Percolation in Cosmological Phase Transitions

ParAs Seminar Presentation

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Outline

Phase transitions

Phase transitions can be roughly categorized as first-order, second-order and crossovers which are similar to second-order ones.

The Standard Model undergoes a crossover at electroweak scale rather than a phase transition, but extensions of it may have a first-order phase transition. [1, 2]

First-order phase transitions happen through nucleation of bubbles of the phase.

- For example water boiling.

Disclaimer, I do not know QFT.

Phase transitions

- Phase transitions usually have **order parameter**, which is a scalar field ϕ.
- There is a jump in the value of this field during phase transition.
- The value of this scalar field in the symmetric phase is $\phi = 0$ and $\phi \neq 0$ in the broken phase. [2]

Phase transitions

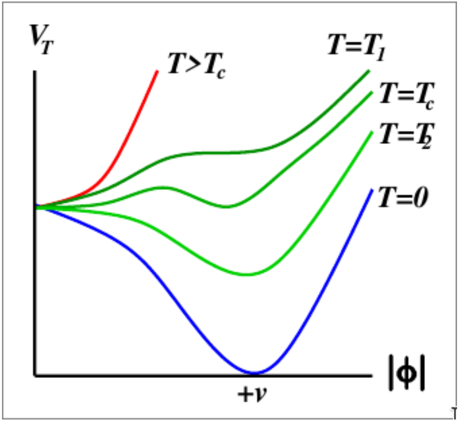


Figure: At T_c the minima are degenerate.[2]

First-order phase transitions involve a discontinuous jump in the field value ϕ between the phases.

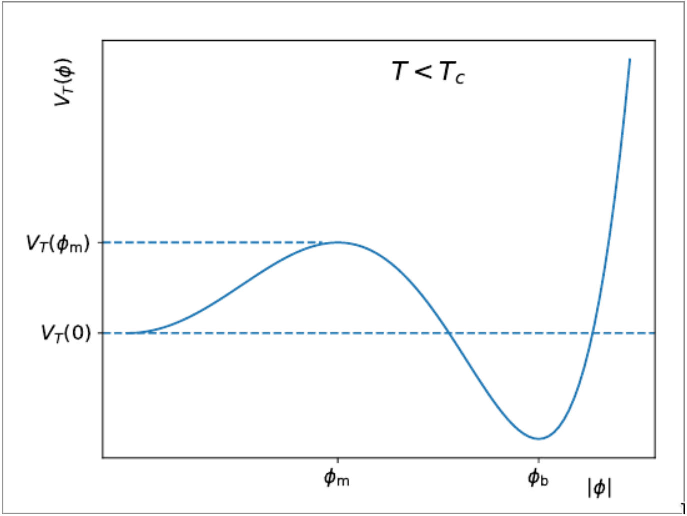


Figure: Minima separated by potential barrier. [2]

First-order phase transitions

Assuming pure universe the jump in the field value happens due to either thermal or quantum fluctuations.

These fluctuations cause bubbles of the new phase to grow, which eventually merge to change the whole phase of the universe.

During this merging gravitational waves can be formed from for example the collisions of the bubbles.

Similar to water, Universe can supercool which slows down the phase transition. [2]

Nucleation of bubbles

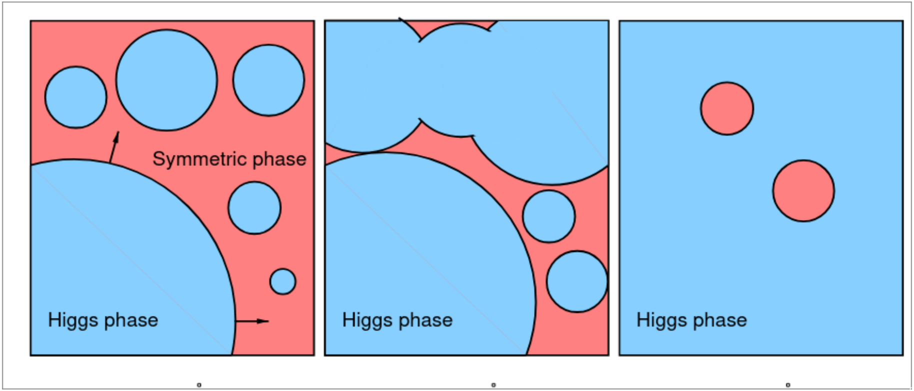


Figure: Growth of bubbles of new phase.[2]

Fractional volume

The bubbles grow to take up some fractional volume[1] ϕ.

As the bubbles only nucleate in the symmetric phase, the nucleation rate effectively slows down as more bubbles nucleate.

