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One-point functions in defect Conformal Field Theories and Integrability

In a conformal filed theory (CFT), the dilatation operator encodes the action of dilatations on the conformal operators, which means operators with definite scaling dimension are eigenstates of the dilatation operator and their scaling dimension are corresponding eigenvalues. For interacting CFTs, the scaling dimension is a function of coupling constant, incorporating quantum corrections known as the so-called anomalous dimension.

There is hidden integrable structure in CFTs such as $\mathcal{N} = 4$ supersymmetric Yang-Mills theory (SYM). In the planar limit, the dilatation operator at one-loop order can be identified as the Hamiltonian of the integrable spin-chain, and the conformal operators can be mapped to the energy eigenstates of the spin-chain, which allows us to solve the spectrum by means of integrability. Bethe ansatz is a fruitful technique to reduce the spectral problem of diagonalizing the dilatation operator to the problem of solving a set of coupled algebraic equations. The sign of integrability in Bethe ansatz is that any amplitude of a many-body scattering process factorizes into a product of a sequence of two-body scattering matrices.

When defects or boundaries are introduced in a conformal field theory, the theory can have non-trivial one-point functions. There exists a domain wall version of $\mathcal{N} = 4$ SYM in which some of the scalar fields acquire a vacuum expectation value. The defect is represented by the so-called Matrix Product State, which is a fixed state in the Hilbert space of the corresponding spin-chain. It turns out that one-point functions in this defect CFT can be express as an overlap between the boundary state and Bethe eigenstate. For the integrable boundary state that is annihilated by all odd conserved charges, the overlap formula exhibits a universal structure, consisting of a pre-factor multiplied by the determinant of the Gaudin matrix. The same structure of the overlap formula can also be observed in other defect CFTs, for instance, the $\mathcal{N} = 6$ superconformal Chern-Simons theory (ABJM).

However, in the case of the domain wall version of ABJM theory, everything becomes significantly more complicated due to the complexity of the ABJM theory itself. The two-loop dilatation operator in the ABJM theory is identified as the Hamiltonian of the $SU(4)$ alternating spin-chain. We employ the nested Bethe ansatz, which results in three sets of coupled equations with three types of Bethe roots. The integrable boundary state yields selection rules under which the overlap is non-vanishing. This leads to the factorization of the Gaudin matrix, and ultimately results in the emergence of the universal structure in the overlap formula.