Lattice measurements of the static quark-antiquark potential and holographic bulk reconstruction

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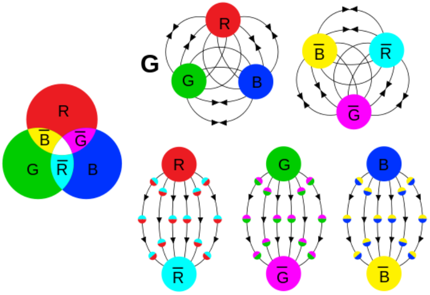
Seminar in Particle Physics and Astrophysical Sciences,
26.3.2024

Structure

Structure

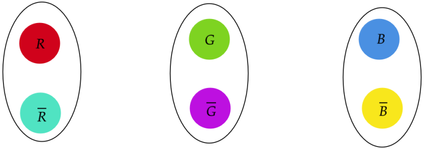
Quantum chromodynamics (QCD)

- QFT of the strong interaction
- 2 types of particles
  - quarks (fermions)
  - gluons (gauge field)
- Interesting properties
  - Gluon self-interactions
  - Color confinement



[Wikipedia]

Quark-antiquark potential shows confinement



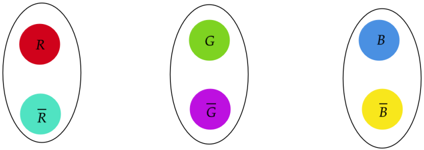
- A static quark-antiquark pair

Cornell potential

$$V(r) = A + \frac{B}{r} + \sigma r$$

- A is a constant

Quark-antiquark potential shows confinement



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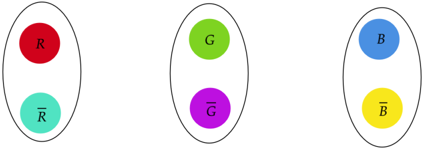

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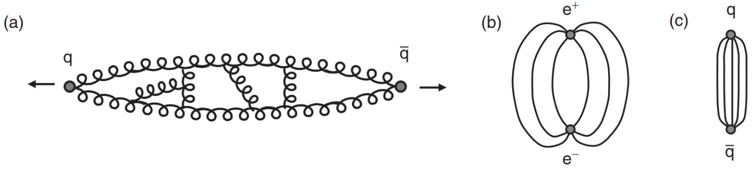
Cornell potential

$V(r) = A + \frac{B}{r} + \sigma r$

- A is a constant
- $\frac{B}{r}$ is the electric potential
- σr from strong interaction

Linear term gives rise to confinement

- The attractive self-interaction of gluons forces the color field into a narrow tube



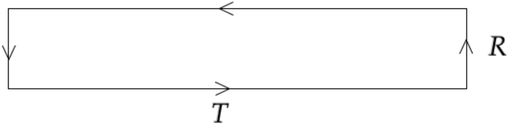
[Mark Thompson, Modern particle physics]

- Constant energy density$\rightarrow$ linear term σr
- Trying to separate the pair requires infinite energy $\rightarrow$ quarks confined

Wilson loop

- The static potential can be related to a QFT operator called the Wilson loop

$$W(\mathcal{C}) = \mathrm{tr}\left[\exp\left(i\int_{\mathcal{C}} A_\mu \mathrm{d}x^\mu\right)\right]$$



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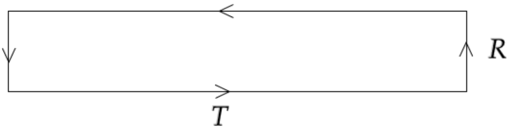

- To compute the expectation value of a QFT operator a path integral needs to be evaluated

$$\langle W(\mathcal{C})\rangle = \int \mathcal{D}A \, W(\mathcal{C}) e^{iS(A)}$$

Structure

Evaluating the path integral

- How can you calculate the path integral?

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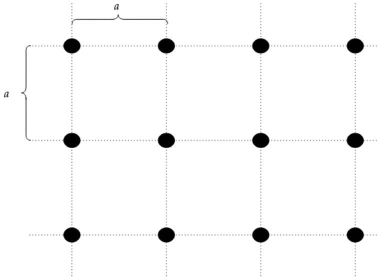
- How can you calculate the path integral?
  - You can't at least not directly
- In perturbation theory integral expanded to a series w.r.t to the coupling
  - requires weak coupling
- Lattice field theory allows estimation in non-perturbative, high coupling regime

Putting QFT on a lattice

- The basic idea of lattice field theory is to replace continuous space-time with a discrete lattice
    - $x_\mu = an_\mu$, $n_\mu = 0, 1, ..., N_\mu$
- Periodic boundary conditions to deal with edges
- No value set for a
- Fermions at lattice site



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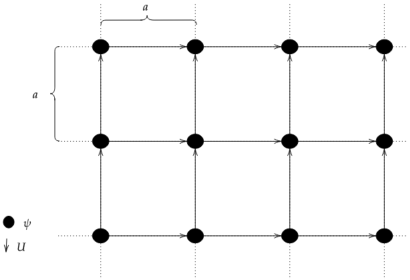

