

Junction Conditions in General Relativity

Shankar Bhandari

12 March 2024

General relativity (GR), a non-linear theory of gravity introduced by Albert Einstein in 1915, is a geometric theory encapsulated by Riemannian geometry. In GR, spacetime is characterised by a metric tensor $g_{\alpha\beta}$, which depends on the coordinate and can vary across different locations. The metric tensor and its derivatives inform us about the curvature of spacetime. The Einstein equation then relates the geometry of spacetime with the energy and matter content of the universe. There are various solutions to the Einstein equation: some of them being the Minkowski space, which is the simplest solution; the Schwarzschild spacetime, which describes a black hole; and the Friedmann-Lemaître-Robertson-Walker spacetime, which describes our universe.

In Newtonian theory, we can always set up a system of well-defined coordinates, which makes calculating any discontinuous system straightforward. However, in GR, the metric $g_{\alpha\beta}$ is influenced not only by the smoothness of the physical conditions but also by the smoothness of the coordinates used on the spacetime manifold. This means that the choice of coordinates is very important and it is not always straightforward to separate the physical discontinuity from the coordinate discontinuity.

In my thesis, I study the topic of junction conditions in general relativity. The junction conditions arise from the glueing of two spacetimes, which leads to discontinuity in matter and spacetime. The glueing is done such that the boundary between the spacetimes is confined in a hypersurface, which is essentially a 3D manifold in a 4D spacetime. Since the intrinsic properties of a hypersurface don't depend on the coordinate of the spacetime it is embedded on, they can be used to formulate coordinate invariant junction conditions. The conditions on these intrinsic properties, for the glueing of spacetimes to be smooth, are called the junction conditions. There are two junction conditions and violating the second junction condition leads to interesting physics, namely singular thin shells at the hypersurface. There are many applications of junction conditions and thin shell formalism; for example, the Oppenheimer-Snyder collapse can be studied using junction conditions.