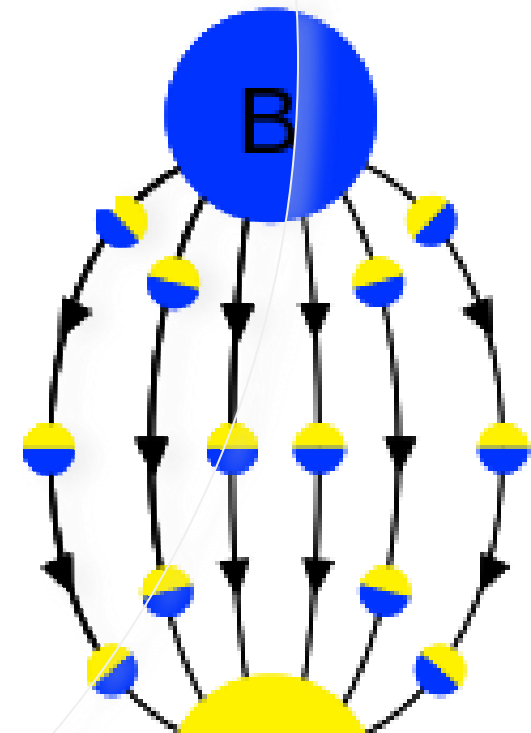
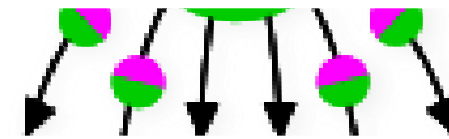
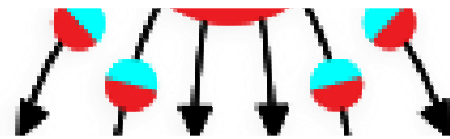
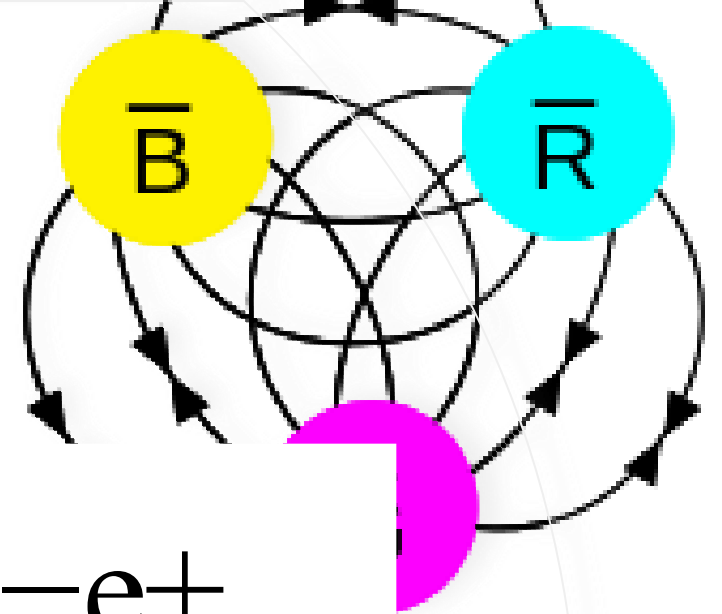
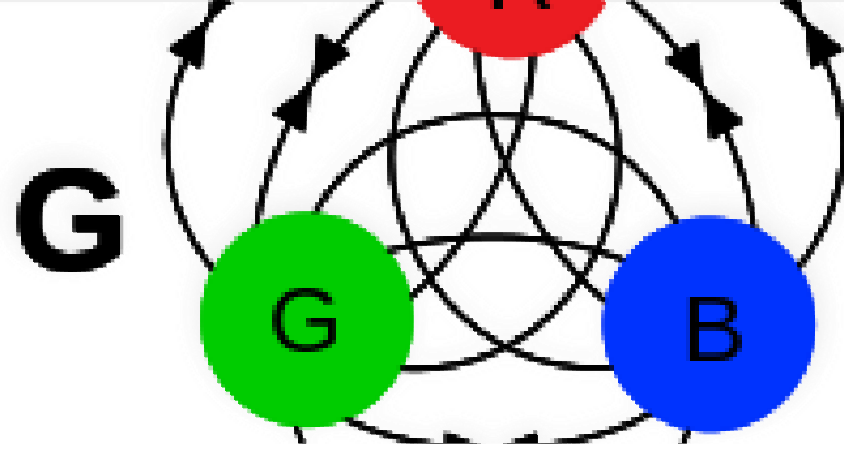