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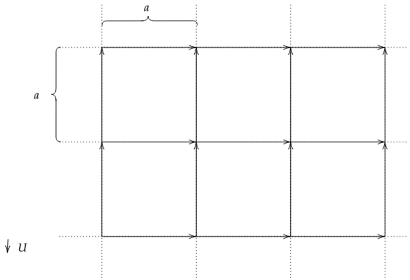
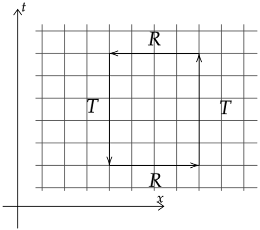


Path integral on the lattice evaluated with Monte Carlo

- Path integral on the lattice is in theory analytically computable
- The amount of parameters makes it impossible in practice
- Monte Carlo sampling can be used to estimate them
  1. Random sample configurations U drawn from dist. $P(U) \propto e^{-S[U]}$
  2. Evaluate measured observables at samples
  3. Take the mean to estimate the expectation value

My lattice simulations

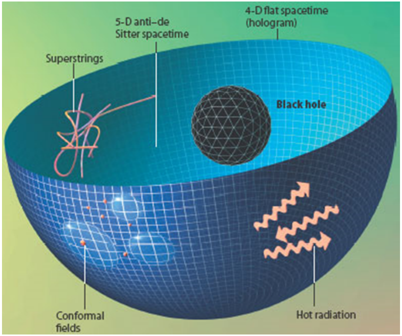


- Lattice of size 16^4 with β between 6.0 and 7.0
- The W.loop is measured in tens of thousand of configurations for separations between $r = 1$ and 7
- The Cornell potential is fitted and compared with experimental data to define a and get continuum units
- Has been measured before to high precision. My implementation purely educational.

Structure

What is holographic duality?

- Equivalence
  - a QFT in 4D flat space-time ↔string theory in higher dimensional curved space-time
- QFT lives on the boundary of higher dimensional space-time
- At strong coupling string theory reduces to supergravity
  - Non-perturbative QFT calculations with "easy" gravity computations



Building a holographic model

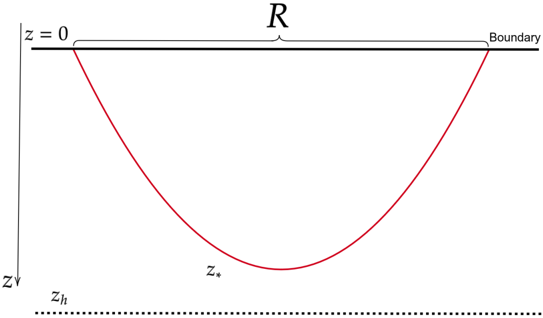
- Traditionally:
  1. Educated guess for gravity side
  2. work out QFT quantities
  3. compare with data
  4. reiterate
- Recently the inverse approach has been adopted:
  1. Start with QFT data
  2. Construct gravity side from data
  3. Computations with constructed gravity dual
  - Called bulk reconstruction

Holographic Wilson loop

- Extend a string from the loop into the bulk

$$\langle W \rangle \approx e^{-S_{NG,min}}$$

- Simply minimization of classical action



Bulk reconstruction from $q\bar{q}$-potential

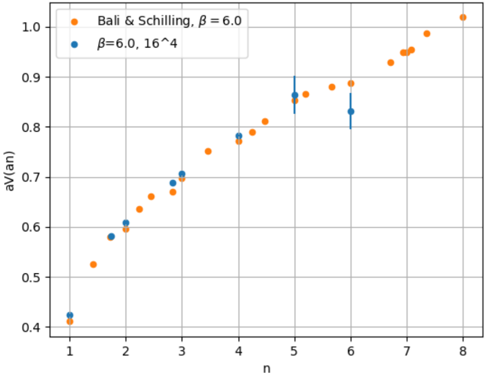
- Metric based on AdS-BH:

$$ds^2 = \frac{L^2}{z^2}(-f(z)dt^2 + g(z)dz^2 + d\vec{x}^2),$$

- $f(z)$ and $g(z)$ from BH multiplied by an exp. of power series
- Using a machine learning method the coeffs. from the metric are optimized to fit the lattice data of $R, V(R)$
- Done with two terms in the power series, no further computations

