STABILITY OF EXPANDING BUBBLES IN THE SOUND SHELL MODEL

OUTLINE

- Cosmological phase transitions
- Hydrodynamics
  - Detonations
  - Subsonic deflagrations
  - Supersonic deflagration (Hybrids)
- Stability of the bubbles
- Spherical perturbations
  - Milne coordinates
  - Peturbed equations

COSMOLOGICAL PHASE TRANSITIONS

- Scalar fields involved in the electroweak symmetry breaking can become metastable. [1, 2]
- In extensions of the standard model, this can be modelled as a first order phase transition. [3]
- Around 10ps after the Big Bang.

HYDRODYNAMICS

- Can be fully described by the conservation of the energy momentum tensor

$$T^{\mu\nu} = \omega u^{\mu} u^{\nu} + g^{\mu\nu} p$$

- Fluid equations, matching conditions, Equation of the interface
- Three types of solutions

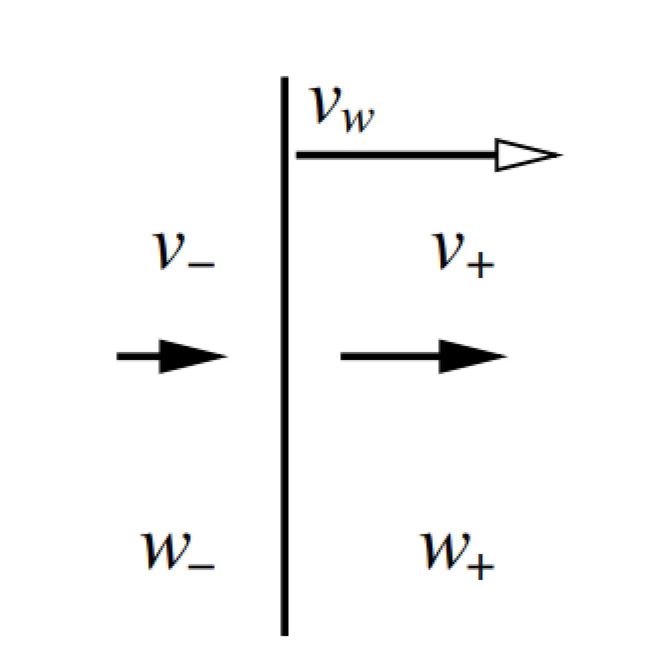

Figure: Velocity v and enthalpy w next to the bubble wall. [4]

SUBSONIC DEFLAGRATIONS

- Wall speed less than the sound speed.

- Fluid is at rest inside the bubble.
- Fluid speed decreases until a shock is encountered