QCD outcomes in e^-e^+ -collision



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PAP301 Seminar in Particle Physics and Astrophysical Sciences



Contents

1. High-energy e^-e^+ -collision leads to different outcomes of electromagnetic and strong force particles
2. Cross section of a specific process describes its interactions and production rate
3. Production of quarks as an example of strong interaction
4. QCD total cross section considers loop corrections and scattering gluons

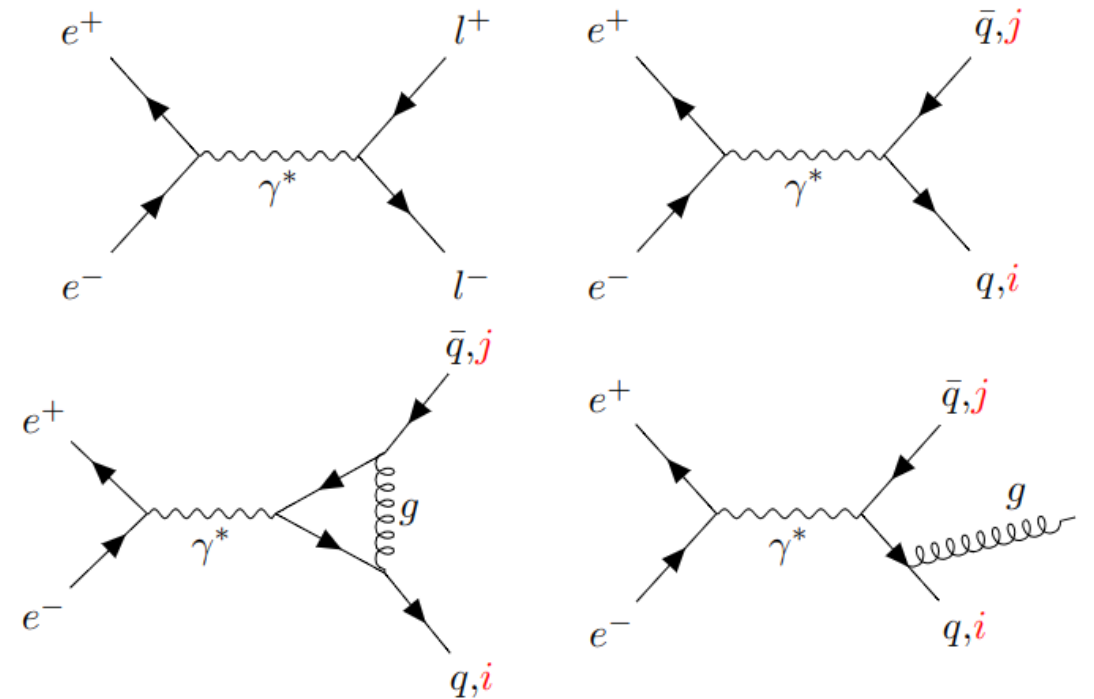


Figure 1: The collision processes, up to next-to leading order, are mediated by a virtual(*) photon.

Particle collisions in high energies

- Particles or groups of particles are collided for experimental results
- Immediately after the collision, in high energy and density, the particles can be observed free
- Colliding particles are electron and positron, annihilating into a photon

