Borrowing constraints and house price dynamics: the case of large shocks [*]

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Abstract

We study how a household borrowing constraint in the form of a down payment require-ment shapes house price dynamics. We consider the fully non-linear dynamics following large aggregate shocks in a calibrated OLG model with standard preferences. We find that the main effect of a down payment constraint is to make house price dynamics asymmetric be-tween large positive and large negative income shocks: Prices increase rapidly following the impact effect of a large adverse income shock but decline slowly following the impact effect of a positive income shock. This asymmetry stems from the fact that the share of borrowing constrained households changes over time. However, the down payment constraint does not substantially magnify the impact effect of adverse income or interest rate shocks.

Keywords: House prices, dynamics, borrowing constraints, down payment constraint

JEL codes: E21, R21

1   Introduction

For highly leveraged households, even a moderate fall in house prices can induce a large reduction in net worth. Stein (1995) was the first to show that these changes in household wealth positions

may feed back into house prices through household borrowing constraints and create a multiplier mechanism that amplifies house price fluctuations.

To see the intuition behind this mechanism, consider a household that has a house worth 100 000 euros, a mortgage loan of 70 000 euros and no other assets or debt. The household wants to move to a bigger house. It can get a mortgage but banks require it to pay a 20% down payment. With its current net worth of 30 000 euros, the household could buy a house worth 150 000 euros, which would be 50% bigger (in a quality adjusted sense) than its current one. Assume then that house prices fall by 10%. This reduces the net worth of the household to 20 000 euros. It could now finance a house worth at most 100 000 euros. Given that house prices have fallen, that would still mean a bigger house than the current one, but only 10% bigger. Hence, a fall in house prices may reduce some households' housing demand. This reduced demand creates further downward pressure on house prices.[1]

In our view, this is an interesting mechanism. One would expect it to be important especially in situations where a substantial fall in house prices reduces the net worth of leveraged households dramatically. A down payment constraint may then become binding for many households.

However, based on the previous literature, it seems fair to say that the quantitative importance of this mechanism is unclear. Stein's model is essentially static, as he assumes that all trade takes place in one period. This alone makes it difficult to assess the quantitative relevance of the mechanism. Ortalo-Magné and Rady (1999, 2006) build a fully dynamic model with two types of dwellings. They are able to characterize how the interplay between aggregate income shocks, homeowners' capital gains or losses and a down payment constraint affects house price dynamics and transaction volume. In addition, they highlight the role of first-time buyers and volatility of young households' income in explaining housing market fluctuations. However, like Stein's analysis, their analyses are qualitative rather than quantitative in nature. For instance, in order to keep the model tractable, Ortalo-Magné and Rady assume preferences that rule out consumption smoothing.

A down payment constraint for households' housing investments is also incorporated in many recent dynamic stochastic general equilibrium (DSGE) models, see for instance Iacoviello (2005), Iacoviello and Neri (2010), and Monacelli (2009).[2] However, the dynamics of the DSGE models are

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[1]The multiplier mechanism discussed in Stein (1995) is similar to the 'credit cycles' mechanism in Kiyotaki and Moore (1997). Cordoba and Ripoll (2004) have analyzed the quantitative importance of that mechanism with a linearized model.

[2]Of course, down payment constraint is an important feature also in many models that do not account for aggregate dynamics. For instance, Gervais (2002) presents an OLG-model where the down payment constraint and the tax system together determine households' choice between owner and rental housing. Ríos-Rull and Sanchez-Marcos (2008) present a calibrated model featuring consumption smoothing motive and idiosyncratic shocks together with a similar property ladder structure as in Ortalo-Magné and Rady (2006).

typically analyzed by (log-)linearizing the model around a steady state. The borrowing constraint is modelled by assuming that there are two types of households: patient and impatient. In the steady state, the impatient households are borrowing constrained while the patient households are not. Restricting analysis to the neighborhood of a steady state is computationally convenient, but it rules out potentially important non-linear dynamics. For instance, the linearization technique implies that the share of borrowing constrained households remains constant over time. That may strongly limit the potential of the multiplier mechanism à la Stein to affect house price dynamics.[3]

As for the empirical literature on house prices and household leverage, Lamont and Stein (1999) and Benito (2006) estimate the effect of income shocks on house price dynamics. Lamont and Stein (1999) relate U.S. city-level house price data to the data on household finances and Benito (2006) uses British Household Panel Survey. Both studies indicate that compared to other regions, house prices react more sensitively to aggregate income shocks more in regions where households are highly leveraged. These results are consistent with the multiplier mechanism, but do not testify to the importance of borrowing constraints because households' asset positions may affect house price dynamics even in the absence of borrowing constraints.

Another issue is that a relaxation of household borrowing constraints have often been associated with a housing boom. Agnello and Schuknecht (2009) list housing booms and busts in 18 industrialized countries between 1970 and 2007. They provide statistical evidence that mortgage market deregulations, which typically imply a relaxation of household credit constraints, indeed tend to trigger a housing boom.[4]

In this paper, we aim to understand how the mechanism stressed by Stein works in a fully dynamic set up with a standard consumption smoothing motive and to evaluate its quantitative importance taking into account non-linear effects that may arise with large aggregate shocks. In addition, we consider the effects of a relaxation of the borrowing constraint. To this end, we build a parsimonious OLG model with owner housing and a life cycle savings decision. In the model, young households need to borrow in order to finance their housing. Differences in household size create large differences in household leverage also among households of the same age. By focusing on completely unanticipated shocks, we are able to solve very accurately the fully non-linear dynamics.

We study the model dynamics in different ways. We begin by applying the model to the recent experience in the Finnish housing market. The Finnish housing market is an interesting example since it was hit by two very large consecutive shocks: financial deregulation and depression. Financial deregulation was associated with a house price boom and the recession led to a dramatic

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[3]Kiyotaki et al. (2011) consider how changes in various fundamentals affect house price dynamics in a model with a down payment constraint. Their analysis does take into account the fact that the share of borrowing constrained households may change over time. Their focus is also very different from ours: They do not compare price dynamics with and without the down payment constraint or analyze asymmetries between positive and negative shocks.

[4]See also Attanasio and Weber (1994) and Hendershott and White (2000).

house price bust. Following the bust, house prices increased again quite rapidly. We find that the calibrated model can explain a large part of the first boom as an equilibrium response to an empirically plausible relaxation of the borrowing constraint. This suggests that the model captures much of the actual relevance of borrowing constraints for aggregate housing demand. The model can also explain a large part of the subsequent price changes with aggregate income shocks that are similar to those experienced in Finland during and after the recession.

We then highlight the role of the down payment constraint for house price dynamics by contrasting two cases: one where household borrowing is unlimited and another where households face a down payment constraint. We find that in some cases the down payment constraint indeed substantially shapes house price dynamics. The most noteworthy effect is that the down payment constraint makes house price dynamics quite asymmetric between large positive and large negative income shocks: Prices increase rapidly after the impact effect of a large adverse income shock but decline slowly after the impact effect of a positive income shock. This sort of asymmetry is an example of non-linear dynamics that cannot be captured by linearized models. In our set-up, the asymmetry stems from the fact that the share of borrowing constrained households changes over time.

Interestingly, we also find that the down payment constraint does not substantially magnify house price effects of the shocks. Hence, the borrowing constraint does not aggravate housing busts. Rather, it makes house prices converge more rapidly towards the steady state level after an aggregate shock. Intuitively, a fall in house prices following a negative income shock effectively tightens the borrowing constraint, much as in the static model introduced by Stein. This effect tends to reduce current housing demand. However, in a dynamic set-up, the tightening of the borrowing constraint has a second effect, namely that it forces households to save more (or borrow less). The latter effect increases future housing demand implying higher future house prices. For a given current house price, higher future house prices increase current housing demand because of the anticipated capital gains. In equilibrium, the effect of this delayed housing demand largely offsets the downward pressure on current house prices caused by tightening of the borrowing constraint.

