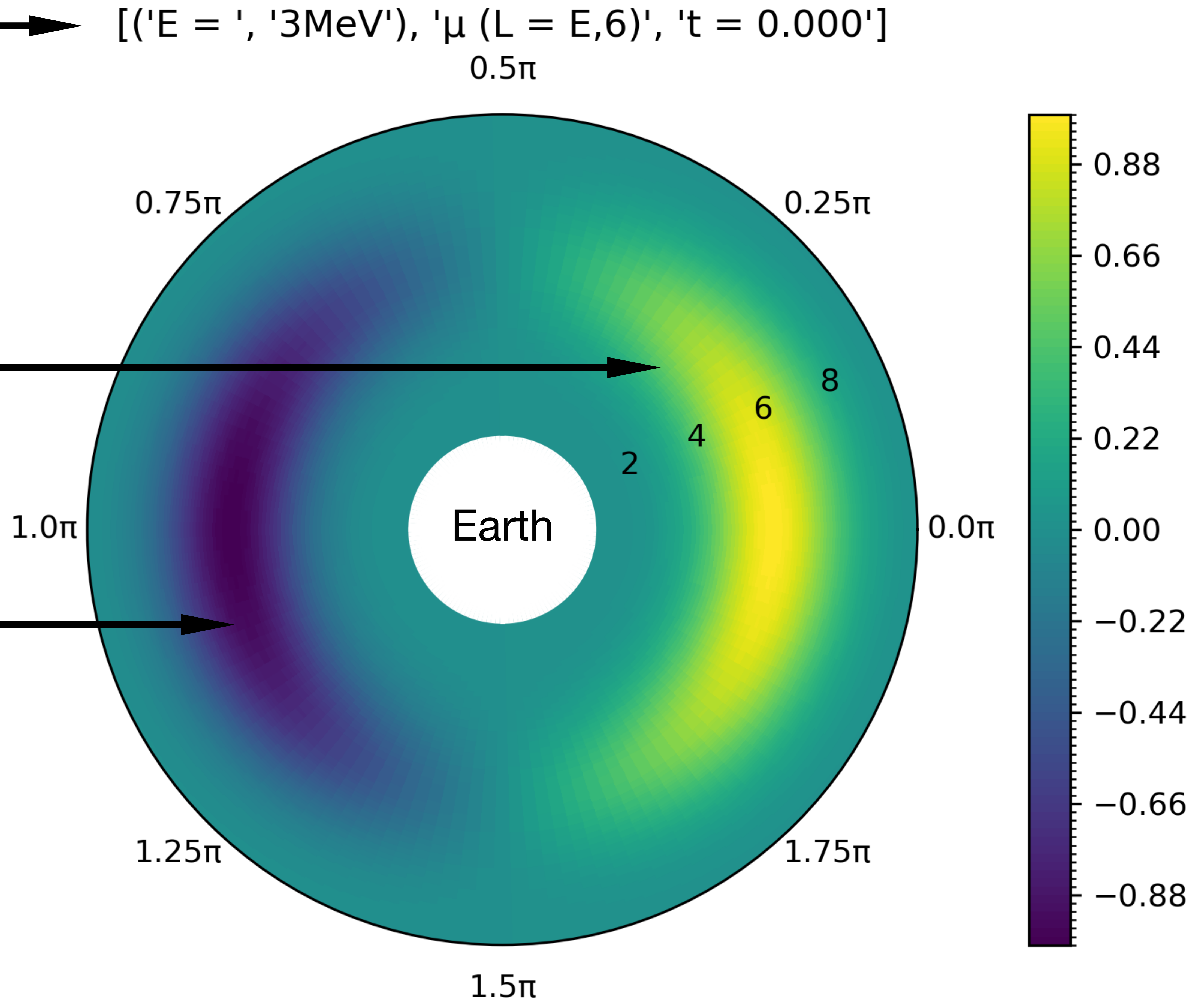
https://dedalus-project.org

# Model setup

energy, fixed magnetic moment & time

distance in Earth radii

initial particle distribution: Gaussian centered around $L = 6$




**GitLab link** for simulations

## Model parameters

- Particle energy and species:

3 MeV electrons

- Field perturbation:

1 & 5 nT

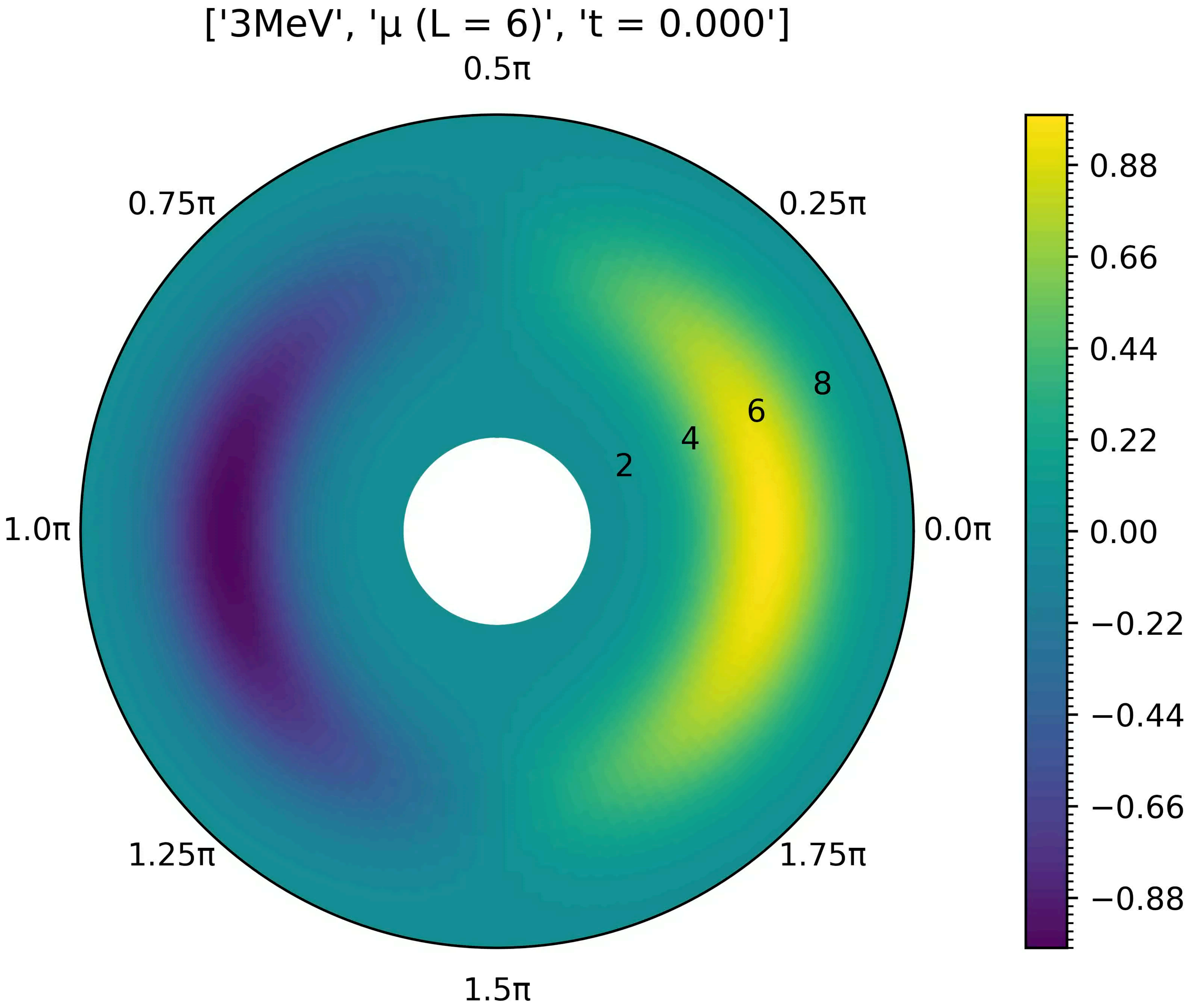
- Slow damping rate
- Wave frequency:

no wave,

non-resonant wave &

resonant wave

# Results: No wave, $\frac{\partial \delta f_m}{\partial t} + im\Omega_d \delta f_m = 0$




- No wave → no field fluctuations or resonance
- Expect to see phase-mixing of the distribution function
- Runtime 15 drift periods, time-step 1/15th of a drift period

✔ Phase-mixing occurs at different rates w.r.t. L

## Results: non-resonant wave, $\frac{\partial \delta f_m}{\partial t} + im\Omega_d \delta f_m = -(\gamma_m - i\omega_m)\frac{\delta B}{B_0} e^{-i\omega_m t + \gamma_m t} r \frac{\partial f_0}{\partial r}$