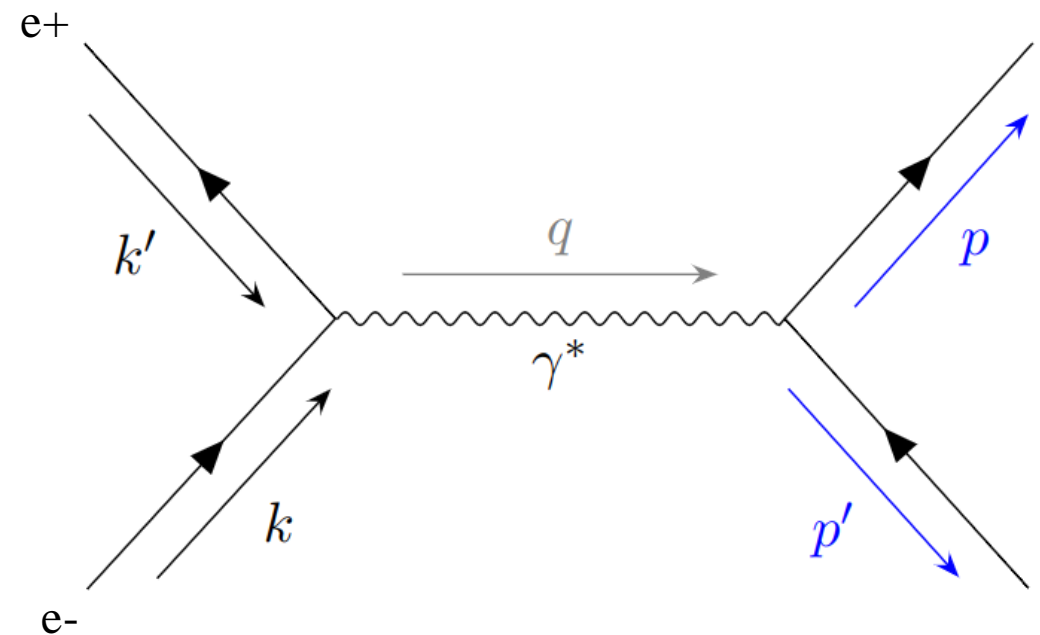


Figure 2: Feynman graph of a 2-to-2 particle collision.

We need both electromagnetic and strong interaction particles

- In QED process, the particles are leptons
- In QCD outcomes, the particles are quarks and gluons = partons
- The six quarks have different masses and charges
- Partons are normally observed as part of a hadron
 - o Proton = 2u+1d

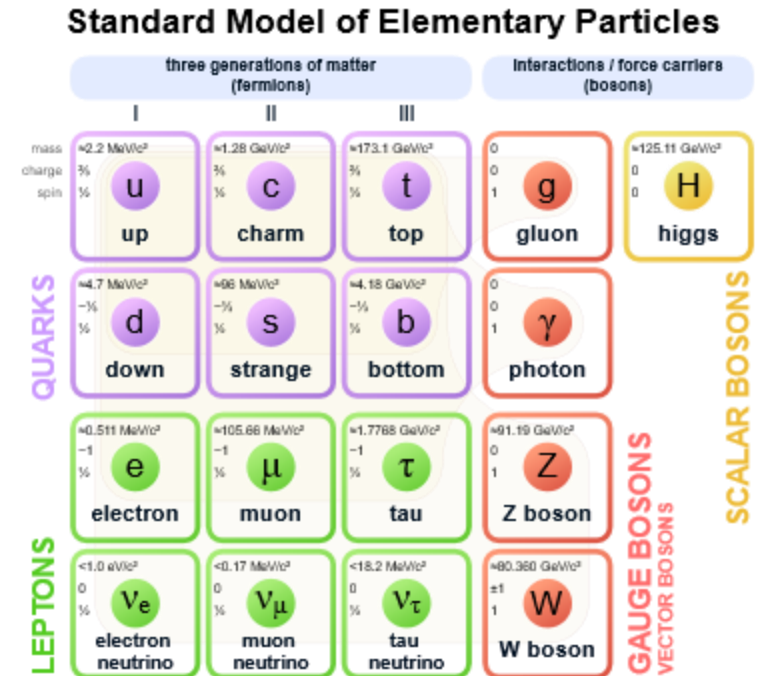


Figure 3: Standard Model.
 [Wikipedia: Standard Model]

e-e+ -collision commonly scatters a leptonic outcome

- The annihilated photon is not the final state, but scatters into different outcomes
- Common outcome is a pair of massive particles μ^- and μ^+
- Hard scattering leads to momentum conservation