In section 2, we describe the model, explain how we solve it, and discuss calibration and the initial steady state. In section 3, we derive some analytical results that help in interpreting our numerical results. In section 4, we present the numerical results. We conclude in section 5.

2 Model

2.1 Set-up

We consider a model economy with overlapping generations of households. Population remains constant over time. Households live for $J + 1$ periods. During the first J periods of their lives, households derive utility from non-housing consumption, c, and the services generated by the housing their own, h. In the beginning of age $J + 1$, households sell the house where they lived in the previous period (say because they need to move to an old age institution) and consume their net worth.

The earnings of households of age j in period t are y_t^j. The price of one unit of housing in period t is p_t. Housing involves some direct costs such as maintenance costs and property taxes. Part of these costs are proportional to the size of the house and part of them (taxes in particular) are proportional to the value of the house.[5] We denote these two costs by η and κ.

Households can also invest in a financial asset, a. The interest the financial asset earns from period $t - 1$ to period t is $R_t - 1$. Households can borrow only against their housing. They have to finance part of their housing with own equity and can borrow only up to fraction θ of the value of their house. Households cannot default.

In each generation, there are I different household types, indexed by $i = 1, 2, .., I$. This intra-generational heterogeneity stems from households getting children at different ages. Households learn their type in the beginning of their lives. Children affect household saving behavior by changing the household size over the life cycle. As we will see, differences in the age at which households get children result in large differences in household leverage. The mass of households of type i is denoted by m_i. We normalize so that $\sum_{i=1}^{I} m_i = 1$.

The periodic utility is determined as $u(c, h; s)$ in ages $j = 1, ..., J$ and $v(b; s)$ in age $J + 1$, where s denotes household size and b is net worth.

2.2 Household problem

We use superscripts to denote household age and subscripts to denote household type and time period so that $c_{i,t}^j$, for instance, denotes non-housing consumption of a household of age j and type i in period t.

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[5]We introduce these two types of costs because they have different implications for equilibrium house prices. For instance, if there are large maintenance costs that are proportional to the size of the house alone, a large part of the user cost is independent of the house price. In this case, a relatively small change in aggregate household income, for instance, implies a relatively large change in the equilibrium house price.

The problem of a household of age $j = 1$ and type i in period t is

$$\max_{\{c^j_{i,t+j-1}, h^j_{i,t+j-1}, a^j_{i,t+j-1}\}^J_{j=1}} \sum_{j=1}^{J} \beta^{j-1} u(c^j_{i,t+j-1}, h^j_{i,t+j-1}; s^j_i) + \beta^J v(b^{J+1}_{i,t+J}; s^{J+1}_i) \tag{1}$$

subject to

$$c^j_{i,t+j-1} + g_{t+j-1} h^j_{i,t+j-1} + a^j_{i,t+j-1} = y^j_{t+j-1} + b^j_{i,t+j-1} \tag{2}$$

$$a^j_{i,t+j-1} \geq -\theta p_{t+j-1} h^j_{i,t+j-1} \tag{3}$$

$$b^j_{i,t+j-1} = R_{t+j-1} a^{j-1}_{i,t+j-2} + p_{t+j-1} h^{j-1}_{i,t+j-2} \text{ for } j > 1 \tag{4}$$

$$b^1_{i,t} = 0, \tag{5}$$

where

$$g_t = p_t + \kappa p_t + \eta.$$

The subjective discount factor is β. The first constraint is the periodic budget constraint. The second constraint is the periodic down payment constraint. The third constraint defines net worth and the last constraint states that the household starts its life without initial assets or debt.

The Lagrangian for the household's maximization problem is:

$$\begin{aligned} L &= \sum_{j=1}^{J} \beta^{j-1} u(c^j_{i,t+j-1}, h^j_{i,t+j-1}; s^j_i) + \beta^J v(b^{J+1}_{i,t+J}; s^{J+1}_i) \\ &+ \sum_{j=1}^{J} \lambda^j_{i,t+j-1}[y^j_{t+j-1} + b^j_{i,t+j-1} - c^j_{i,t+j-1} \\ &- g_{t+j-1} h^j_{t+j-1} - a^j_{i,t+j-1}] \\ &+ \sum_{j=1}^{J} \gamma^j_{i,t+j-1}(a^j_{t+j-1} + \theta p_{t+j-1} h^j_{t+j-1}), \end{aligned} \tag{6}$$

where $\lambda^j_{i,t}$ are the Lagrange multipliers for the budget constraints and $\gamma^j_{i,t}$ are the Kuhn-Tucker multipliers for the borrowing constraints.

2.3 Aggregate consistency

The economy is small and open in the sense that the interest rate and the wage level are exogenously given. The only aggregate consistency condition is the market clearing condition for the housing market. Since our focus is entirely on the demand side of the housing market, we take the housing supply as fixed at $\overline{H}$.[6] The market clearing condition for period t reads as

$$\sum_{i=1}^{I} \sum_{j=1}^{J} m_i h^j_{i,t} = \overline{H}. \tag{7}$$

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[6]See e.g. Davis and Heathcote (2005) and Kiyotaki et al. (2011) for different ways of introducing housing supply and land into dynamic macromodels.

2.4 Solving the model

We find the transitionary dynamics following an aggregate shock by solving the set of non-linear equations that consists of the household first-order conditions, the budget constraints, and the housing market equilibrium conditions taking into account the complementary slackness conditions related to the borrowing constraint. Assuming that it takes up to T periods for the economy to converge to a new steady state, we have the following system of equations and complementary slackness conditions for $t = 1, 2, ..., T$ and $i = 1, 2, ..., I$.[7],[8]

$$\beta^{j-1} u_{h_{i,t}^j} + p_{t+1} \lambda_{i,t+1}^{j+1} = g_t \lambda_{i,t}^j - \gamma_{i,t}^j \theta p_t \text{ for } 1 \leq j < J \tag{8}$$

$$\beta^{J-1} u_{h_{i,t}^J} + \beta^J p_{t+1} v_{b_{i,t+1}^{J+1}} = g_t \lambda_{i,t}^J - \gamma_{i,t}^J \theta p_t \tag{9}$$

$$\beta^{j-1} u_{c_{i,t}^j} = \lambda_{i,t}^j \text{ for all } j \tag{10}$$

$$-\lambda_{i,t}^j + R_{t+1} \lambda_{i,t+1}^{j+1} + \gamma_{i,t}^j = 0 \text{ for } 1 \leq j < J \tag{11}$$

$$\beta^J R_{t+1} v_{b_{i,t+1}^{J+1}} - \lambda_{i,t}^J + \gamma_{i,t}^J = 0 \tag{12}$$

$$\gamma_{i,t}^j \left(a_{i,t}^j + \theta p_t h_{i,t}^j \right) = 0 \text{ for all } j \tag{13}$$

$$c_{i,t}^j + g_t h_{i,t}^j + a_{i,t}^j = y_t^j + b_{i,t}^j \text{ for all } j \tag{14}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} m_i h_{i,t}^j = \overline{H} \tag{15}$$

$$\gamma_{i,t}^j \geq 0 \text{ and } a_{i,t}^j + \theta p_t h_{i,t}^j \geq 0. \tag{16}$$

In practice, we combine (13) and the complementary slackness conditions in (16) into one (highly non-linear) equation and use Matlab's fsolve function to solve the system. In our calibrated model, the system consists of about 3000 equations. Nevertheless, we find that the system can be solved quite reliably.[9]

The fact that we can solve directly the equilibrium allocation and prices allows us to find the non-linear dynamics relatively fast and very accurately. In this respect, the model features two key simplifications. The first is that we consider only perfect foresight dynamics following completely unanticipated aggregate shocks. With aggregate uncertainty, we would have to use recursive methods with the distribution of households over their asset positions (or at least some moments describing it) as a state variable.

