Early Universe Production of Dark Matter

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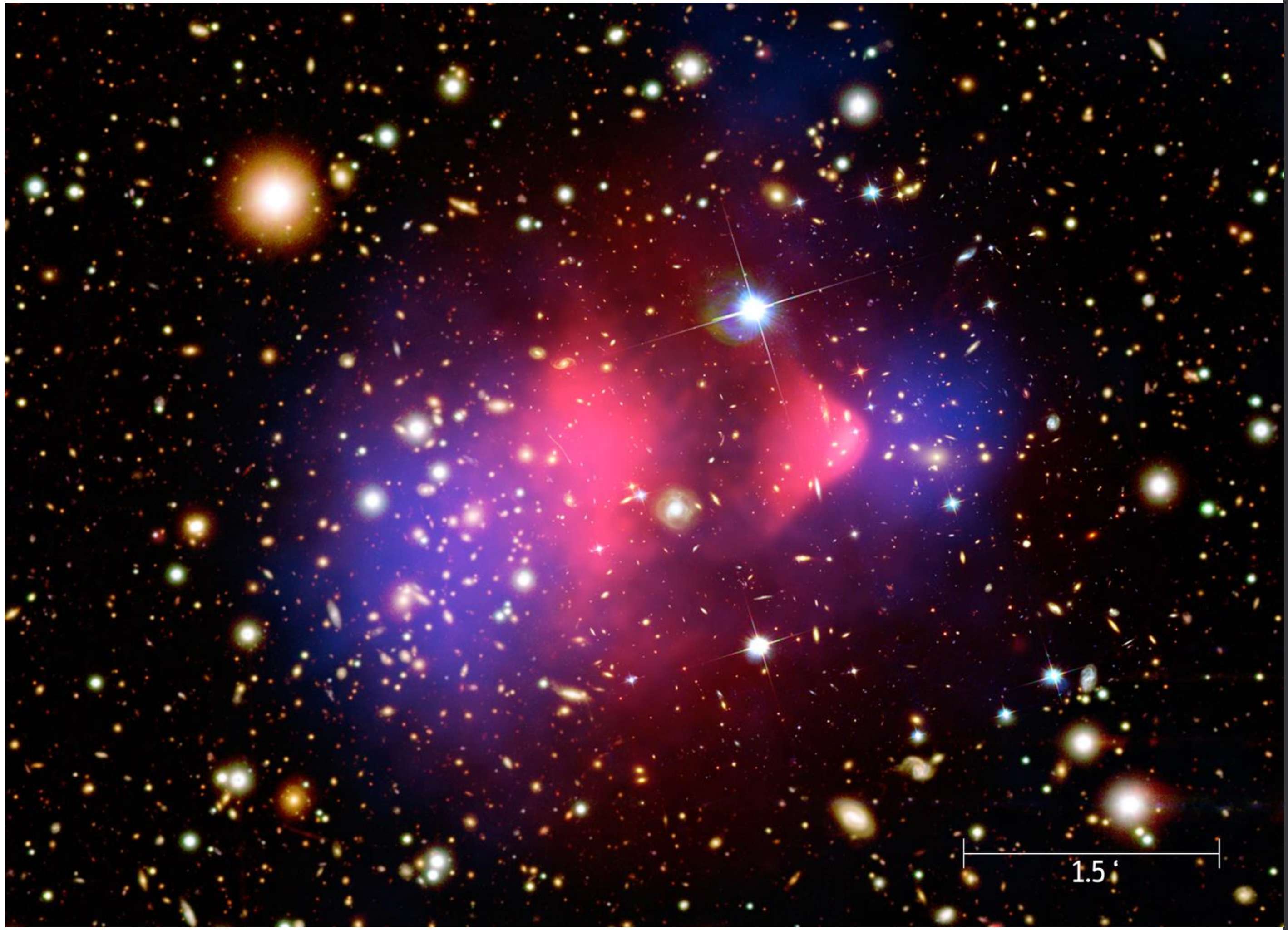


The Bullet Cluster. Taken from Wikipedia.