$$e^-(k) + e^+(k') \rightarrow l^-(p) + l^+(p')$$

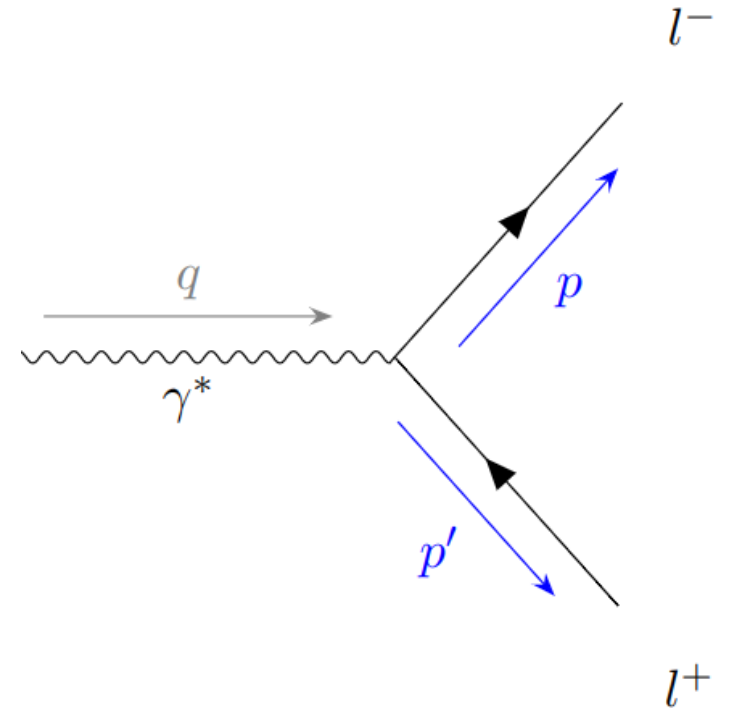


Figure 4: A photon scattering into a leptonic outcome.

e-e+ -collision can also produce QCD outcomes

- The photon scatters into a quark and an antiquark
= partonic outcome
- In pre-hadronic state the partons are observed only briefly as free
- The outcomes include leading order, virtual corrections (loop) and real corrections (gluon scattering)

$$e^{-}(k) + e^{+}(k') \rightarrow q(p) + \bar{q}(p') + (g(l))$$

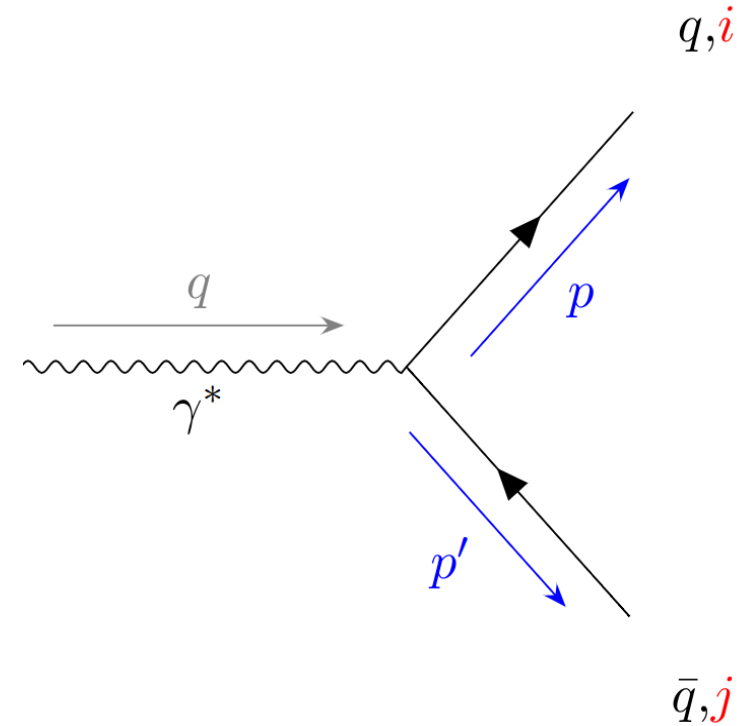


Figure 5: A photon scattering a partonic outcome.