The second simplification is that there are no transaction costs. Realistic non-convex transaction costs would make the household problem non-convex and would probably also force us to have a continuum of households in different situations in order to make the aggregate demand for

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[7] We need to check that the solution is not affected by our guess for T.

[8] We denote partial derivatives by subscripts. For instance, $u_{c_{i,t}^j}$ denotes the marginal utility of non-housing consumption of household of age j and type i in period t.

[9] All the Matlab programs needed to solve the model are available from the authors upon request.

housing a smooth function of house prices. The absence of transaction costs means that we cannot consider the dynamics of the transaction volume. It also means that households generally adjust their housing position every period, which is not realistic if the model period is relatively short. However, as we show below, since the demand for housing in our model is affected by changes in household size, the model nevertheless has the realistic feature that households undertake major adjustments to their housing only a few times over their life cycle.

2.5 Calibration

We base our calibration on the 2004 Wealth Survey conducted by Statistics Finland. The survey includes portfolio information from 3455 Finnish households. We consider only homeowners (2450 households).[10] In the survey, they were asked, among other things, to give an estimate of the current market value of their house.

The model period is four years and households' economically independent life lasts for 12 periods, that is $J = 12$. We interpret model age 1 as real ages 25-28. Model age 12 then corresponds to real ages 69-72.

These choices are somewhat arbitrary, of course. Because the model does not feature transaction costs related to moving, a relatively long model period seems more appropriate than a model period of, say, one year. As we will discuss below, a long model period also allows us to partly capture maturity constraints with the borrowing constraint. On the other hand, a model period of four years suffices to describe the main house price changes in Finland during the recent boom-bust-boom experience: The boom that begun in the late 1980s lasted for roughly four years and it took four years for house prices to fall from peak to bottom in the early 1990s.

We divide the households into five groups, that is $I = 5$. For the first four household types, type indicates the model age of getting children. Households of type 5 never get children. We use the Wealth Survey to construct the shares of different household types as follows: First, we calculate from the data the share of households having children in age groups 25-28, 29-32, and 33-36. These shares are 25%, 55%, and 68%, respectively. Based on these figures, we set

$$m_1 = 0.25, \ m_2 = 0.30, \text{ and } m_3 = 0.13.$$

The survey does not contain information on the share of households not getting children. However, according to the Family Federation of Finland, in 2004, 16% of females of age 45-49 did not have children. We therefore set $m_5 = 0.16$ and determine m_4 as a residual. This means that $m_4 = 0.16$.

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[10]About 30% of all households in the survey are renters. However, most of the rental dwellings in Finland are part of social housing where rents are regulated and tenants are selected on the basis of social and financial needs. Only 10% of the households in the survey have rented from the private rental market.

In the model, a household always consists of two adults and possibly two children. The children live within the household for five model periods (or 20 years). We compute the corresponding household sizes using the OECD scale for household consumption units. For instance, for $i = 2$, this means that

$$s_2^1 = 1.7, \ s_2^j = 2.7, \text{ for } j = 2, 3, 4, 5, 6, \text{ and } s_2^j = 1.7 \text{ for } j \geq 7.$$

We use the Wealth Survey to construct the age-income profile. We first compute the average annual non-capital income (after taxes and including transfers) for model ages $j = 1, ..., 9$. We then scale the profile so that average income for model ages $j = 1, ..., 9$ is equal to one. For model ages $j = 10, 11, 12$, households are assumed to receive a pension which is 60% of the average income.[11] This figure is close to the current replacement rate of the Finnish pension system. The resulting income profile is

$$\{y^j\}_{j=1}^J = \{0.93, 0.99, 1.09, 1.11, 1.11, 1.01, 1.01, 0.85, 0.84, 0.60, 0.60, 0.60\}. \tag{17}$$

The Wealth Survey also allows us to determine the housing related cost parameters, κ and η. Except for single family houses, the legal structure for home ownership in Finland is a limited liability housing company. Homeowners own shares of the housing company which give them the possession of a specific apartment.[12] The company is responsible for the management and upkeep of the building. To that end, it collects management fees which are proportional to the size of the apartment. The Wealth Survey includes information about this management fee. In addition, households were asked to estimate how much they spend on maintenance operations in their own apartment. Together these costs were annually on average 2.5% of the reported house value. To our understanding, the only component of this cost that is related to the value of the house is the property tax, which is included in the management fee. The tax rate varies by municipality but its average rate is only about 0.5% of the house value. As a result, given our four year model period, we set $\kappa = 0.02$ and $\eta = 0.08$.

We assume that the periodic utility $u(c, h; s)$ is determined by a CES-CRRA utility function:

$$u(c, h; s) = \begin{cases} s\frac{[\varphi(c,h;s)]^{1-\sigma}}{1-\sigma}, \text{ for } \sigma > 0 \text{ and } \sigma \neq 1 \\ s \log \varphi(c, h; s), \text{ for } \sigma = 1. \end{cases} \tag{18}$$

where

$$\varphi(c, h; s) = \begin{cases} [(1 - \alpha_h)(c/s)^\rho + \alpha_h(h/s)^\rho]^{\frac{1}{\rho}}, \ \rho \leq 1 \text{ for } \rho \neq 0 \\ (c/s)^{1-\alpha_h}(h/s)^{\alpha_h}, \text{ for } \rho = 0. \end{cases} \tag{19}$$

For households of age $J + 1$ utility is

$$v(b; s) = \begin{cases} s\alpha_b\frac{(b/s)^{1-\sigma}}{1-\sigma}, \text{ for } \sigma > 0 \text{ and } \sigma \neq 1 \\ s\alpha_b \log(b/s), \text{ for } \sigma = 1. \end{cases} \tag{20}$$

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[11] According to the Finnish Center for Pensions, the median retirement age in 2004 was 60.1.

[12] Naturally, the shares are treated as private property and can be used as collateral for mortgage loans.

The average yearly real after tax interest rate on mortgage loans during the period 2000-04 was 1.95%. We therefore set the interest rate term at $R = 1.08$. Finally, we set the borrowing constraint parameter at $\theta = 0.75$. This means that a household is required to make a down payment of 25% of the value of the house. We think of this as a realistic borrowing constraint in Finland after the credit market liberalization in the late 1980s.

We are then left with the preference parameters, σ, ρ, β, α_h, and α_b. We set σ, the parameter governing intertemporal elasticity of substitution, at $\sigma = 2$ which is a relatively conventional value. The elasticity of substitution between housing and non-housing consumption, which is determined by ρ, should be important for house price dynamics. Empirical estimates of this elasticity vary a lot. Using a structural life cycle model, Li et al. (2009) find an elasticity of substitution equal to 0.33. On the other hand, much of the related literature uses Cobb-Douglas preferences implying an elasticity of substitution equal to 1. Davis and Ortalo-Magné (2011) provide empirical support for that assumption. We set $\rho = -1$, which implies an elasticity of substitution equal to 0.5. Later we also experiment with different values for ρ. Finally ,we choose parameters β, α_h, and α_b to match the following targets:

   i) Average net worth-to-house value (NWHV) ratio equal to 0.82.

   ii) Average NWHV ratio in age J equal to 1.05.

   iii) Average net worth-to-income ratio equal to 0.86.