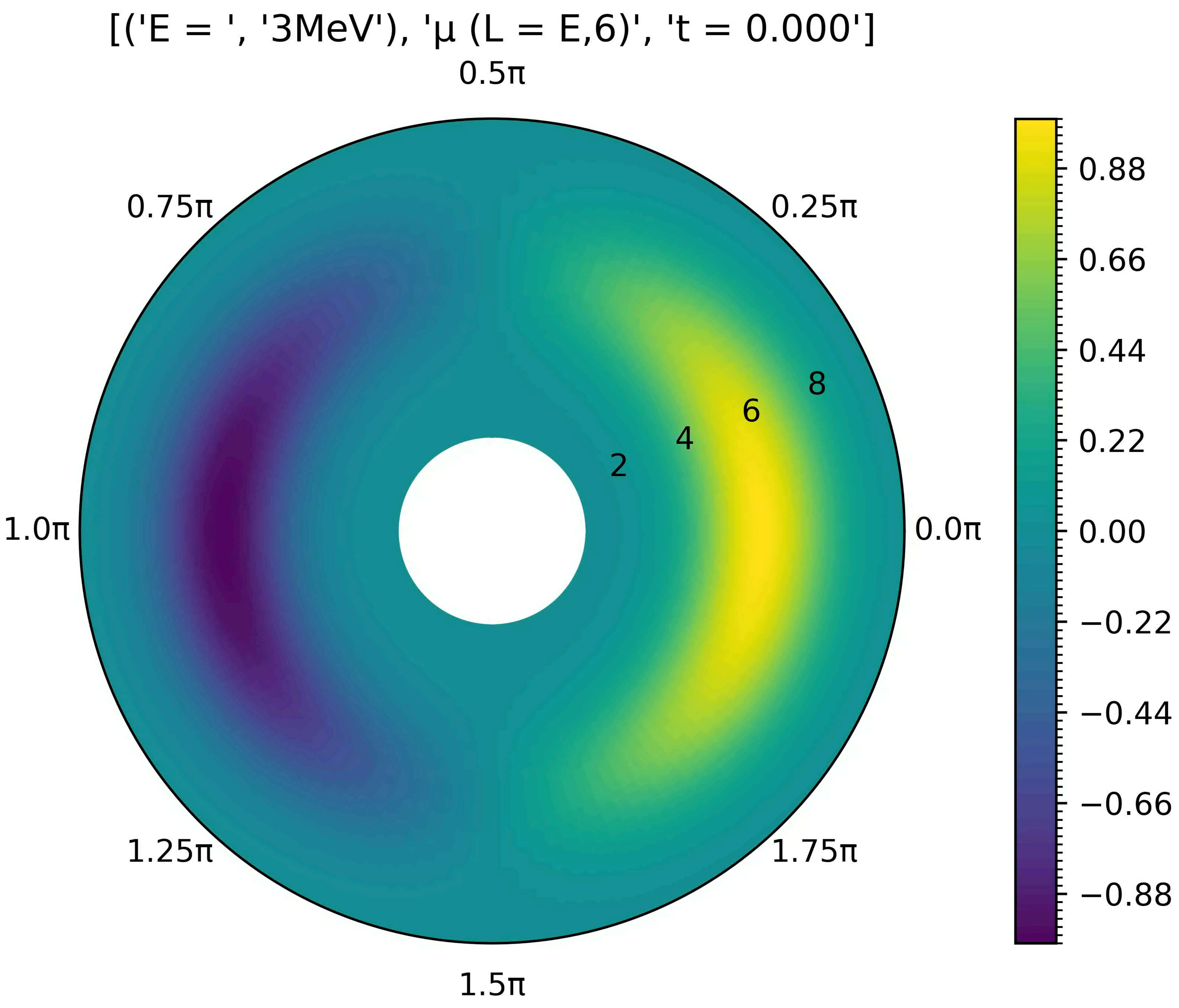
- The perturbed distribution function can be solved exactly:

$$\delta f_m(r,t) \sim \frac{\delta B}{B_0} r^5 (1 - cos\Omega_d t)$$

- A time-delayed oscillation $\sim r^5$ appears in the distribution function

- For small field perturbations the wave term vanishes → recover the same phase-mixed solution as before