Strong interaction increases with distance

- Theory of QCD describes strong interaction, that binds together quarks and gluons as hadrons
- QCD has a few important properties:
 - Quarks appear in three colors
 - Color confinement
 - Asymptotic freedom in short distances and large energies

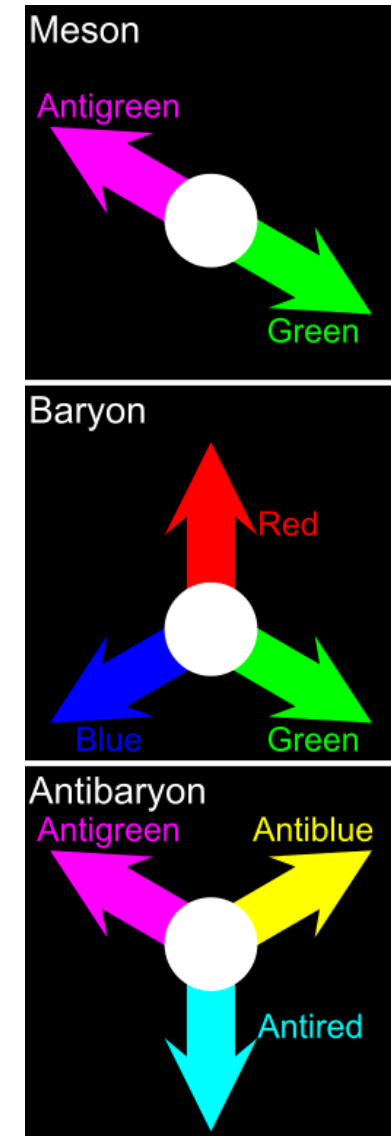


Figure 6: Color neutral hadrons are made of quarks. [Wikipedia: Quark]

Calculating cross sections gives us information about the possible outcomes

- Cross section of any final state gives us the probability of that outcome to be observed
- Why calculate cross sections ?
 - The difference between QED and QCD outcomes
 - QCD NLO corrections

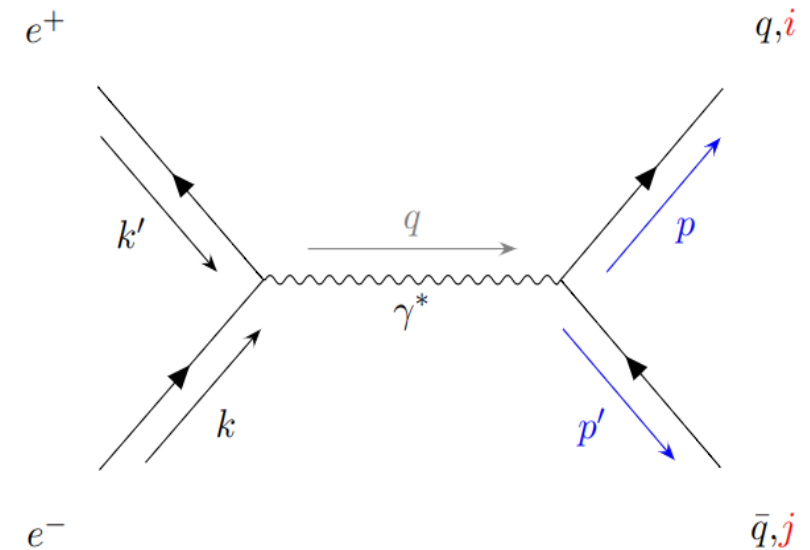


Figure 7: The e^-e^+ annihilation into a quark-antiquark pair.

QCD outcome cross section from e-e+ -collision differs from a similar QED outcome

- Experimental observations have shown that there is a difference between the QED and QCD outcomes
 - o Particles of the strong interaction carry a color charge
- The main result is a ratio of the two cross sections:

$$R = \frac{\sigma^{e^+e^- \rightarrow \text{hadrons}}}{\sigma^{e^+e^- \rightarrow l^+l^-}} = 3 \sum_q Q_q^2$$

- o Upper cross section indicates the QCD process and lower the QED process

Quarks have color charge and various flavors

- The ratio of the two processes includes three possible quark colors and a sum over quark flavors Q_q
- Photon as an electromagnetic particle, can't scatter particles with no electric charge – quarks have a charge
- Quark flavors have different masses and color charges
 - $Q_u, Q_c, Q_t = 2/3$
 - $Q_d, Q_s, Q_b = -1/3$

$$3 \sum_q Q_q^2$$

mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	u up	c charm	t top
	d down	s strange	b bottom