The targets are based on the Wealth Survey. Net worth is defined as the sum of the market value of household's residential property and its financial assets less all debt. The first target is based on the median NWHV ratio for households of age 25-72 in the data. The second target is the median NWHV ratio for households of age 69-72. The third target is based on the median net worth-to-annual income which is 3.45 in the data. Since the model period is four years, we divide this ratio by four.

Some of our experiments consist of comparing the price dynamics in the economy described above to the price dynamics in an economy with unlimited borrowing. For this comparison, we recalibrate the benchmark model using the same targets and exogenously calibrated parameter values that were discussed above but assuming unlimited borrowing. The resulting parameter combinations with and without the borrowing constraint are shown in Table 1. In order to get the same average NWHV ratio in both cases, we have to choose a higher discount factor with unlimited borrowing. Alternatively, when comparing the dynamics with and without the borrowing constraint, we could keep all other parameter values fixed. This would mean, however, that the initial distributions of household leverage would be very different in the two cases.

	β	α_h	α_b
Constr. $(\theta = 0.75)$	0.94	0.21	0.94
No constr. $(\theta = \infty)$	0.95	0.21	0.93

Table 1: Parameter combinations in the benchmark calibrations.

2.6 Steady state

The importance of borrowing constraints should crucially depend on household leverage. We characterize household leverage with NWHV ratio. The lower the NWHV ratio of a household, the more highly leveraged it is in the sense that it has more debt or less assets relative to the value of its house. The median NWHV ratios in the data and the average NWHV ratios in the model for different age groups are shown in Figure 1.
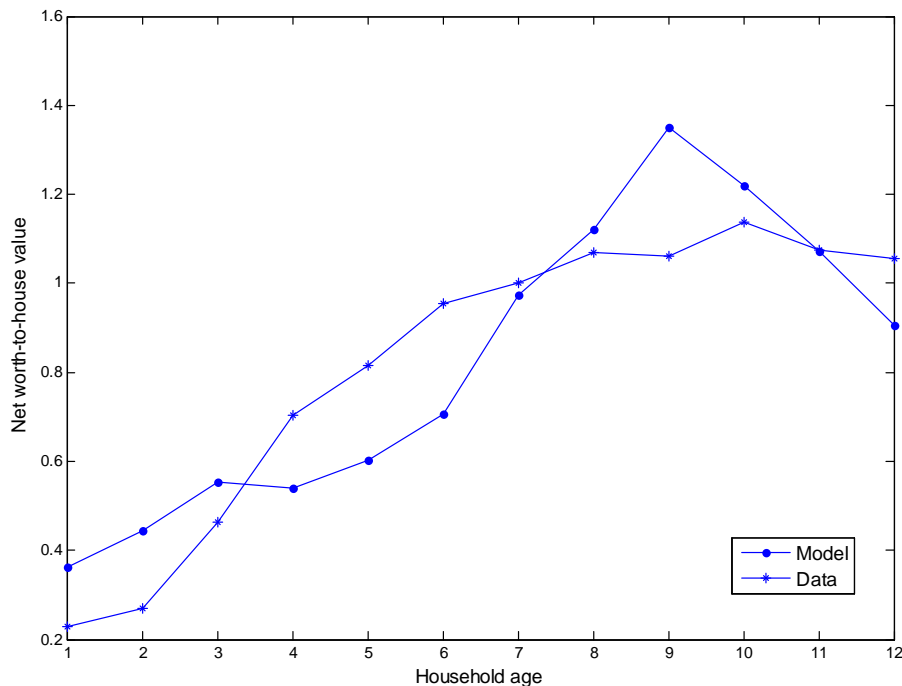

Figure 1: NWHV ratio in different age groups in the data and in the model.

Clearly young households are much more leveraged than older households. In the data, the median NWHV ratio increases from about 0.25 among households of age 25-29 to about 1.1 among households of age 69-72.

Figure 2 displays the steady state housing profiles, h_i^j, for the different household types (these profiles are not scaled by household size, s). Non-housing consumption profiles (not shown) are similar. Consider first households of types 3 and 4. These households are never borrowing constrained and hence their housing follows closely household size. They move to a bigger house when they get children (at model age 3 or 4) and move to a smaller house when children move out (at

model age 8 and 9). In contrast, households of type 1 are borrowing constrained until model age 5. This distorts their housing (and non-housing) consumption profiles over the life cycle. Households of type 2 are borrowing constrained at model age 5. Finally, households of type 5 are borrowing constrained at model age 1. In the absence of the borrowing constraint they would immediately move to a bigger house. The share of borrowing constrained households is 7/60 in steady state.[13]
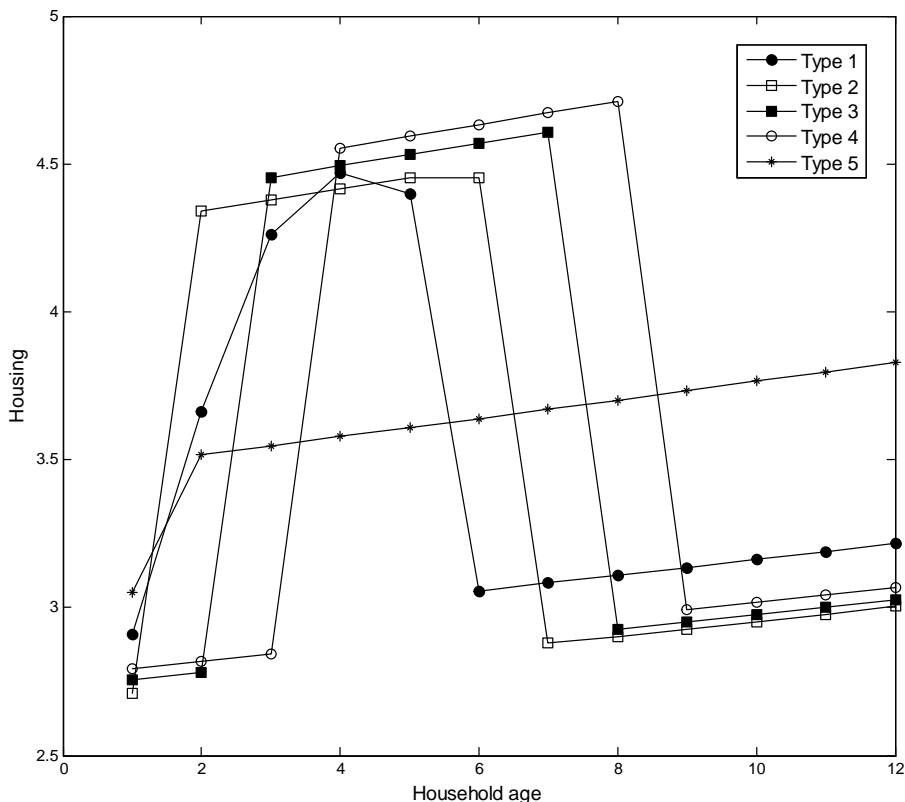

Figure 2: Housing profiles over the life cycle in steady state.

Table 2 compares the distribution of household leverage in the model to the data when $\theta = 0.75$. For the table, we have divided the households into four groups according to their NWHV ratios and calculated the share of households in each group. As the table shows, the distribution is more dispersed in the data than in the model. The main difference is that in the data, some households report to have NWHV ratio less than 0.25, which is the lowest NWHV ratio we allow for in the model. This is somewhat surprising given that prior to 2004 house prices had been increasing steadily for several years. We suspect that these households have underestimated their net worth.