The fractional volume in the new phase can be shown to follow

$$\phi(\tau) = 1 - \exp(-e^{\tau})$$

Where τ is a new time variable that characterizes the phase transition. At $\tau = 0$ roughly $\phi = e^{-1}$, so roughly 64% of the universe is in the new phase. [3, 2].

[1] Not to be confused with the other ϕ...

Strongly supercooled phase transitions

If the phase transition is fast enough we can neglect the expansion of Universe.

However with supercooling Universe remains in the metastable phase for extended time beyond the critical temperature T_c.

In the case of strongly supercooled phase transition the expansion rate affects the dynamics of the phase transition among other things. [4]

Strongly supercooled phase transition

In case of strongly supercooled phase transition we can make some approximations[1, 4]:

- The vacuum energy dominates until percolation.
- Bubbles grow at speed of light.
- Initial bubble radius is negligible.

Intermission

Percolation: Basics

To illustrate percolation, consider two dimensional case of bubbles

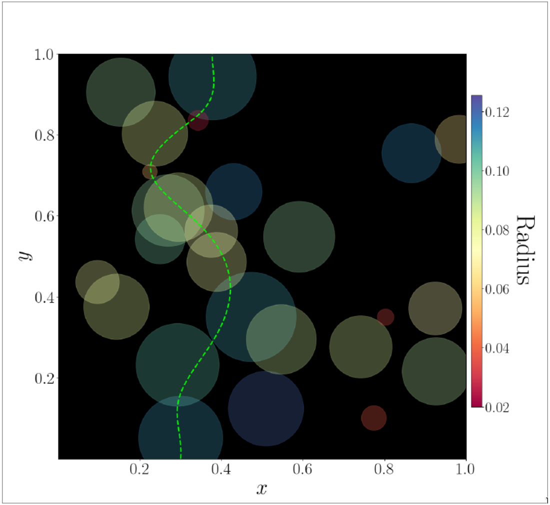


Figure: Example of percolation [4].

Why we are interested in percolation.

Percolation marks an important time when majority of the bubbles are colliding.

- One parameter to consider when predicting GWs.
- Percolation is connected to the reheating of the Universe, which happens before radiation dominated epoch [1].

Percolation

We are interested in continuum percolation, as opposed to percolation in a discrete space.

- Basically in real $\mathbb{R}^3$-space

Percolation

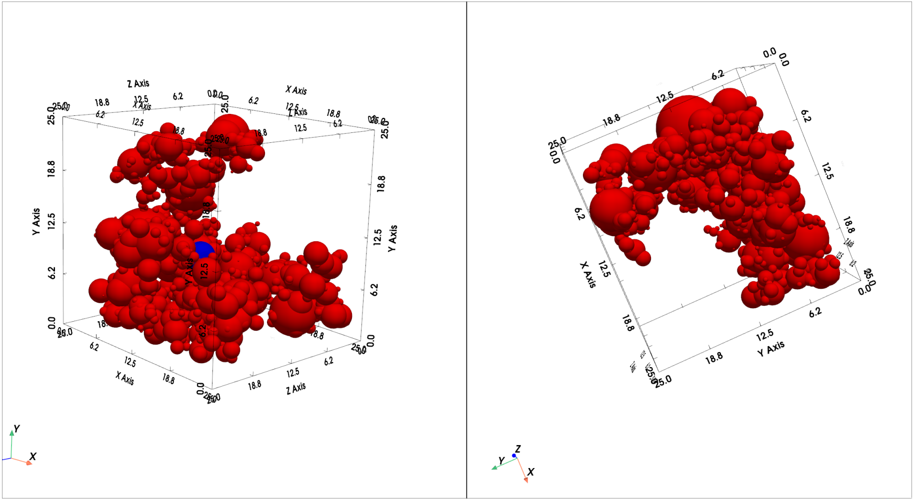


Figure: Percolating cluster from simulation with container size $L = 25$, blue dot represents the center of mass of the cluster.

Percolation

When the system percolates we have a cluster that spans the entire system.

- For infinite volume this happens at some critical value of fractional volume ϕ_c.[5]
- For finite, especially small volumes there is some probability for percolation which varies respect to ϕ.

What direction(s) do we require for the cluster to span?

- Surprisingly does not matter with equal size spheres. [6]

Simulation

- Actual bubble placing algorithm I won't go over, covered in [3].
- The algorithm for finding clusters of bubbles.

Prerequisites

How can we get the ϕ_c value for infinite volume?