QUARKS

The scattering quarks are a proof of QCD

- Partons can be observed free in short distances and high energies
- Cross sections of the QED and QCD outcome 2-to-2 processes are similar and can be compared
- The main result is ratio R – quarks have color charge
- The probability of QCD outcomes can be calculated up to the NLO correction

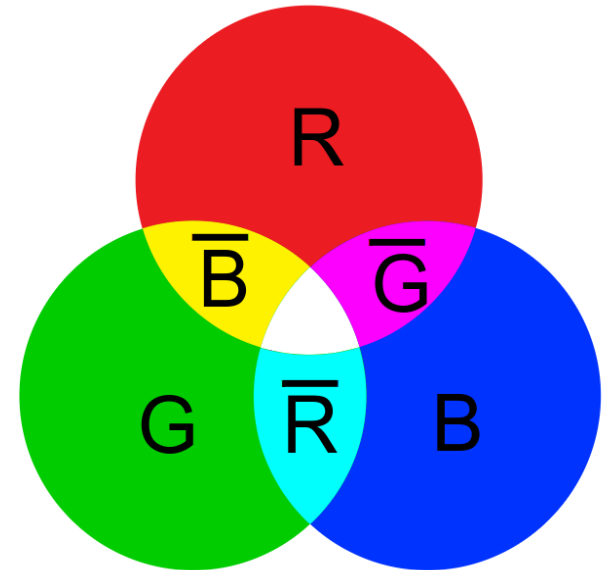


Figure 8: Quark colors. [Wikipedia: Quark]

Total cross section consists of LO and NLO corrections

$$\begin{aligned}
 \sigma &= \left| \text{tree} + \text{virtual} + \text{real} + \text{virtual} + \dots \right|^2 \\
 &= \left| \text{tree} \right|^2 + 2 \operatorname{Re} \left(\text{virtual} \right) \left(\text{tree} \right)^* \\
 &\quad + \left| \text{real} + \text{virtual} \right|^2 = \sigma_0 \left(1 + \frac{\alpha_S}{\pi} \right)
 \end{aligned}$$

Figure 9: Total cross section of the corrections up to the NLO include virtual and real gluons. The cross section depends on the total amplitude squared. σ_0 signifies the leading order QCD cross section. [Tramontano, 2019]

Calculations of NLO: perturbation theory

- Perturbative calculation includes summation of the orders, starting from the lowest order
- The collision can produce a lot of different outcomes, not only two-particle final state

$$\sigma = \sigma_0 + \sigma_{\text{virtual}} + \sigma_{\text{real}}$$

- Each term increases the result's accuracy
- The terms have their own probabilities for the specific outcome

Calculations of NLO: a 2-to-2 particle process

- NLO virtual correction includes a gluon loop
- Virtual loop: two outgoing particles
 - Gluon still affects the cross section result
- The loop calculation – with dimensional regularization and Passarino-Veltman reduction
- Includes terms involving only parton interactions