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[13]We are not aware of a study that would estimate the share of borrowing constrained households in Finland using household level data. Kilponen (2009) estimates, among other things, the *consumption share* of borrowing constrained households using a two-agent model and aggregate time series data. According to his point estimates, the consumption share of borrowing constrained households in period 1995-2008 was almost 50%. However, the confidence interval for this share is very large.

	Net worth-to-house value			
	< 0.25	$0.25 - 0.5$	$0.5 - 1.0$	> 1.0
Data	8.0%	9.2%	28%	54%
Model ($\theta = 0.75$)	0%	25%	33%	42%

Table 2: Share of households with different NWHV ratios.

3 Analytical results

As we discussed in the introduction, the multiplier mechanism in Stein (1995) is essentially a link between house prices and buyer liquidity. In this section, we will analyze in detail how this link works in a dynamic set-up with a consumption smoothing motive. The main purpose of this exercise is to develop intuition for our numerical results by disentangling the different channels through which current house price affects housing demand.

Let us consider a household of age $1 < j < J$. For notational convenience, we drop here time, age, and type indices. Housing and financial assets of the household in the beginning of the current period are denoted by h_{-1} and a_{-1} and its housing and financial assets in the beginning of the next period by h and a. Current house price is p, next period house price is p', and the interest rate is constant. Periodic utility is separable between consumption and housing, that is $u_{ch} = 0$.

The problem of the household can now be formulated as:

$$\max_{c,h,a} \{u(c, h) + \beta V(b)\} \tag{21}$$

subject to

$$c + (p + \kappa p + \eta) h + a = y + ph_{-1} + Ra_{-1} \tag{22}$$

$$b = Ra + p'h \tag{23}$$

$$a \geq -\theta ph. \tag{24}$$

where $V(b)$ denotes remaining life time utility. As long as the household has a consumption smoothing motive, $V_{bb} < 0$.

We first ask how the current housing demand depends on the current house price, given a_{-1} and h_{-1}. In the unconstrained case, the effect of a marginal change in the current house price on current housing demand is (see Appendix for details)

$$\frac{\partial h}{\partial p} = \frac{1}{D} \left[\underbrace{\frac{(1 + \kappa) u_c u_{cc}}{R}}_{\text{negative}} + \beta R \left(\underbrace{V_{bb} (1 + \kappa) u_c}_{\text{negative}} + \underbrace{P V_{bb} u_{cc} (h_{-1} - (1 + \kappa) h)}_{\text{negative if } (1+\kappa)h > h_{-1}} \right) \right] \tag{25}$$

13

where $D > 0$ and $P = p + \kappa p + \eta - \frac{p'}{R}$. We assume here that $P > 0$.

The overall effect consists of three terms reflecting the standard substitution and income effects. The first term is related to the intratemporal resource allocation: An increase in the current house price makes current housing more expensive relative to current non-housing consumption. This reduces housing demand. The other two terms depend on V and are related to the intertemporal resource allocation. The first of them is always negative: An increase in the current house price makes current housing more expensive relative to future housing and non-housing consumption. The last term is related to an endowment effect: An increase in the current house price makes the household "wealthier" if $h_{-1} > (1 + \kappa) h$, that is, if it is downsizing fast enough. In that case, the third term works to increase housing demand when the current house price increases.

When the household faces a binding borrowing constraint, the effect of a marginal change in current house price on current housing demand is (see Appendix for details)

$$\frac{\partial h}{\partial p} = \frac{1}{D^c} \left[\underbrace{-u_c \left(1 + \kappa \right)}_{\text{negative}} + \underbrace{u_{cc} T \left(\left(1 + \kappa \right) h - h_{-1} \right)}_{\text{negative if } (1+\kappa)h > h_{-1}} \right] + \frac{\theta}{D^c} \left[\underbrace{-u_{cc} T h}_{\text{positive}} + \underbrace{u_c}_{} \underbrace{-R\beta V_b}_{\text{negative}} \underbrace{-S\beta V_{bb} R h}_{\text{positive}} \right] \quad (26)$$

where $D^c > 0$ and $T = p + \kappa p + \eta - \theta p > 0$ and $S = p' - R\theta p$. We assume here that $S > 0$.

The first two terms represent the substitution and endowment effects and have the same interpretation as in the unconstrained case. The second term of the unconstrained case is missing here. This is because with a binding borrowing constraint a higher current house price does not induce the household to substitute future housing or non-housing consumption for current housing.

Compared to the unconstrained case, there are hence four additional terms. The first term shows the direct link between house prices and the borrowing constraint: as the current house price goes up, the household can borrow more which increases the demand for housing. This effect creates the multiplier effect in Stein's (1995) model. The second term is also positive and closely related to the first one: A borrowing constrained household can only increase its housing demand by giving up more current consumption. When the house price increases, the household can borrow more for each unit of housing and hence must give up less current consumption. This induces the household to buy more housing. We will refer to these two terms together as the *liquidity effect*.

The last two terms are related to the fact that for a borrowing constrained household, the amount of housing it buys today directly determines its next period net worth. The last two terms show how a change in the current house price affects housing demand via this savings motive. First, an increase in the current house price makes saving more expensive which reduces savings. On the other hand, as the current house price increases, for a given housing demand, the household has less savings in the future. The last term shows that this effect induces the household to demand more housing.

In equilibrium, the current house price depends on both current and future housing demand. Future housing demand must in turn depend positively on household's next period net worth. We now consider how next period's net worth is affected by a change in the current house price.

The Appendix shows that with unlimited borrowing

$$\frac{\partial b}{\partial p} = \frac{1}{D} \left[\underbrace{-u_{cc} P \left(1 + \kappa\right) u_c}_{\text{positive}} + \underbrace{u_{cc} u_{hh} \left(h_{-1} - \left(1 + \kappa\right) h\right)}_{\text{positive if } h_{-1} > (1+\kappa)h} \right] \tag{27}$$

where again $D > 0$. A sufficient condition for a higher house price to increase savings is that $h_{-1} > (1 + \kappa) h$. It is straightforward to show that with $u(c, h) = \log(c) + \log(h)$, for instance, a higher current house price always increases savings.

When a household faces a binding borrowing constraint, it follows that

$$\frac{\partial b}{\partial p} = S \frac{\partial h}{\partial p} - R\theta h, \tag{28}$$

where $\frac{\partial h}{\partial p}$ is given by (26). Recall that if the *liquidity effect* is not strong enough to dominate in the demand response, $\frac{\partial h}{\partial p} < 0$. In that case, equation (28) shows that an increase in the current house price will reduce savings.

All in all, changes in current house price affect the housing demand of the borrowing constrained households through several channels. Necessary conditions for the multiplier mechanism to work are that the liquidity effect dominates in the demand response of the individual households and that the share of these households is large enough for the liquidity effect to shape also the aggregate demand response.

However, it is also important to understand that house price changes affect the savings decisions of the households. In particular, a fall in house price is likely to make unconstrained households save less and borrowing constrained households save more. It is therefore important to model properly households' life cycle savings problem by taking into account the consumption smoothing motive. Indeed, as we will see, the "delayed demand" effect caused by increased savings of the borrowing constrained households turns out to be important in shaping the price dynamics after certain types of shocks.

4   Numerical results

In this section, we analyze numerically the dynamics of the model. In the first subsection, we use the model to mimic the Finnish experience of the late 1980s and 1990s. After that we consider different shocks starting from the steady state calibration presented in section 2.5 representing year 2004. The aim of these experiments is to illustrate in more detail how the down payment constraint

shapes house price dynamics. In all cases, we compare the price dynamics with and without the down payment constraint. We first consider positive and negative income and interest rate shocks to illustrate non-linearities in house price dynamics. We then discuss the role of leverage. Finally, we study the potential for multiple equilibria.