- Periodic boundary conditions.
- Generate system of size L^3 and plot the probability of percolation against ϕ.
- Estimate critical value $\phi_c(L)$ for this system from fitting a function that approaches a step function.
- Fit a scaling $\phi_c(L) - \phi_c \propto L^{-1/\nu}$ to extract ϕ_c. [5]

Disclaimer about scaling not being proven possibly.

Finding cluster

Basic idea is as follows:

- Start from some bubble, check if the bubble touches other bubbles.
- Add the bubbles that touch this one into a queue.
- Once all bubbles we need to check have been checked move onto the next bubble in queue.
- Repeat until nothing in queue.
- **Outcome:** we have identified a cluster of bubbles that touch each other

Optimization methods, dividing the volume into cells and adding checks to not loop over same bubbles twice. Method is probably not the fastest but seems to work fast enough.

Quick Example

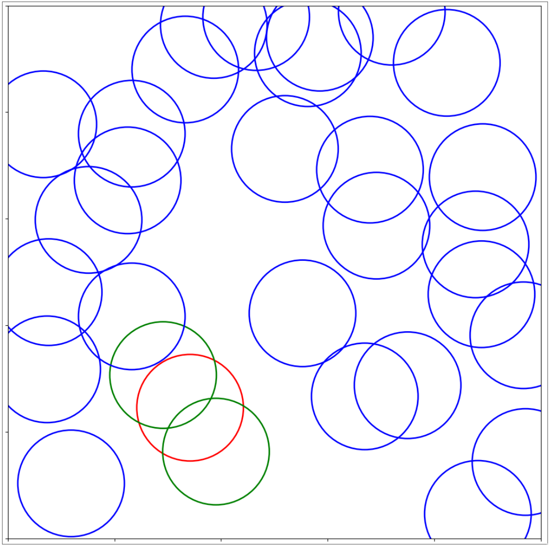
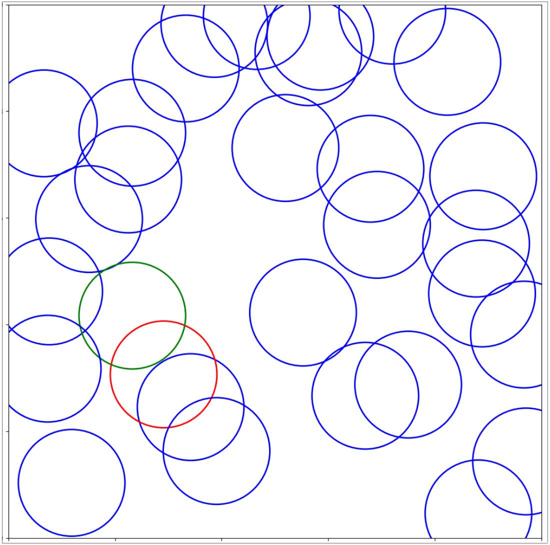


Figure: Illustrative example

Quick Example



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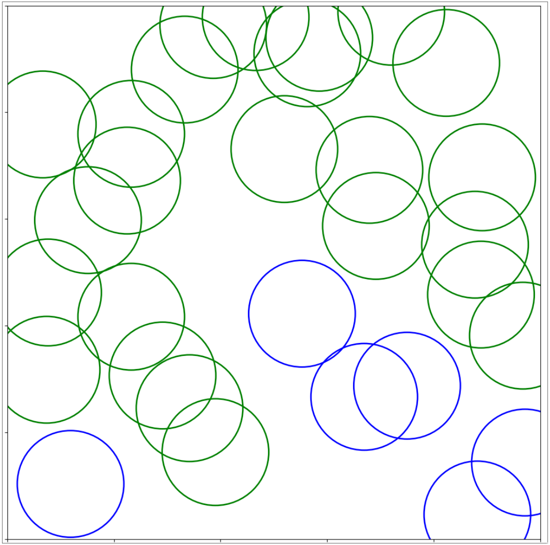

Figure: Illustrative example

Checking for percolation

Checking for percolation after this is simpler since we just have to correctly identify the bubbles near the boundary that wrap around periodically.

Testing the simulation

I tested the simulation against the known result of equal size spheres $\phi_c \approx 0.29$. [5, 7]

My simulation gives results that agree with this.

- Error bars might not be entirely accurate.

No certain results for different sized bubbles yet, preliminary results suggest that $\phi_c = 0.28 - 0.3$.