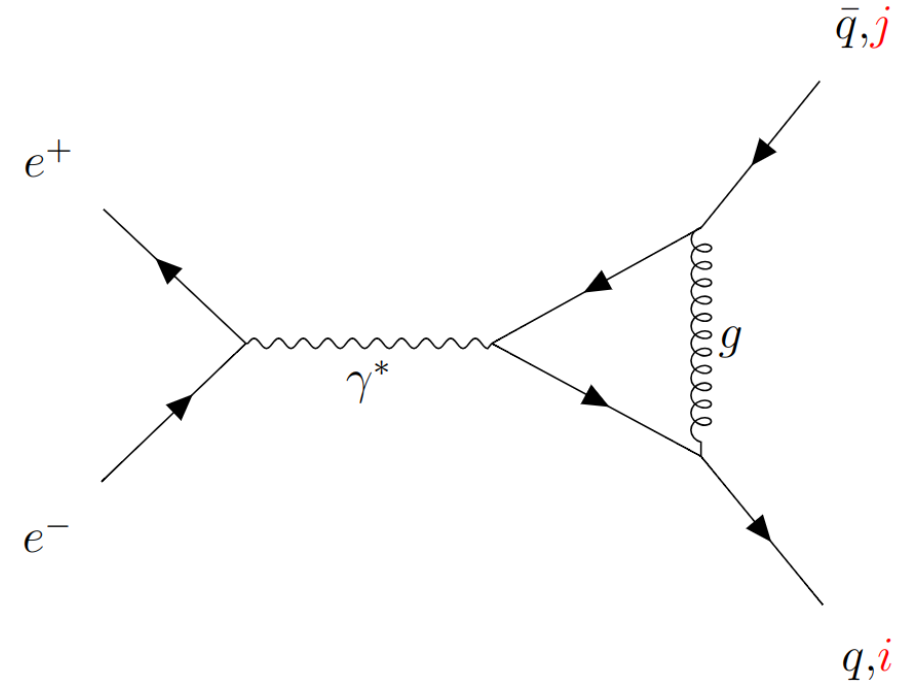


Figure 10: Feynman graph of a 2-to-2 process with a virtual gluon correction.

Calculations of NLO: a 2-to-3 particle process

- NLO real correction includes a gluon scattering
- The gluon is a third outgoing particle
- The real correction cross section is calculated over all three final particles
- Includes strong interactions between partons

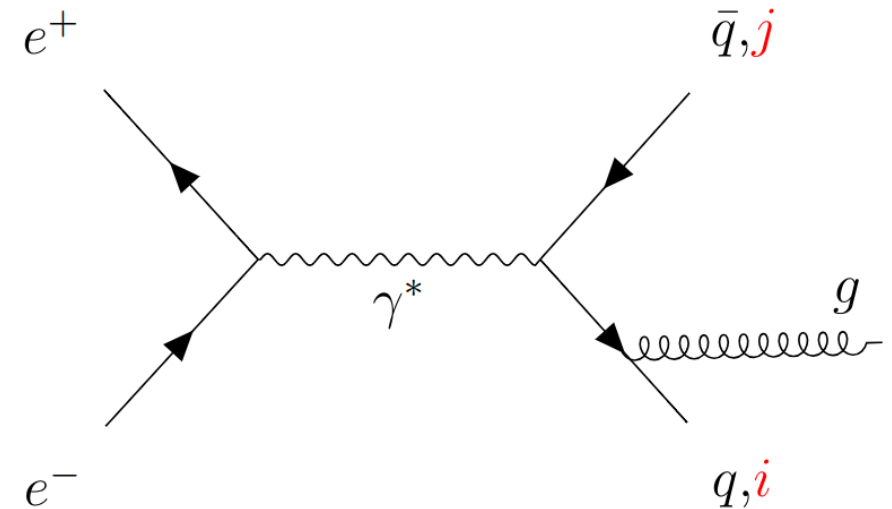


Figure 11: Feynman graph of a 2-to-3 process.

Strong interaction coupling term arises from Feynman diagrams in NLO

- Let's take a closer look at the ratio in NLO:

$$R = \frac{\sigma^{e^+e^- \rightarrow \text{hadrons}}}{\sigma^{e^+e^- \rightarrow l^+l^-}} = 3 \sum_q Q_q^2 \left(1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right)$$

- The first correction term corresponds to NLO level
- The correction in NLO is finite and α_s can be seen as effective force of the coupling

Effective coupling α_s depends on the exchange of the momentum

- Coupling strength depends on the energy scale Q
- Q can be interpreted as the momentum exchange between the initial and final particles
- What to take from this: the coupling is weaker in higher energies

