Percolation of bubbles in cosmological phase transitions

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Abstract

The Laser Interferometer Space Antenna or LISA is set to launch in the next decade with the goal of measuring gravitational waves. Some of these gravitational waves we could measure with LISA could potentially be generated from very early universe. These sort of gravitational waves contribute to so called gravitational wave background (GWB) which in a way is analogous to the cosmic microwave background (CMB). These signals can be generated from numerous sources, one of which are first-order phase transitions in the early universe.

The phase transitions are a common feature of many gauge field theories, but not all of them feature first-order phase transitions for example such a transition is absent in the Standard Model (SM) but present in many extensions of it at electroweak scale. The first-order phase transition proceeds through nucleation nucleation of bubbles of new phase. One of the ways GWs can be generated during first-order phase transitions is through the collision of the growing bubbles of new phase. The time at which significant number of the bubbles collide coincides with when the system percolates, and as such the time at which percolation occurs is important in predicting GWs. First-order phase transitions can happen fast or be delayed in the case of supercooling.

Percolation is best illustrated by the movement of fluid through a porous material. A system is said to percolate when we can move across the whole volume through the pores of the material for example water moving through earth. In our case we ask when we can traverse across bubbles of new phase in an infinite volume. We call these connected regions clusters and the question is at what fraction of volume must the bubbles take before we get a cluster that spans the entire system. In our case we are interested in percolation in all coordinate directions and as such we consider our system to percolate only when we can traverse through the volume in any direction. Below percolation threshold such clusters do not exist, so our goal is to find the critical volume fraction ϕ_c after which percolation occurs.

Finding the percolation threshold is a very complex analytically to calculate and as such we resort to numerical simulations, which in the case of strongly supercooled phase transitions are comparatively easy to perform. The basic recipe for simulation is to first simulate the nucleation of bubbles in some physical volume up to some time t which is then repeated to form a sample of probability of percolation at time t. From this we can calculate the critical fractional volume ϕ_c at which the percolation occurs for some finite-sized volume. After this we repeat the simulation for increasingly bigger volumes from which we can extrapolate the ϕ_c for the case of infinite sized volume.