4.1 Model dynamics vs. the Finnish boom-bust-boom cycle

4.1.1 The Finnish experience

Figure 3 displays the real house price index of Statistics Finland from 1980 to 2008. Real house prices first increased by about 50% from 1986 to 1989 and then fell by almost as much from 1989 to 1993[14] After 1996 or so, house prices started to increase again quite rapidly.

The main explanation usually put forward for the boom of the late 1980s is the gradual deregulation of the financial system that started in the early 1980s (for details on the timing of the different measures, see, Vihriälä, 1997). Until mid 1980s, both deposit and lending rates were administratively set. Together with loan volume control, this resulted in very tight credit rationing. Bank lending to households was liberalized in 1986. This eased the borrowing constraints on households and induced a huge growth of credit (see e.g. Koskela et al. 1992, Berg 1994, Laakso 2000, and Oikarinen 2009).
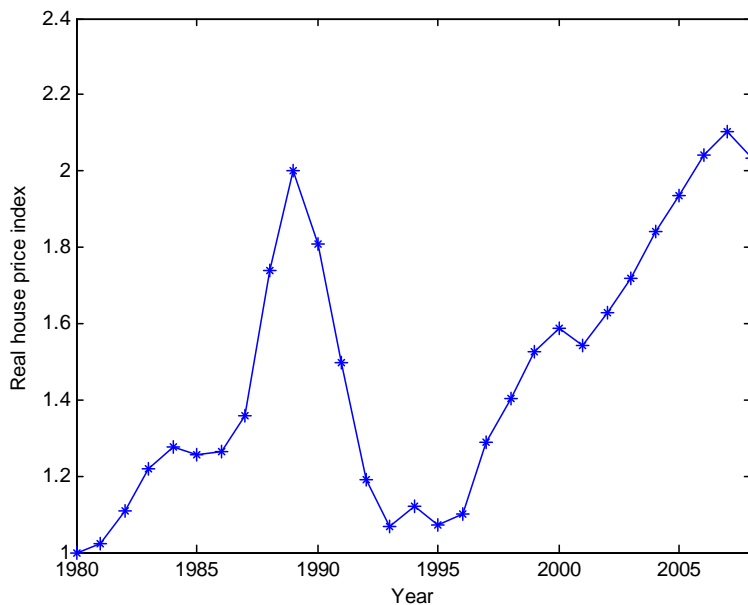

Figure 3: Real house prices in Finland 1980-2008.

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[14]It should be noted that despite the drastic house price fall, there were very few defaults on mortgages. This is because in Finland mortgages are always full recourse loans.

The housing market bust in turn coincided with the depression of the early 1990s. Real GDP decreased by over 10% from 1990 to 1993. Among the factors that contributed to the depression were a banking crisis and the collapse of demand from the former Soviet Union.[15] After the depression, the Finnish economy grew relatively fast during several years reaching its pre-depression growth path by 2005 or so. Figure 4 shows the yearly growth rate of real GDP per capita from 1980 to 2008.
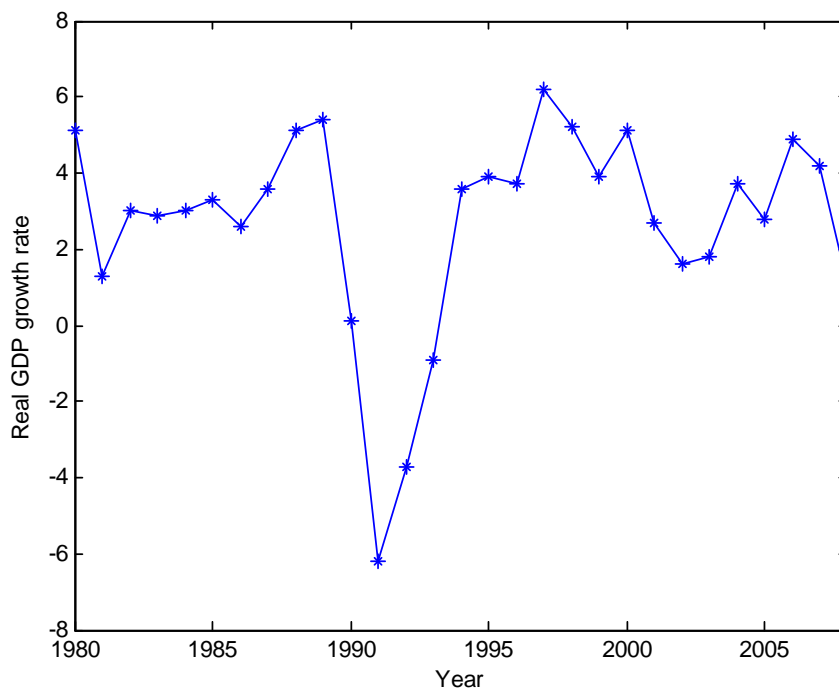

Figure 4: Real GDP growth rate in Finland 1980-2008.

We next study to what extent the model can explain the huge house price fluctuations in Figure 3 as a response to an empirically plausible relaxation of the borrowing constraint and income shocks that are similar to those in Figure 4.

4.1.2 Mimicking the Finnish experience

In order to use the model to study the Finnish experience we assume that the economy is initially in a steady state with a very tight borrowing constraint. We then consider a series of shocks where the borrowing constraint is first suddenly relaxed and after that households are hit by adverse income shocks.

We first need to specify the effect of financial deregulation on the household borrowing constraint in the model. Before the financial deregulation, households were constrained by very short

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[15]See Honkapohja et al. (2009) for a comprehensive discussion of the Finnish depression and the subsequent recovery.

mortgage maturities. Typical maturity was 7-9 years. In addition, households needed to pay a down payment of around 30% of the house value (see Loikkanen and Salo 1992, and Koskela et al. 1992).

Although we do not formally have a maturity constraint in the model, we can partly capture it by lowering the borrowing constraint parameter θ. A mortgage maturity of 8 years together with a down payment constraint of 30% means that a household needs to pay about 80% of the value of its new house during the first four years. In the model, this translates into $\theta = 0.20$. We therefore model the financial deregulation as a sudden and permanent increase of the down payment constraint parameter from $\theta = 0.20$ to $\theta = 0.75$.

As for the income shocks, we consider the difference between the actual path of household disposable monetary income calculated by the Statistics Finland and its trend growth path. We compute the trend growth path based on the average growth in household income between 1975 and 1990 which was very close to 2% annually. Partly because of government transfers, during the recession household disposable income started to decline later than GDP. In periods 1990-93, 1994-97, and 1998-2001, household disposable income was on average 5.1%, 8.9%, 4.5%, respectively, below its trend growth path. By 2005 it had converged back to its trend growth path.

Of course, a crucial assumption concerns households' expectations about future income. At one extreme, we could assume that in the beginning of the depression, households learn the true future income path. That is, they realize that while their income will temporarily fall far below the level they had expected, it will also converge relatively quickly back to its original trend growth path. Alternatively, we can assume that households expected the depression to lower their income permanently.

We find the latter assumption more realistic. Finnish households may have expected the depression to be followed by some convergence towards the pre-depression growth path, but the brisk recovery in the late 1990s must have taken most people by surprise. International econometric evidence also shows that shocks to income growth tend to have a very persistent or even permanent effect on the level of output (Campbell and Mankiw 1987, IMF 2009 chapter 4). We therefore mimic the depression by hitting the model economy with a sequence of income shocks all of which households expect to be permanent.