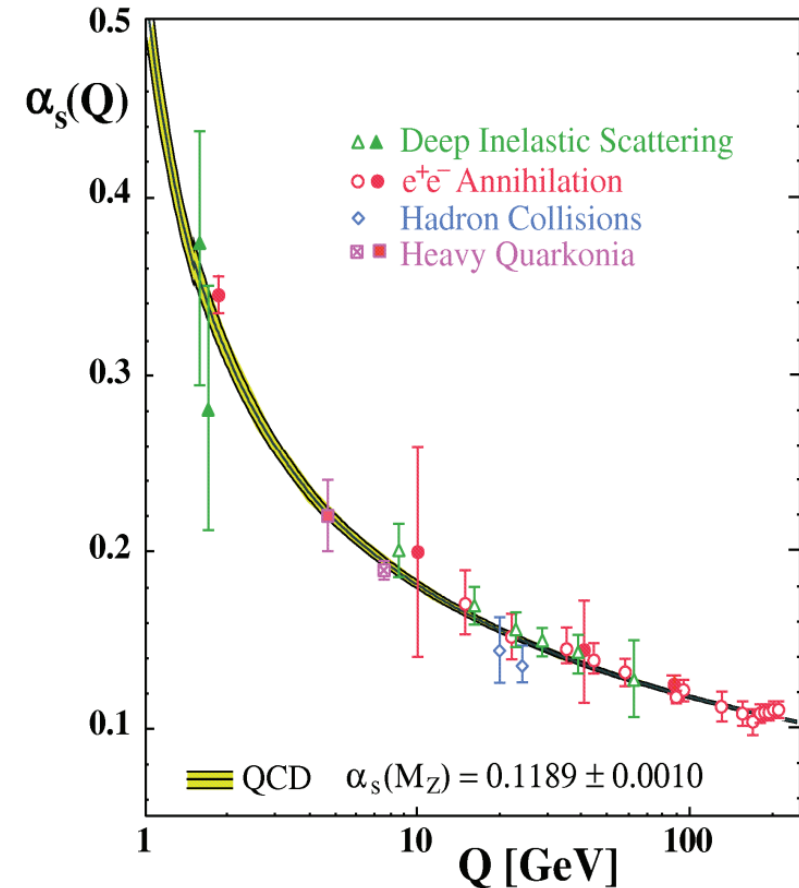


Figure 12: The curve represents the theoretically calculated coupling. [Chaudhuri, 2012]

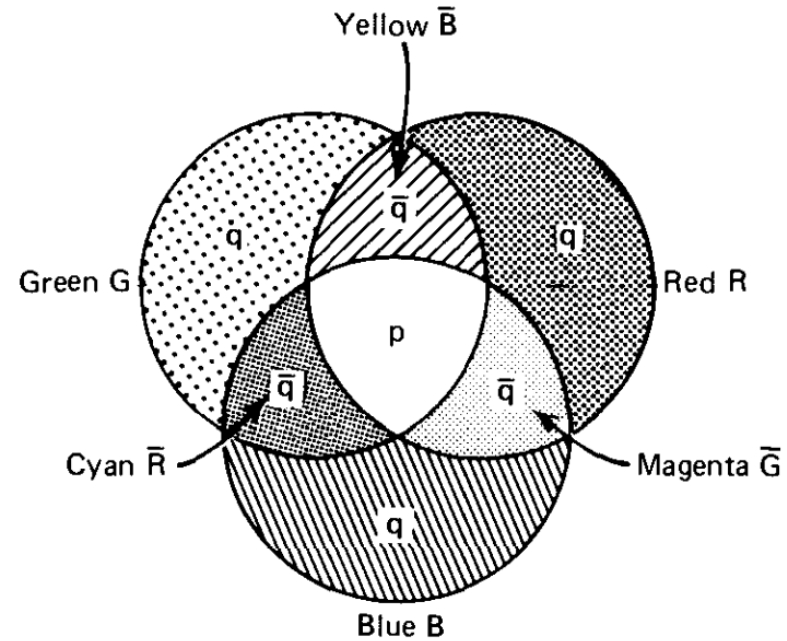
Results of NLO+ introduce higher powers of the effective coupling

- Higher than NLO cross sections are harder to calculate
 - o Include infinite terms that need to be renormalized
- Outcomes from the photon scattering will have more loops or extra scattered particles
- Smaller correction to the effective coupling means smaller probability of the specific outcome

Summary

- e-e+ collision can produce both leptonic and hadronic outcomes
- Cross section σ is calculated up to NLO for the probability of any QCD outcome in that level
- Ratio between the hadronic and leptonic processes gives evidence of the color coefficients of QCD
- NLO correction with the strong coupling is simple and finite: $\frac{\alpha_s}{\pi}$

Thank you!



- Quarks and Leptons: An Introductory Course in Modern Particle Physics, Halzen, F. and Martin, Alan D. (1984)