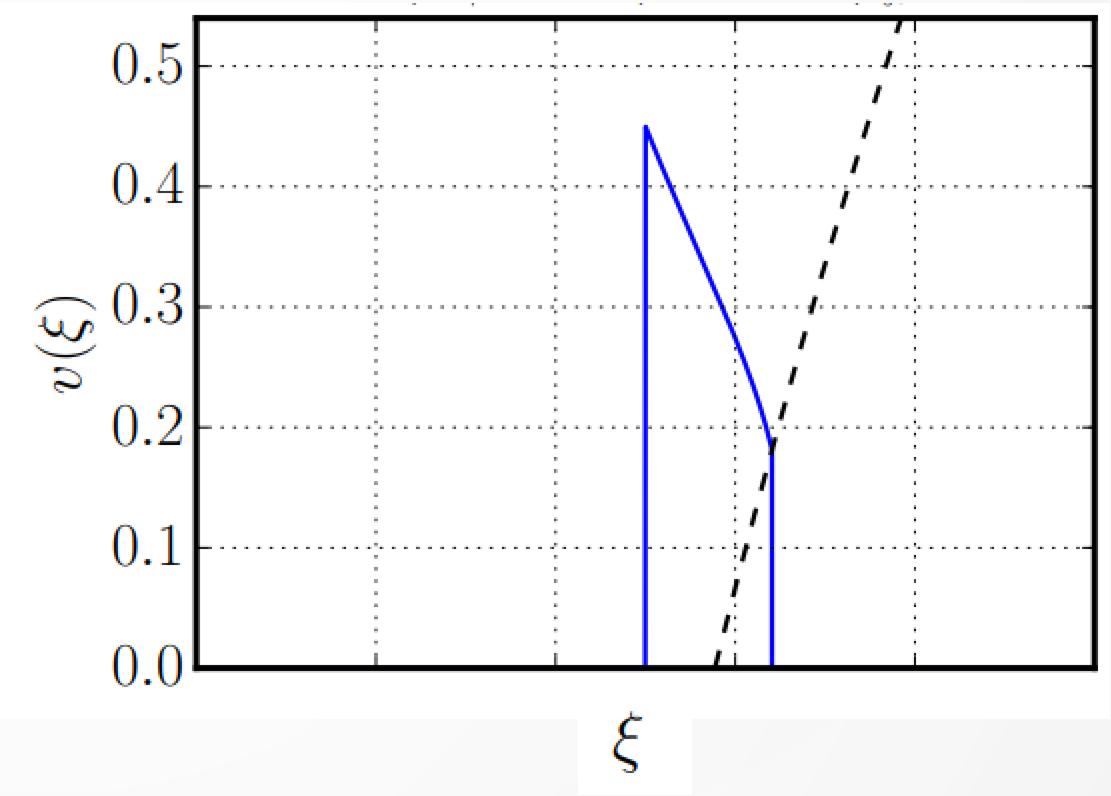

Figure: Deflagration at wall speed 0.5 [4]

SUPERSONIC DEFLAGRATION (HYBRIDS)

- Wall speed greater than the speed of sound, but less than the Chapman-Jouguet speed
  - The Chapman-Jouguet speed is the theoretical velocity at which a detonation travels through a reactive medium

- Fluid speed starts increasing at the sound
- Decreases after the wall, until the shock is encountered