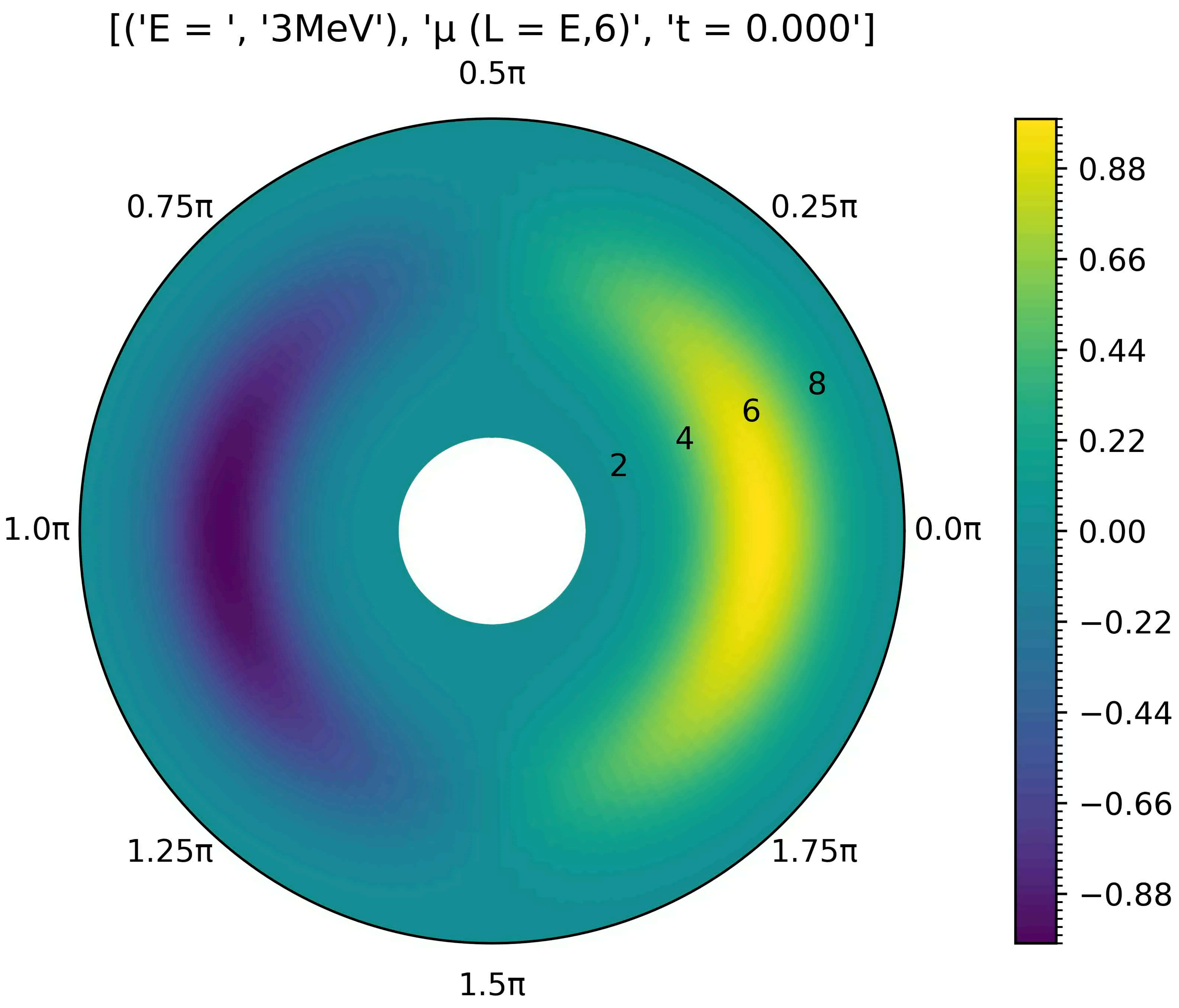
# Non-resonant wave, large field perturbation




- $\delta B = 5$ nT
- Non-resonant frequency (10 mHz)
- Runtime 25 drift periods, time-step 1/40th of a drift period

✔ Phase-mixing and non-local transients in $r$

# Non-resonant wave, small field perturbation




- $\delta B = 1$ nT
- Non-resonant frequency (10 mHz)
- Runtime 25 drift periods, time-step 1/40th of a drift period