The experiment is thus the following. We first solve for the steady state with a tight borrowing constraint $\theta = 0.20$. Other parameter values correspond to the calibration presented in section 2.5. Thus, the endogenously determined parameter values are those of the first row of Table 1. We refer to this initial steady state as period 0. In the beginning of period 1, representing years 1986-89, the borrowing constraint is relaxed to $\theta = 0.75$. In period 2, households are hit by an income shock that lowers their income by 5.1% compared to the initial steady state. Households expect this income shock to be permanent. Similarly, in periods 3 and 4 households are hit by

income shocks that change their income so that it is, respectively, 8.9% and 4.5% below the initial steady state income level. The last income shock, in period 5, increases household income back to the initial steady state level. In each period, households make their decisions after learning about the shock. When solving for the transitionary dynamics following each income shock, we start from the distribution determined by the first period decisions of the previous transition.

We also consider the sensitivity of the price dynamics with respect to the elasticity parameter ρ. In addition to the benchmark case with $\rho = -1$, we considered cases with $\rho = 0$ and $\rho = -2$. These parameter values correspond to intratemporal elasticities of substitution equal to 1 and 1/3, respectively. As discussed in section 2.5, both of these values are in the range of empirical estimates. In both cases, the model is recalibrated so that the final steady state (with $\theta = 0.75$) replicates the same calibration targets as in the benchmark case.

Figure 5 displays the house price dynamics that result from this exercise. For the benchmark calibration, it also presents the price dynamics associated with the relaxation of the borrowing constraint alone. The final steady state house price is equal to one by construction. The initial steady state price, in contrast, depends on the elasticity parameter. This is because the extent to which tightening the down payment constraint reduces aggregate housing demand depends on the elasticity of substitution between housing and non-housing consumption.
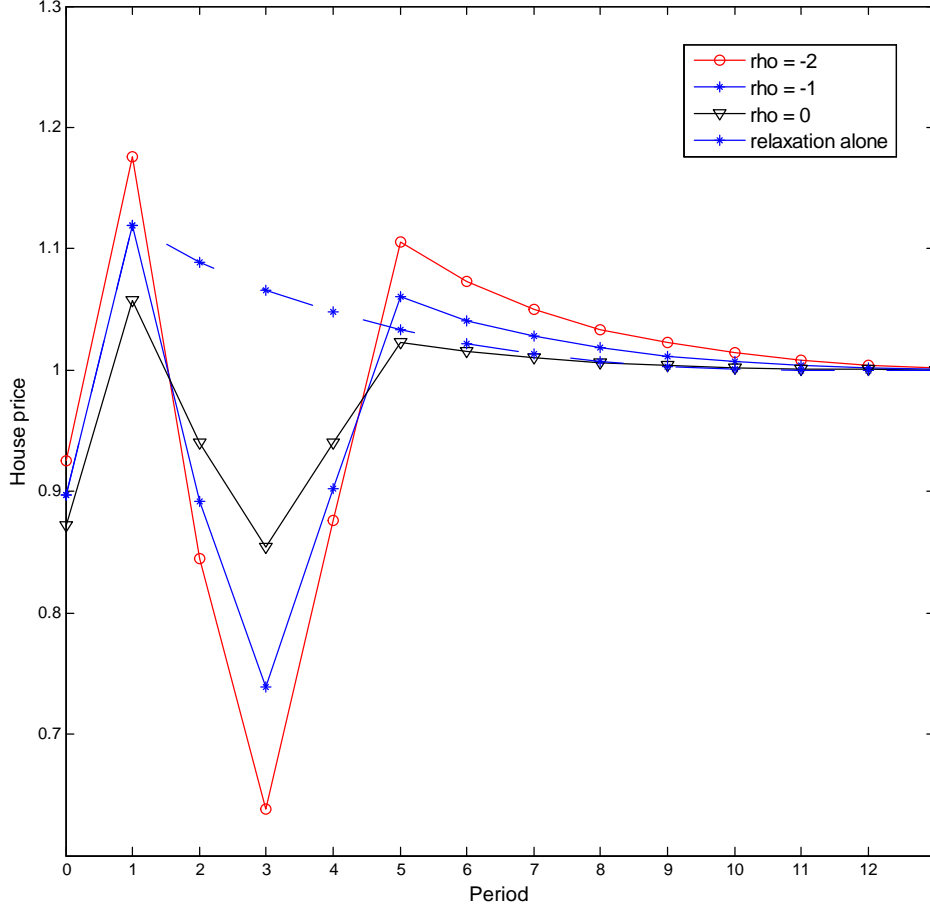
Figure 5: Relaxation of borrowing constraint and income shocks with different intratemporal elasticities of subsitution.

The figure shows, first of all, that the model can account for a large part of the huge house price fluctuations depicted in Figure 3. Recall that in the data (Figure 3) house prices first increase and then decrease by about 50%. In the benchmark calibration, house prices first increase about 25% following the relaxation of the down payment constraint and then fall by about 35% from period 1 to period 3. Assuming an intratemporal elasticity of substitution that is at the lower end of empirical estimates further magnifies these fluctuations.[16],[17]

It is interesting to note that the relaxation of the borrowing constraint alone leads to a substantial overshooting in house prices. As can be seen from Figure 5, about half of the increase in house prices that is associated the relaxation of the borrowing constraint is temporary. The intuition for overshooting should be clear: On impact, the only thing that changes is that households can borrow more against housing. Hence, the house price must go up. However, the laxer borrowing constraint also lowers the household savings rate which in the future shows up as lower

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[16]Of course, it should be kept in mind that the housing supply is perfectly inelastic in the model.

[17]We also experimented with different values for σ ($\sigma = 1$ and $\sigma = 3$) which determines intertemporal elasticity of substitution. Compared to the benchmark calibration, the differences in house price dynamics were very small.

household net worth which in turn reduces housing demand. Therefore, house prices must fall after the impact effect.

The fact that the model can explain a large part of the first boom as an equilibrium response to an empirically plausible relaxation of the borrowing constraint suggests that it captures much of the actual relevance of borrowing constraints for aggregate housing demand. However, Figure 5 does not reveal how the remaining down payment constraint influences the effect of different aggregate shocks on house price dynamics. The following subsections study this issue in detail.

4.2 Illustrating non-linearities

We now analyze the behavior of the model economy after different income and interest rate shocks. In order to get a good overall picture of the importance of borrowing constraints, we consider both permanent and temporary shocks and both positive and negative shocks. We consider relatively large shocks in order to highlight the non-linearities of the model.

In order to isolate the effect of the borrowing constraint, we compute the house price dynamics with and without it. The parameter values are given in Table 1. The economy is initially in a steady state and by construction the initial house price is equal to 1.

The income shocks affect all households equiproportionally. Let y_0 denote the initial age-income profile specified in equation (17). In the case of a permanent shock, we decrease or increase all households' income by 10%. That is, we set $y_t^j = 0.9y_0^j$ or $y_t^j = 1.1y_0^j$ for all j and $t \geq 1$. The temporary shocks last for four periods. The negative shock is specified as $y_1^j = 0.80y_0^j$, $y_2^j = 0.85y_0^j$, $y_3^j = 0.89y_0^j$, $y_4^j = 0.95y_0^j$ and $y_t^j = y_0^j$ for all j and $t \geq 5$, and the positive shock as $y_1^j = 1.20y_0^j$, $y_2^j = 1.15y_0^j$, $y_3^j = 1.10y_0^j$, $y_4^j = 1.05y_0^j$ and $y_t^j = y_0^j$ for all j and $t \geq 5$.

Figure 6 displays the house price dynamics following the different income shocks. Top-left panel relates to a temporary negative shock, top-right panel to a permanent negative shock, bottom-left panel to a temporary positive shock, and bottom-right panel to a permanent positive shock.