Part I: Background

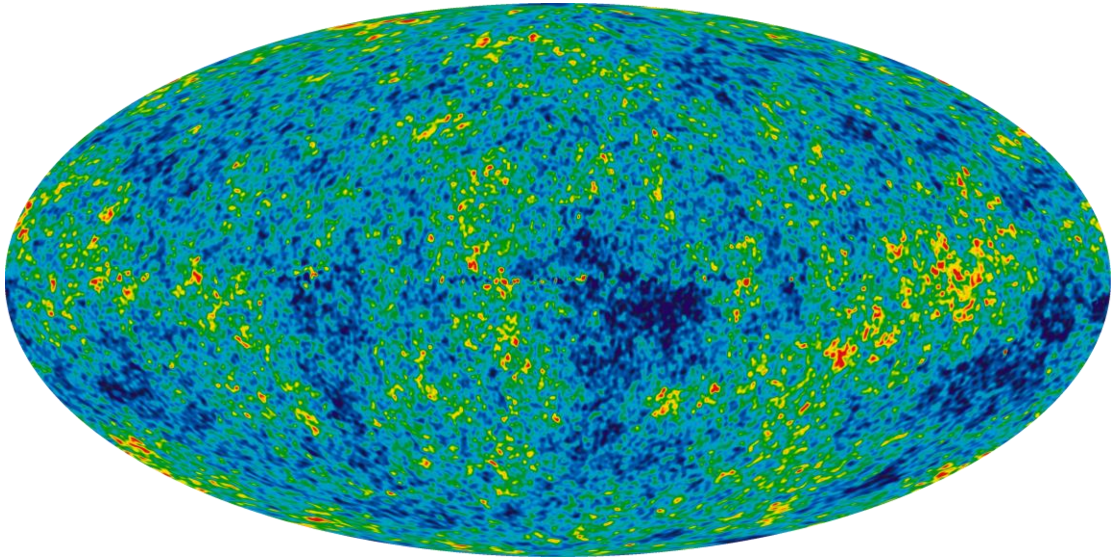
- **Part I: Background**
  - **Basic cosmological concepts**
  - **Thermodynamics of the early Universe**
  - **Why dark matter?**
  - **What is dark matter?**

- Part II: Machinery
  - Cross section
  - Number density
  - Boltzmann equation

- Part III: Application
  - Freeze-out mechanism
    - WIMPs
  - Freeze-in mechanism
    - FIMPs

Basic cosmological concepts

- The Hot Big Bang model
  - Initially, a very small region of hot, dense plasma, which began to rapidly expand
  - Preceded by inflation?
- Structure formed out of primordial density perturbations
  - Baryonic matter clumped into gravitational wells due to dark matter (DM)
- Cosmic Microwave Background (CMB)
  - Nearly isotropic black-body radiation from the early Universe
  - Gives data for the cosmological parameters



CMB. $T_{avg} \approx 2.725\ K$
Profile shows $\pm 200 \mu K$.
Taken from Wikipedia.

The ΛCDM model

- The standard model of cosmology
  - Λ = Cosmological constant (dark energy)
  - CDM (cold dark matter)

- Major components:
  - Ordinary matter
  - Cold dark matter
  - Dark energy

- The cosmological parameters depend on the background cosmology
  - For example, the abundance of dark matter
  - Background cosmology e.g. Friedmann-Lemaître-Robertson-Walker (FLRW)

Density parameter

- From the Einstein field equation

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

we can obtain the 1st Friedmann equation for a flat Universe

$$H^2 = \frac{8\pi G_N}{3}\rho$$

from which we can get the critical density

$$\rho_{cr} = \frac{3H^2}{8\pi G_N}$$

and finally define the density parameter Ω_i for species i:

$$\Omega_i \equiv \frac{\rho_i}{\rho_{cr}}$$

$G_{\mu\nu}$ = Einstein tensor
$T_{\mu\nu}$ = energy-momentum tensor
G_N = Newtonian gravitational constant
H = Hubble parameter
ρ = energy density

- Describes the relative abundance of the species
- Modern estimate for CDM:

$$\Omega_{CDM} \approx 26.4\ \% \qquad \frac{\Omega_{CDM}}{\Omega_{matter}} \approx 84.4\ \%$$

Thermodynamics of the early Universe (1/2)

- Universe was in (an approximate) thermal equilibrium in the past
  - But non-equilibrium dynamics important
- Condition for local thermodynamic equilibrium:

$$\Gamma \gtrsim H$$

  - Γ = interaction rate of those interactions that maintain the equilibrium
  - H = expansion rate of the Universe
- Once Γ falls below H, the species has decoupled
  - No significant amount of production / annihilation of the species

Thermodynamics of the early Universe (2/2)