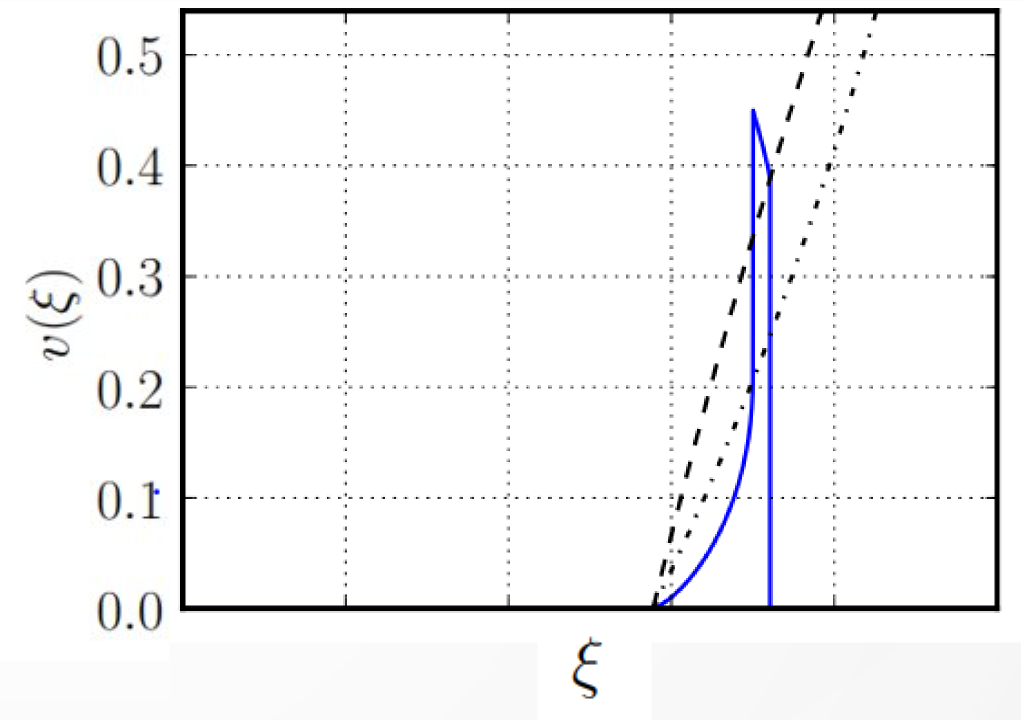

Figure: Hybrid at wall speed 0.7 [4]

DETONATIONS

- Wall speed greater than the Chapman-Jouguet speed and the speed of sound
- Speed starts increasing at the sound speed
- Drops to zero at the wall

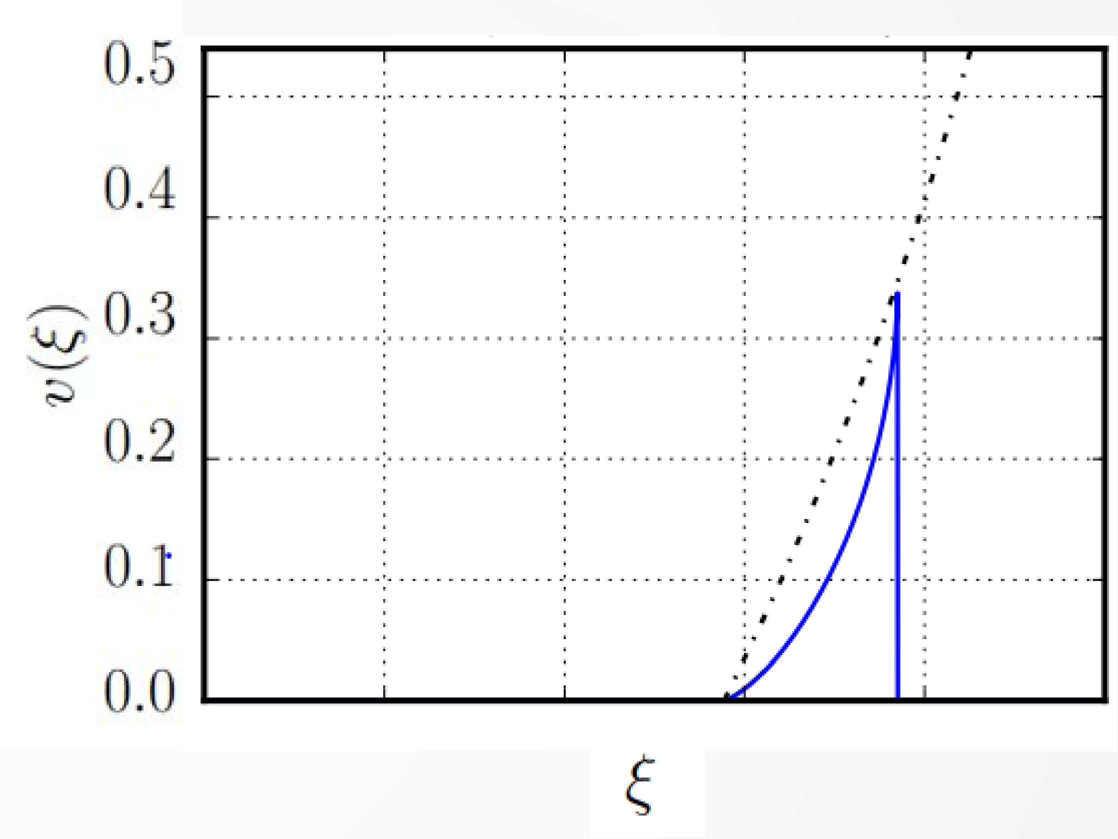

Figure: Detonation at wall speed 0.77 [4]