✔ Weak transients, similar to a system with no wave

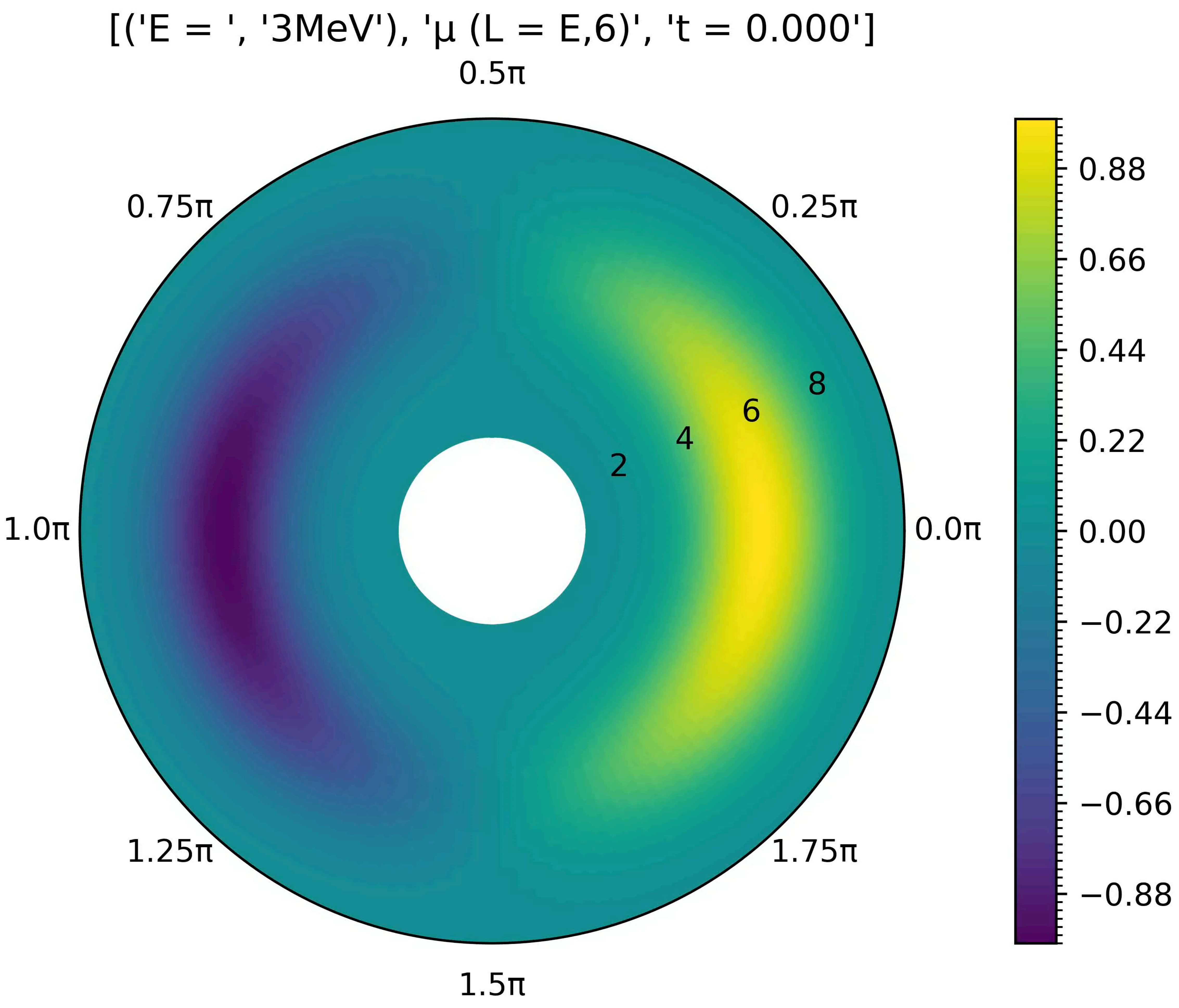
# Results: resonant wave, $\frac{\partial \delta f_m}{\partial t} + im\Omega_d \delta f_m = -(\gamma_m - i\omega_m)\frac{\delta B}{B_0} e^{-i\omega_m t + \gamma_m t} r \frac{\partial f_0}{\partial r}$

- The resonant response shows exponential decay

$$\delta f_m^L = -\frac{\delta B}{B_0} r^5 \left[ \underbrace{e^{i\omega_m t + \gamma_m t}}_{\text{growth/damping}} - \underbrace{e^{-im\Omega_d t}}_{\text{zebra stripes}} + \underbrace{m\Omega_d e^{-im\Omega_d t} \left( \frac{e^{im\Omega_d t - i\omega_m t + \gamma_m t} - 1}{\omega_m - m\Omega_d + i\gamma_m} \right)}_{\text{wave-particle resonance}} \right]$$

- Same coefficient as in the non-resonant case → **transients!**
- Transients will hide resonance for large $\delta B$, for smaller perturbations the transients disappear but the width of the resonant region shrinks → identifying resonance may be difficult

# Resonant wave




- $\delta B = 1$ nT
- Resonant frequency (25 mHz)
- Runtime 25 drift periods, time-step 1/40th of a drift period

✗ No amplification of the wave or clear spreading of the distribution function

✓ Weak transients

# Significance & future research

- Analytical and numerical results are in excellent agreement with each other
- The resulting model does not violate Liouville's theorem, makes no assumptions of the nature of the wave signal & requires minimal computational tools
- The model is limited to equatorially trapped electrons → introduce angular dependence
- Further data analysis required to identify drift resonance
- Adding multiple waves to the system would allow for testing of current diffusion models