- The energy and entropy densities:

$$\rho = \frac{\pi^2}{30} g_*(T) T^4 \qquad s = \frac{2\pi^2}{45} g_{*S}(T) T^3$$

    - $g_{*(S)}$ is the (entropy) number of degrees of freedom present
    - T is the temperature of the bath

- Adiabatic expansion approximation
    - Entropy per comoving volume is conserved
    - It follows that $T \propto a^{-1}$ (i.e. T falls as Universe expands)

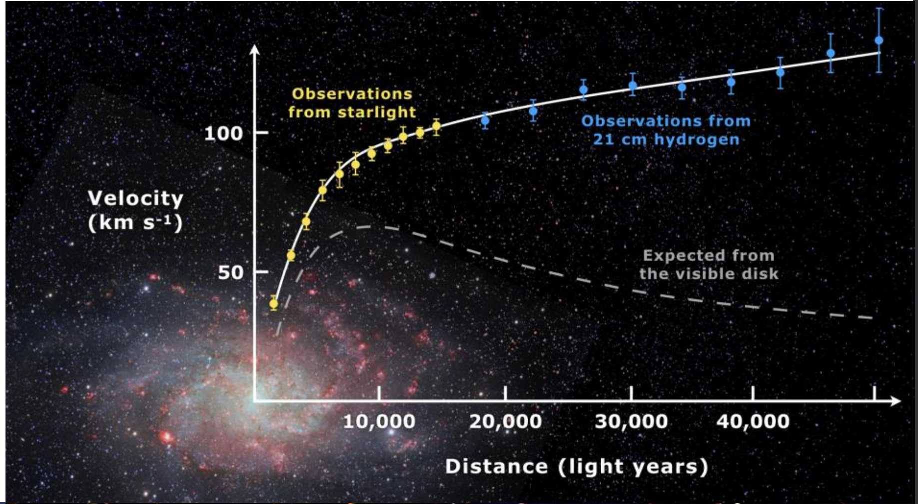
Why dark matter?

- Observational evidence points to ‘missing mass’
- ‘Dark’ because interactions with standard matter weak at best
  - Strong & EM interactions effectively ruled out
  - Doesn’t interact with e.g. photons → Difficult to observe
  - Only gravitational interactions for certain, possibly some other weak interaction
- Used/needed in e.g.:
  - Structure formation models
  - Explaining observed angular power spectrum of CMB