STABILITY OF THE BUBBLES

- Introduce perturbations, if a solution for the perturbed equation can be found the bubble is not stable
- Critical radius predicts the behavior

- So far has only been calculated for non-spherical perturbations
  - Critical radius significantly less than in simulations [5, 6]

SPHERICAL PERTURBATIONS

- Partial differential equations, separability helpful
- Milne coordinates

$$\chi = \tanh^{-1}\frac{r}{t}, \quad \tau = \sqrt{t^2 - r^2}$$

- Conformal time

$$d\tau\eta = d\eta$$

- Ansatz: ~ $\exp[\omega\eta]$

DEFLAGRATIONS (MILNE COORDINATES)

- Fluid is at rest inside the bubble.
- Fluid speed decreases until a shock is encountered

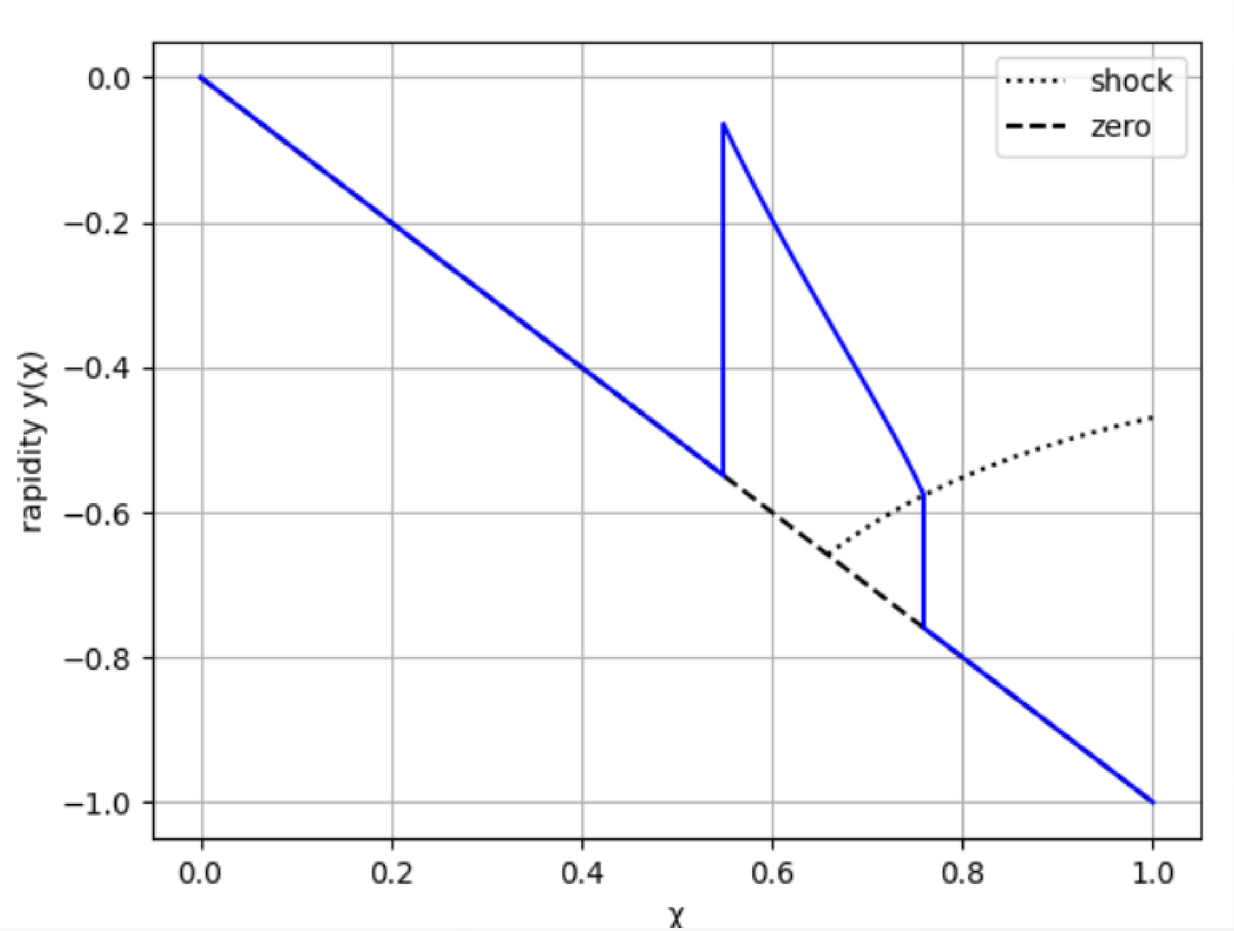

Figure: Deflagration at wall speed 0.5

HYBRID (MILNE COORDINATES)

- Fluid speed starts increasing at the sound
- Decreases after the wall, until the shock is encountered

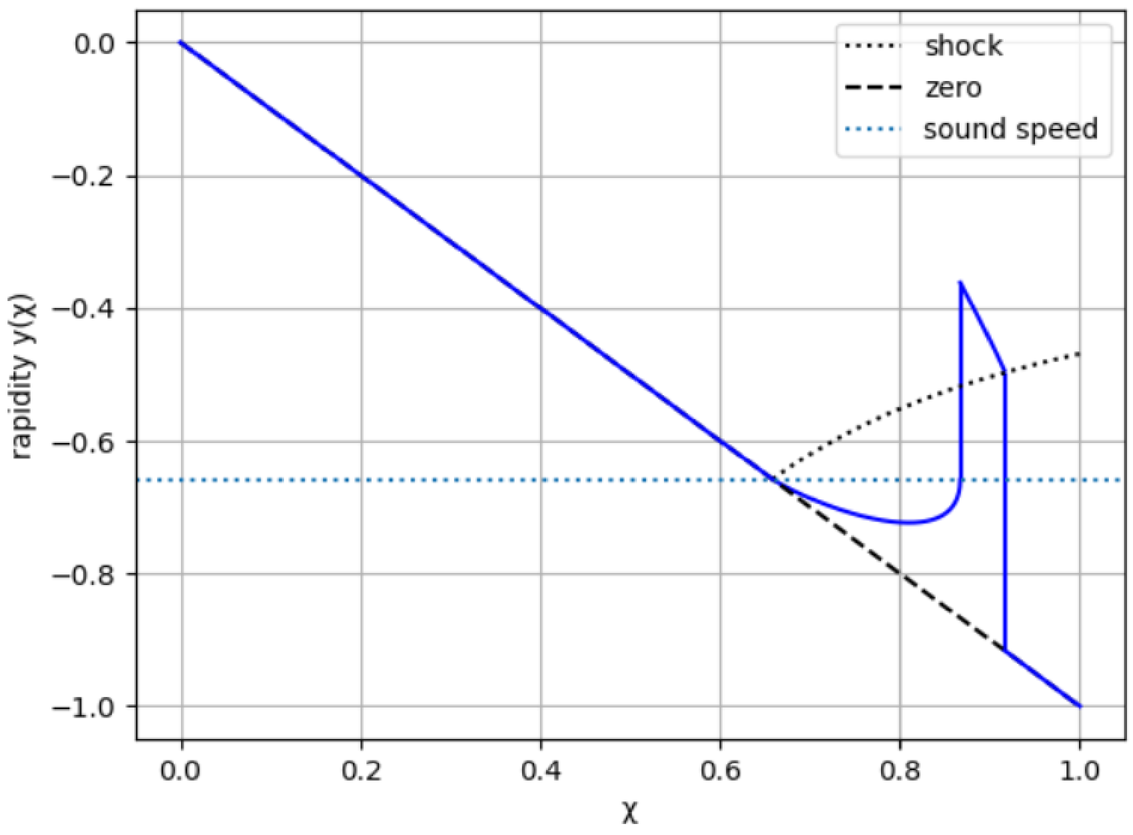

Figure: Hybrid at wall speed 0.7

DETONATIONS (MILNE COORDINATES)