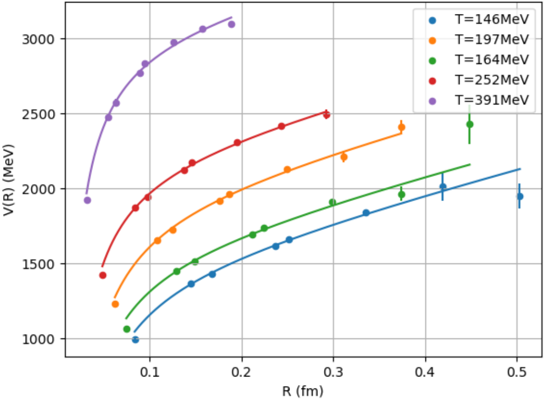
Structure

Unscaled lattice data in good agreement with previous results



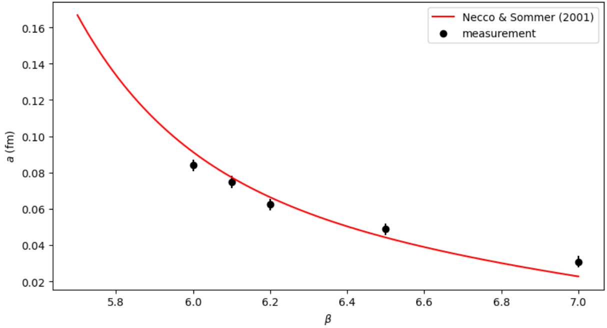
- Unscaled data in agreement with (Bali & Schilling (1992))
- Differences & uncertainties grow as separation increases

Scaled potential at different temperatures



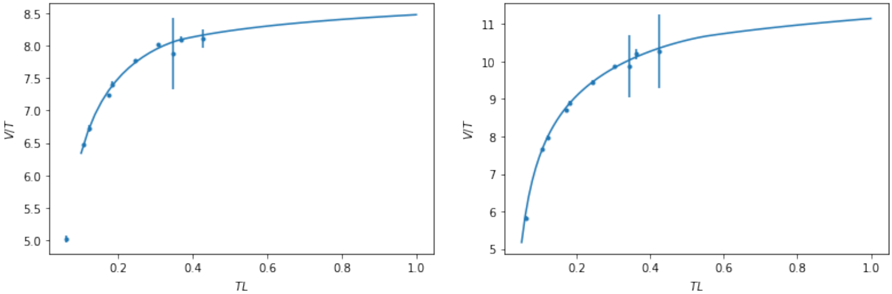
- Energy increases with temp.
- *a* gets smaller as temp. increases

Scaling has correct form but slight deviations



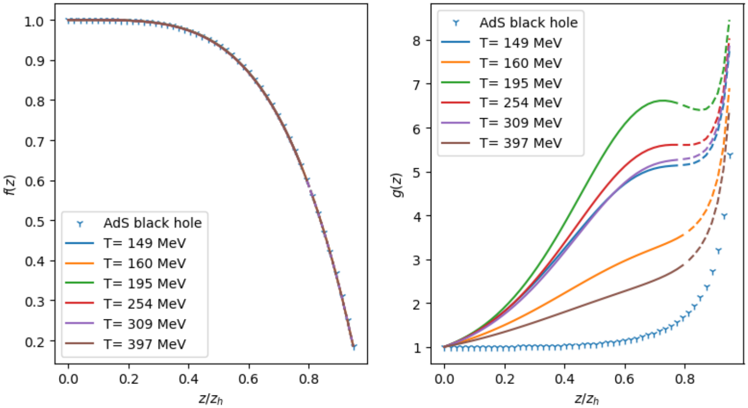
- General form similar to (Necco & Sommer, hep-lat/0108008)
- Deviations likely due to limited data

Holographic model for static potential



- The constructed holographic models fit the data well
- Form follows the Cornell potential with the extension beyond the data being linear
- Method seems to work

Reconstructed metric behaviour as temperature grows



- very little change in $f(z)$
- No clear temperature behaviour

Structure

Future works

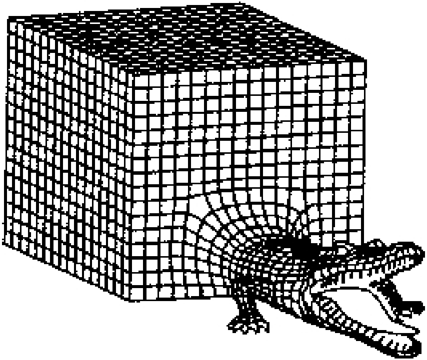
- Ways to improve both parts
  - Lattice measurements
    - more configs., more R, bigger system, additional terms etc.
  - Bulk reconstruction
    - errors, different base metric, bigger data set
- Using the constructed metrics compute different QFT quantities
  - Sensitive to deconfinement phase transition
- Reconstruction can also be made using these observables
  - More difficult to obtain lattice data
  - Progress in entanglement entropy
- Holography has entered a new era of precision

Summary

- Linear term in potential gives rise to confinement
- Lattice field theory used to evaluate path integrals and measure static potential
- From lattice data corresponding holographic models constructed $\rightarrow$ new predictions
- Lattice measurements had correct form, errors due to limited data set
- In constructed metrics f from BH, no clear temperature behaviour in g

Thank you!!

Questions??



[Karl Jansen, homepage]