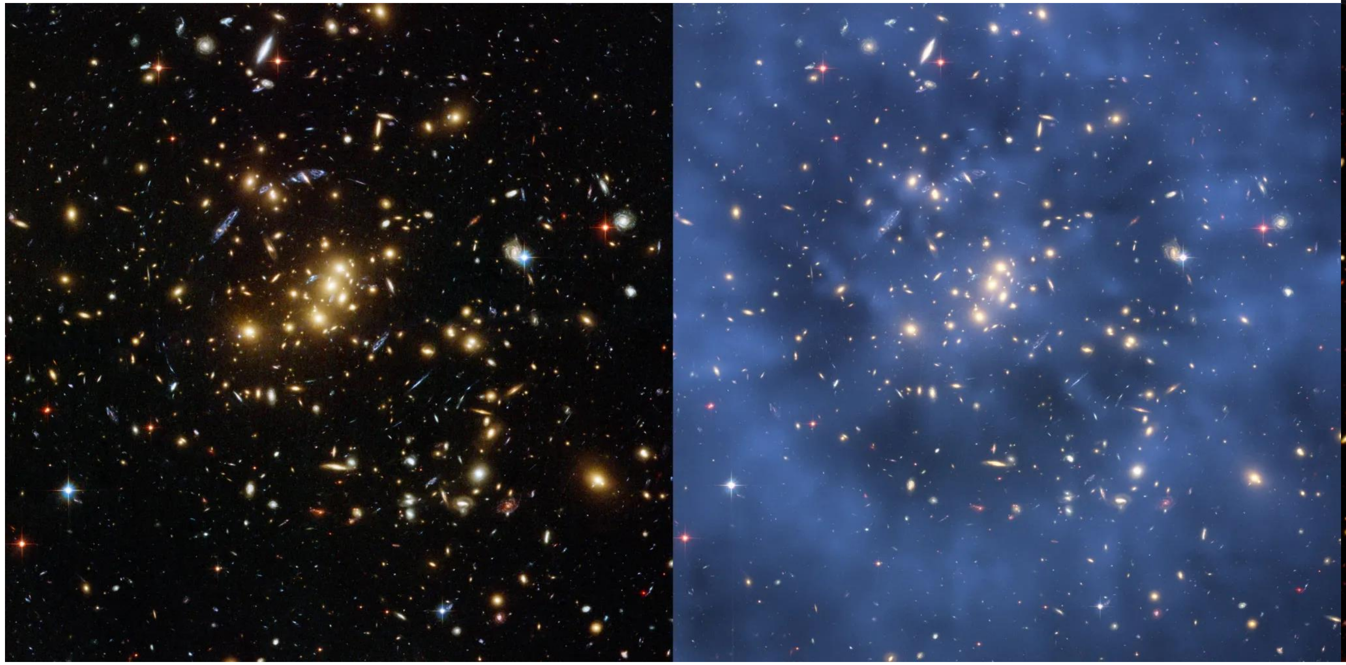
Evidence for dark matter

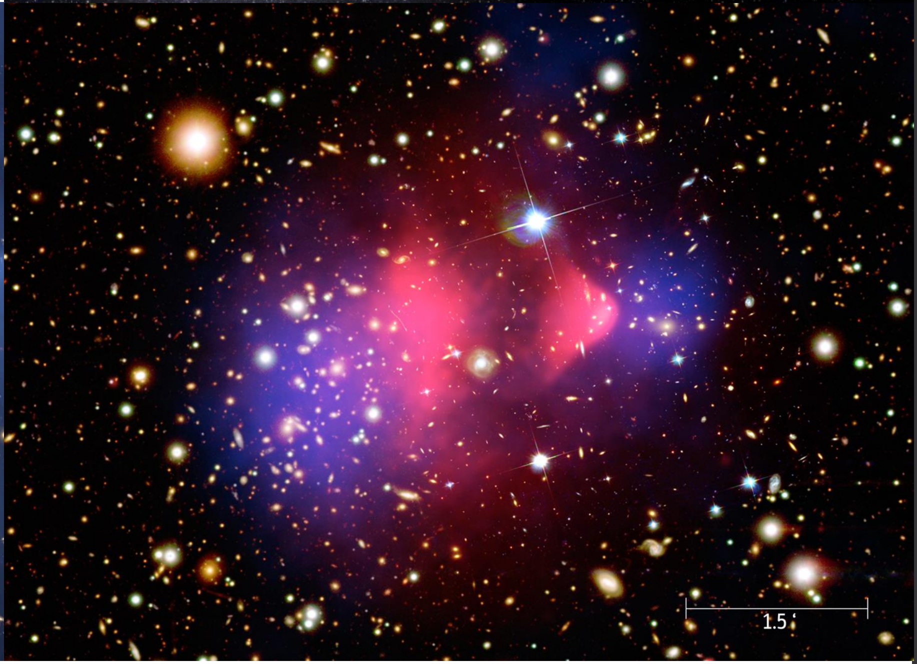
- Velocity curves of galaxies
- Gravitational lensing
- Bullet Cluster
- CMB

Velocity curve. Taken from Wikipedia under CC 4.0.



DM distribution from lensing. [NASA, ESA, M.J. Jee, and H. Ford (Johns Hopkins University)]





What is dark matter? (1/2)

- No confirmed observations
  - Some models predict it could be detectable already
- Various models to explain observed phenomena
  - New particle(s)?
  - Modified gravity?
  - Primordial black holes?

What is dark matter? (2/2)

- No significant EM or strong interactions with standard matter (SM)
- Interacts with SM at best on the weak scale
  - Not necessarily via *the* weak interaction itself
  - Possibly not at all (other than gravitationally)
- Non-relativistic at matter-radiation equality (cold DM)
  - Structure formation reasons
- Stable over cosmological time scales

Part II: Machinery

- Part I: Background
  - Basic cosmological concepts
  - Thermodynamics of the early Universe
  - Why dark matter?
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- **Part II: Machinery**
  - **Cross section**
  - **Number density**
  - **Boltzmann equation**

- Part III: Application
  - Freeze-out mechanism
    - WIMPs
  - Freeze-in mechanism
    - FIMPs