- Speed starts increasing at the sound speed
- Drops to zero at the wall

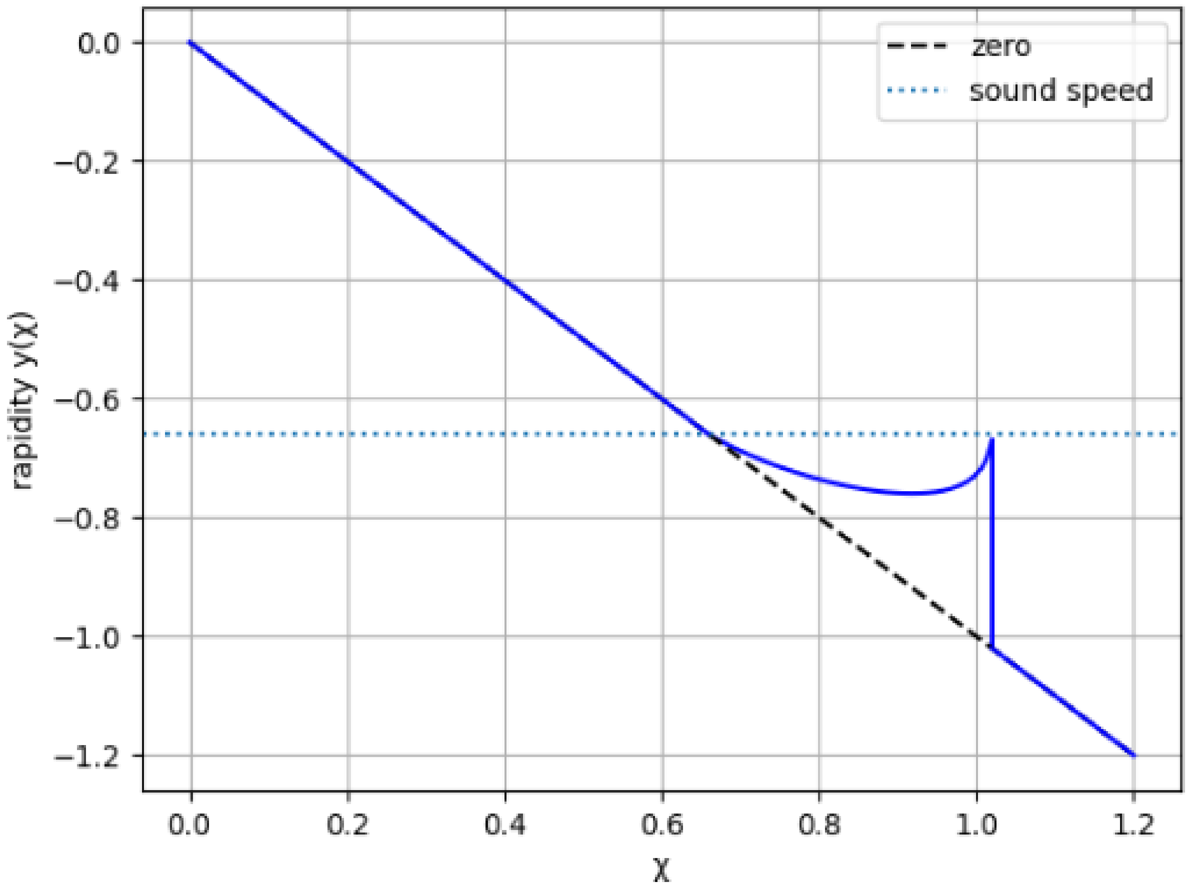

Figure: Detonation at wall speed 0.77

PERTURBED SPHERICAL EQUATIONS

Fluid equations

$$[A\partial_\chi + \Omega B + C]\overline{V} = 0$$

$$\overline{V} = \begin{bmatrix} \Pi \\ R^\chi \\ R^+ \\ R^- \end{bmatrix}$$

PERTURBED SPHERICAL EQUATIONS

Fluid equations

$$[A\partial_\chi + \Omega B + C]\overline{V} = 0$$

$$A = \begin{bmatrix} u_\chi & -w\gamma & 0 & 0 \\ \gamma & -u_\chi & 0 & 0 \\ 0 & 0 & w\gamma & 0 \\ 0 & 0 & 0 & w\gamma \end{bmatrix}$$