Equal size 3D continuum percolation

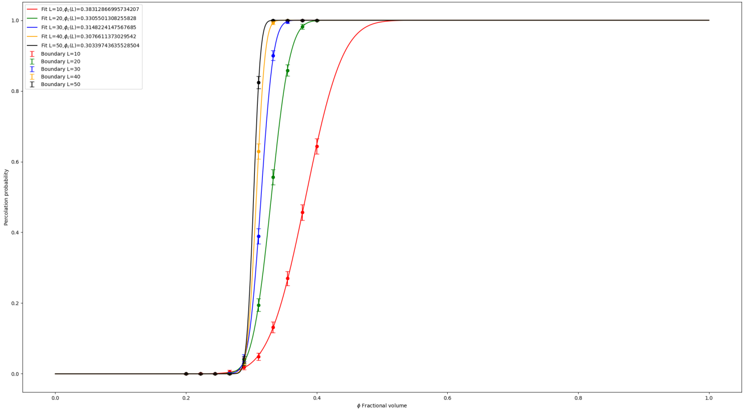

Figure: Percolation probability for different sized systems

Fit

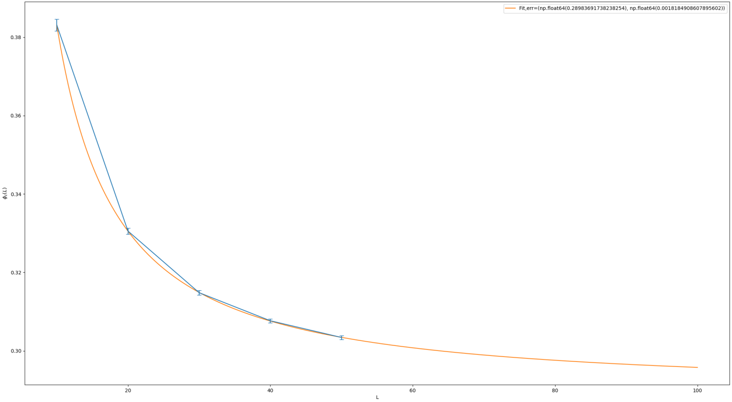

Figure: Fitted scaling function to get ϕ_c

Same with OR

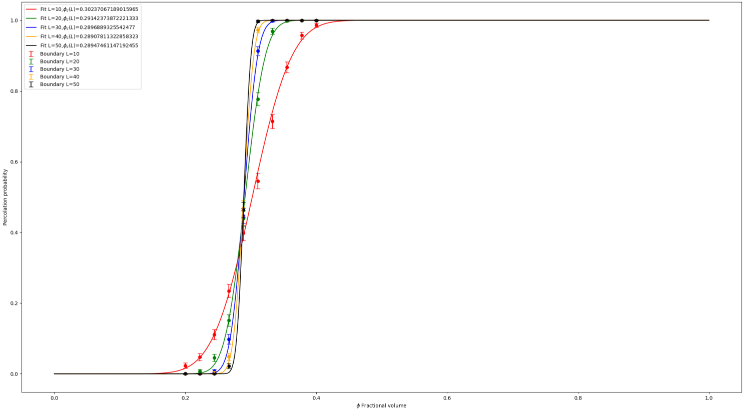

Figure: Percolation probability for different sized systems

Thank you for listening

References I

[1] John Ellis et al. "Gravitational wave energy budget in strongly supercooled phase transitions". In: *Journal of Cosmology and Astroparticle Physics* 2019.6 (June 10, 2019), pp. 024–024. DOI: 10.1088/1475-7516/2019/06/024.

[2] Mark B. Hindmarsh et al. "Phase transitions in the early universe". In: *SciPost Physics Lecture Notes* (Feb. 15, 2021), p. 24. DOI: 10.21468/SciPostPhysLectNotes.24.

[3] K. Enqvist et al. "Nucleation and bubble growth in a first-order cosmological electroweak phase transition". In: *Physical Review D* 45.10 (May 15, 1992), pp. 3415–3428. DOI: 10.1103/PhysRevD.45.3415.

[4] Peter Athron, Csaba Balázs, and Lachlan Morris. "Supercool subtleties of cosmological phase transitions". In: *Journal of Cosmology and Astroparticle Physics* 2023.3 (Mar. 1, 2023), p. 006. DOI: 10.1088/1475-7516/2023/03/006.

References II

[5] Mingqi Li, Huisu Chen, and Jianjun Lin. “Numerical study for the percolation threshold and transport properties of porous composites comprising non-centrosymmetrical superovoidal pores”. In: *Computer Methods in Applied Mechanics and Engineering* 361 (Apr. 2020), p. 112815. DOI: 10.1016/j.cma.2019.112815.

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[7] Don R. Baker et al. “Continuum percolation threshold for interpenetrating squares and cubes”. In: *Physical Review E* 66.4 (Oct. 30, 2002), p. 046136. DOI: 10.1103/PhysRevE.66.046136.