Cross section

- Needed in our calculations for e.g. 2 ↔ 2 processes
- Thermally-averaged cross section (times velocity) can be expressed as:

$$\langle \sigma v \rangle = \sigma_0 x^{-n}$$

  - σ_0 is a constant
  - $x = m/T$ is a new ‘time variable’ (mass / temperature)
  - n refers to specific wave annihilations (e.g. 0 refers to s-wave)
- Relevant for our DM particle candidate:

$$\sigma \propto g^4/m^2$$

    - g is the coupling strength
    - m is mass
  - Note: great freedom in both g and m if not constrained by other means
    - Or given by some model

Number density (1/2)

- The quantity of interest: abundance of DM
- Particle species in thermal equilibrium with a bath:
  - Non-relativistic number density equation (CDM):

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

    - n = number density
    - g = number of degrees of freedom
    - m = mass of the species
    - T = temperature of the bath
- Particle species in an expanding volume:
  - Number density in a comoving volume element: $N^{\mu} = nu^{\mu} = (n, 0,0,0)$
  - Demanding conservation $\left(\nabla_{\mu} N^{\mu} = 0\right)$ leads to:

$$\dot{n} + 3Hn = 0$$

    - H = Hubble parameter

Number density (2/2)

- Finally, consider the process:

$$\chi + \bar{\chi} \leftrightarrow \psi + \bar{\psi}$$

  - We can keep track of the evolution of species of interest as follows:

$$\frac{1}{a^3}\frac{d\left(n_\chi a^3\right)}{dt} = -\langle\sigma v\rangle\left[n_\chi^2 + \left(n_\chi^{eq}\right)^2\right]$$

    - χ ($\bar{\chi}$), ψ ($\bar{\psi}$) DM and SM particles (bar: antiparticle)
    - a is the scale factor
    - $\langle\sigma v\rangle$ is the annihilation rate factor
    - eq refers to equilibrium abundance

The Boltzmann equation (1/4)

- We can now move on to consider the main machinery of our work:
  - The covariant relativistic version of the Boltzmann equation

- With it, we can calculate non-equilibrium dynamics of a system
  - Here the comoving number density of a DM particle species in an expanding Universe

The Boltzmann equation (2/4)

- In its simplest form:

$$\hat{\boldsymbol{L}}[f] = \boldsymbol{C}[f]$$

  - $\hat{\boldsymbol{L}}$ is the Liouville and $\boldsymbol{C}$ the collision operator
  - f is the relevant distribution function
- Upon integration over a volume in phase space:
  - LHS: Time evolution of the number density:

$$\frac{g}{(2\pi)^3}\int \hat{\boldsymbol{L}}[f]\,\frac{d^3p}{E} = \dot{n} + 3Hn$$

  - RHS: Includes interactions:

$$\frac{g}{(2\pi)^3}\int \boldsymbol{C}[f]\frac{d^3p}{E} = -\int\left\{\left(\prod_j d\Pi_j\right)(2\pi)^4\delta^{(4)}\left(\sum_i p_i - \sum_f p_f\right)\left[\left(|M|^2_{i\to f}\right)\left(\prod_i f_i\right)\left(\prod_f\left(1 \pm f_f\right)\right) - (i \leftrightarrow f)\right]\right\}$$

The Boltzmann equation (3/4)

- The equation can be greatly simplified by assuming:
  - T-invariance
  - Maxwell-Boltzmann -statistics
  - Final-state particles coupled strongly to plasma
- Also introducing the thermally-averaged cross section, we get:

$$\dot{n} + 3Hn = -\langle \sigma v \rangle \left[n_\chi^2 - \left(n_\chi^{eq} \right)^2 \right]$$

The Boltzmann equation (4/4)

- We can modify this further still with:
  - Comoving number density $Y = n/s$
    - s is the entropy per comoving volume
  - Dimensionless variable $x = m/T$
  - Parameter λ

- Now the equation reads:

$$Y' = -\lambda x^{-j-2}\left(Y^2 - Y_{eq}^2\right)$$

$$Y_{eq} = 0.145\left(\frac{g}{g_{*s}}\right)x^{3/2}e^{-x}$$

    - j = 0,1, ... refers to s,p,…-wave annihilation
  - This is a so-called Riccati equation
    - No analytical solutions
    - ○ Initial value = equilibrium value

Part III: Application

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    - **WIMPs**
  - **Freeze-in mechanism**
    - **FIMPs**

WIMPs (1/2)