$$B = \begin{bmatrix} \gamma & -wu_\chi & 0 & 0 \\ u_\chi & -\gamma & 0 & 0 \\ 0 & 0 & wu_\chi & 0 \\ 0 & 0 & 0 & wu\chi \end{bmatrix}$$

$$\overline{V} = \begin{bmatrix} \Pi \\ R^\chi \\ R^+ \\ R^- \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & w\bar{u}_\mu \bar{u}\cdot\nabla u^\mu + u\cdot\nabla p & 0 & 0 \\ 0 & -\bar{u}\cdot\nabla e - \nabla\cdot\bar{u} & \frac{\sqrt{l(l+1)}}{\sqrt{2(1-\tanh\chi^2)}} & -\frac{\sqrt{l(l+1)}}{\sqrt{2(1-\tanh\chi^2)}} \\ \frac{\sqrt{l(l+1)}}{\sqrt{2(1-\tanh\chi^2)}} & 0 & \nabla\cdot u_\chi & 0 \\ -\frac{\sqrt{l(l+1)}}{\sqrt{2(1-\tanh\chi^2)}} & 0 & 0 & \nabla\cdot u_\chi \end{bmatrix}$$

PERTURBED SPHERICAL EQUATIONS

Matching conditions

$$\Delta[w\gamma^2(1-v_\chi^2)(-\Omega+\gamma^2 R^\chi)+(1+c_s^{-2})\gamma^2 v_\chi \Pi]=0$$

$$\frac{\sigma}{\tau^2}(\partial_\eta^2-\partial_\perp^2)\zeta+\Delta[2w\gamma^4 v_\chi R^\chi+(1+(1+c_s^{-2})\gamma^2 v_\chi^2\Pi]=0$$

$$\Delta[R^\perp+v_\chi\Omega]=0$$

Equation of the interface

$$\frac{\sigma}{\tau^2}(\partial_\eta^2-\partial_\perp^2)\zeta=F_{dr}-\tilde{\eta}\langle\gamma(v_\chi-\Omega)\rangle$$

PERTURBED SPHERICAL EQUATIONS

- Fluid equation

$$[A\partial_\chi + \Omega B + C]\overline{V} = 0$$

- Boundary values from the interface and matching conditions
  - Example detonation

