

## Entanglement Entropy in the $O(N)$ Model

Entanglement is a fundamental feature of quantum mechanics and plays an important role in characterizing many-body systems and quantum phase transitions. Entanglement entropy provides a quantitative measure of quantum correlations between subsystems. We study the entanglement entropy of an interacting quantum field theory, the  $O(N)$  model.

In local quantum field theories, the leading-order contribution in the scaling of entanglement entropy obeys an area law. This dominant term is governed by short-distance physics near the entangling surface and depends on the ultraviolet regularization, making it non-universal. Beyond this leading scaling behavior, entanglement entropy can contain subleading universal contributions, which encode intrinsic information about the underlying quantum field theory.

We calculate one such universal term: the correlation-length coefficient  $r$ , which appears as a correction to the entanglement entropy slightly away from the quantum critical point. The analysis is carried out in  $D = 2 + 1$  spacetime dimensions for an infinite cylindrical geometry. We study the model in the vicinity of criticality using the large- $N$  expansion, which enables controlled analytical calculations.

Our main result is that the coefficient  $r$  scales linearly with the number of field components  $N$ . This behavior is consistent with the interpretation of  $r$  as a universal quantity associated with the degrees of freedom of the theory. The calculation is based on the replica method and renormalization group arguments, which are presented in detail. Finally, we discuss the physical meaning of the result and possible methods for verifying it.