- WIMP = Weakly Interacting Massive Particle
  - (Most) popular model of particle dark matter
  - Interacts weakly with SM
  - Mass range generally from a few GeV to ~100 TeV
- Should be detectable even with current detectors
  - Failure to do so at this point a disappointment

WIMPs (2/2)

- Relic abundance via the freeze-out mechanism
- Estimated abundance gives the cross section of the weak scale
- Popular candidate: lightest neutralino of supersymmetry
  - Combination of superpartners of neutral gauge bosons and Higgs particles

Freeze-out (1/3)

- Not known by which mechanism DM abundance came to be
  - Not even known if DM was coupled to visible sector

- Freeze-out: a thermal production mechanism
  - The mechanism by which e.g. electrons obtained their relic abundance

- Particles initially in thermal equilibrium with primordial plasma
  - Nonzero initial abundance
  - Particles and antiparticles continuously created and annihilated

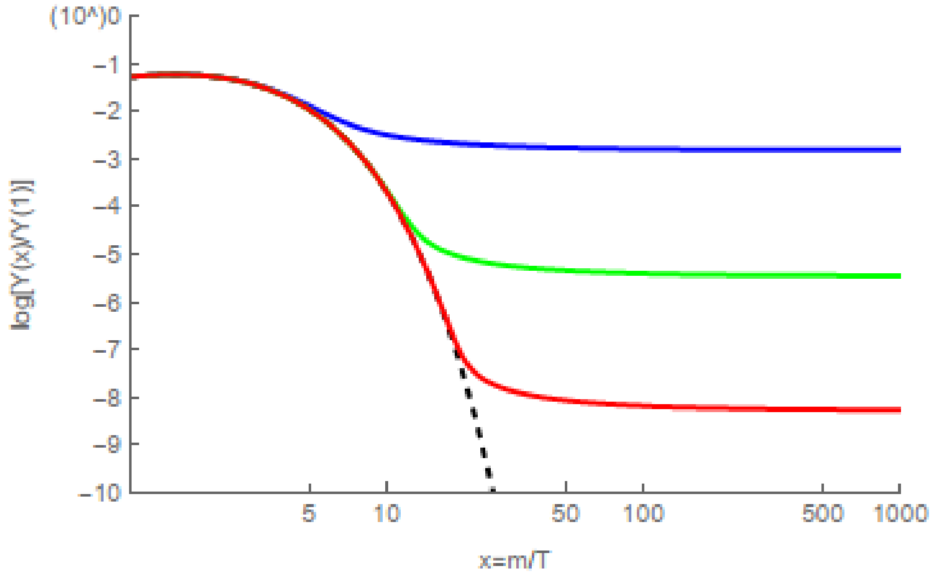
Freeze-out (2/3)

- As Universe expands and temperature falls, particles decouple from plasma
  - Low T: No significant production (Boltzmann-suppressed)
  - Expansion: Distances greater, no pair to annihilate with

- Relic abundance approaches a constant value
  - Assuming no other significant processes
  - If species remained in equilibrium, they would 'die out’
    - Recall exponential decay

Freeze-out (3/3)

- A figure might help to visualize the general trend:
  - Colored curves: Y for various cross sections (red highest)
  - Dashed curve: equilibrium curve for Y
  - Note the trends mentioned
    - Asymptotic behavior
    - Exponential decay



FIMPs

- FIMP = Feebly Interacting Massive Particle
- Small coupling strength
- Relic abundance via the freeze-in mechanism
- ‘Somewhat like neutrinos’

Freeze-in

- 'Opposite' of freeze-out
- Particles initially *not* in thermal equilibrium
  - (Near-) zero initial abundance
- DM particles now accumulate over time
- But as T falls, the processes by which they are created become suppressed
  - Relic abundance again asymptotically approaches a constant value

Summary

- A lot of indirect evidence for dark matter exists, but exact nature is unknown
- The covariant relativistic Boltzmann equation
  - Allows us to calculate non-equilibrium dynamics (e.g. DM abundance)
- Freeze-out
  - Thermal production mechanism
  - Nonzero initial abundance
  - Particles decouple from plasma
- Freeze-in
  - Non-thermal production mechanism
  - (Near-) zero initial abundance
  - Particles accumulate over time
- In both mechanisms:
  - Relevant process becomes Boltzmann-suppressed as Universe expands and cools
  - Abundance asymptotically approaches a constant value

Thank you!

NASA, ESA, M.J. Jee, and H. Ford (Johns Hopkins University)