$$\overline{V}_{shock} = \overline{0}$$

$$\overline{V}_w = \overline{V}_w(\Omega)$$

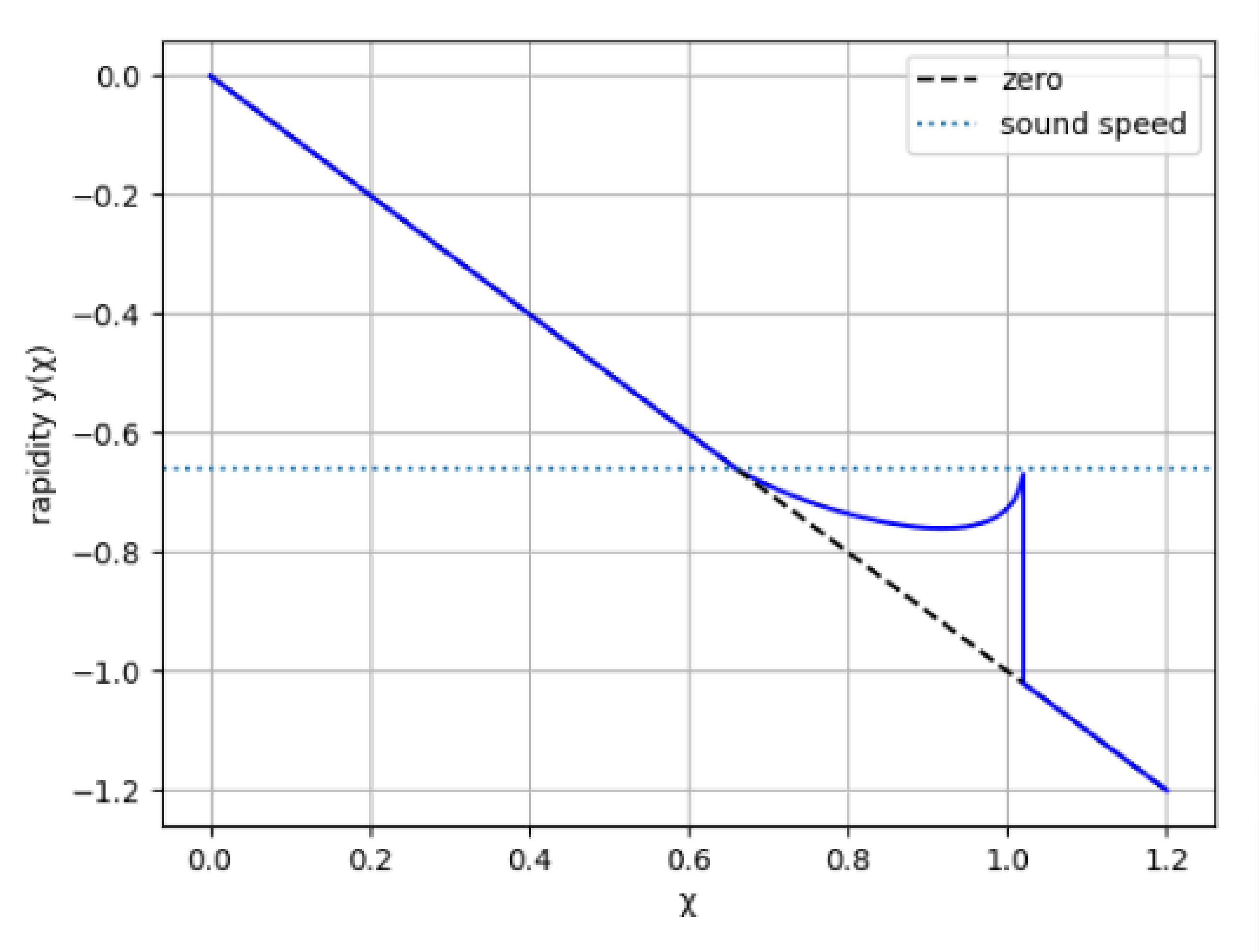

Figure: Detonation at wall speed 0.77

REFERENCES

[1] D. Kirzhnits and A. D. Linde, Symmetry Behavior in Gauge Theories, Annals Phys. 101 (1976) 195–238.

[2] S. R. Coleman, The Fate of the False Vacuum. 1. Semiclassical Theory, Phys. Rev. D15 (1977) 2929–2936. [Erratum: Phys. Rev.D16,1248(1977)].

[3] A. D. Linde, Decay of the False Vacuum at Finite Temperature, Nucl.Phys. B216 (1983) 421.

[4] Mark Hindmarsh and Mulham Hijazi, Gravitational waves from first order cosmological phase transitions in the Sound Shell Model, NORDITA-2019-083, HIP-2019-29/TH.

[6] Ariel Megevand and Federico Agustin Membiela, Stability of cosmological deflagration fronts, Phys. Rev. D 89, 103507 (2014)

[7] Daniel Cutting, Mark Hindmarsh and David J. Weir, Vorticity, kinetic energy, and suppressed gravitational wave production in strong first order phase transitions Phys. Rev. D 89, 103507 (2014)