The permanent income shock, for instance, induces a steady state price effect of about 30%. The reason why the price effect is so much bigger than the income shock is twofold. First, we assume a rather low elasticity of substitution between housing and consumption. Second, the user-cost of housing includes a component that is independent of the house value. This makes the user-cost of housing relatively insensitive to the house price (see footnote 4).

We are interested in how the borrowing constraint shapes house price dynamics. In this respect, the first thing to note from Figure 6 is that the borrowing constraint seems to matter only following negative income shocks. After positive shocks, the price dynamics are remarkably similar with and without the borrowing constraint. This asymmetry stems from the fact the share of borrowing constrained households is different after different types of shocks. For instance, following the permanent positive shock, the share of borrowing constrained households decreases from 7/60 in

the initial steady state to 2/60 in period 1. As a result, the borrowing constraint cannot matter much for house price dynamics. In contrast, after the negative permanent shock, the share of borrowing constrained households increases to 13/60 in period 1.

Consider then the top panels that display the price dynamics after negative income shocks. Two interesting observations can be made. First, the differences in the impact effect of the shocks are not large. In the case of a temporary shock, on impact, the house price falls by about 15% without the borrowing constraint and by about 18% with the borrowing constraint. In the case of a permanent shock, the impact effect is almost the same with and without the borrowing constraint. Second, with the down payment constraint house price increases more rapidly towards the new steady state. This seems to be the most important way in which the down payment constraint shapes house price dynamics.
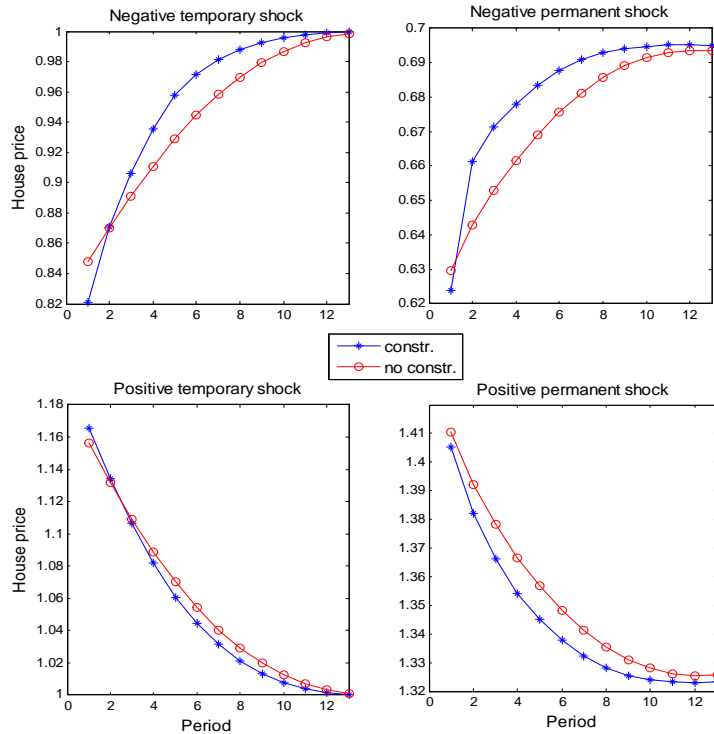

Figure 6: House price dynamics following different income shocks.

Intuitively, the fall in the house price effectively tightens the borrowing constraint and increases the share of borrowing constrained households. This reduces period 1 housing demand through the liquidity effect that we discussed in section 3. For the housing market to clear in period 1, the liquidity effect must be offset by sufficiently large anticipated capital gains to housing from period 1 to period 2. As we showed in section 3, by effectively tightening the borrowing constraint, a fall in the current house price forces the borrowing constrained households to save more. This increases future housing demand. Hence, the anticipated capital gain needed to offset the liquidity effect is created by a relatively high house price in period 2, rather than by a very low house price

in period 1. This mechanism explains why the borrowing constraint makes house prices converge more rapidly towards the steady state level.

We next study the effect of different interest rate shocks on the price dynamics in the same manner as above. We consider a permanent increase from $R = 1.08$ to $R = 1.10$ and a permanent reduction to $R = 1.06$. The temporary shocks last for two periods: In the case of an increase in the interest rate, we set $R_1 = R_2 = 1.16$. In the case of a decrease in the interest rate, we set $R_1 = R_2 = 1.0$. Figure 7 displays the house price dynamics following the four different shocks.
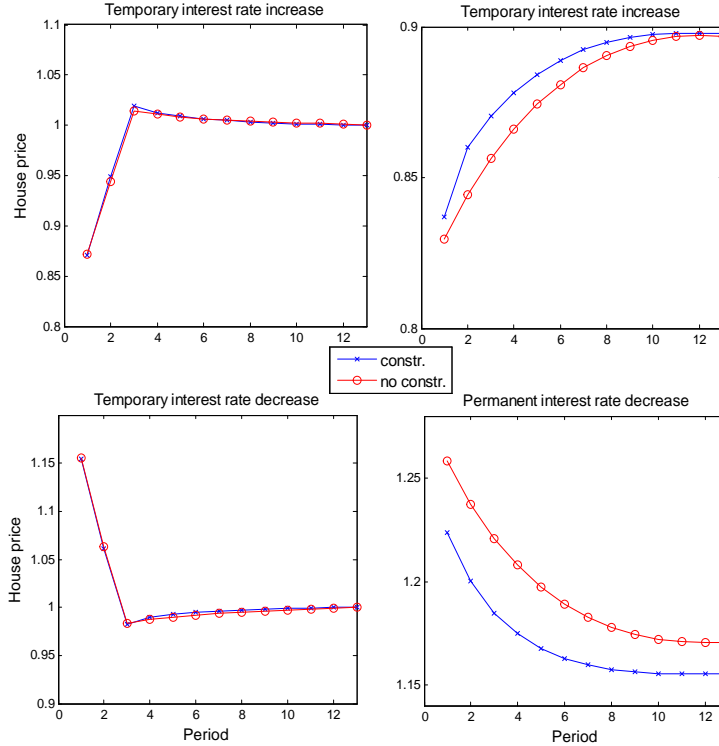

Figure 7: House price dynamics following different interest rate shocks.

The price dynamics are almost identical with and without the borrowing constraint after temporary interest rate shocks. The most noteworthy effect of the borrowing constraint after interest rate shocks seems to be that after permanent reduction in the interest rate, the new steady state price level is lower than in the absence of borrowing constraint. This is because a permanent reduction in the interest rate increases housing demand less in the case where household borrowing is limited by the borrowing constraint. However, apart from this steady state effect, the borrowing constraint seems to have little influence on the price dynamics even after permanent interest rate shocks.

The reason why the borrowing constraint is relatively unimportant in shaping house prices after interest rate shocks is that changes in the interest rate mitigate the liquidity effect. For instance, while an increase in the interest rate effectively tightens the borrowing constraint by inducing a house price fall, it also directly decreases households' willingness to borrow. The latter effect

makes the borrowing constraint less likely to bind. Similarly, while a fall in the interest rate relaxes the borrowing constraint by pushing up the house price, it also makes borrowing more attractive thereby making the borrowing constraint more relevant.

4.3 Leverage and house price dynamics

In this subsection we study how the importance of the borrowing constraint for price dynamics depends on initial household leverage. This experiment is conducted as follows: We change the benchmark calibration with the borrowing constraint by lowering the subjective discount factor from $\beta = 0.94$ to $\beta = 0.88$ without changing other parameters. This implies that the steady state average NWHV ratio, the average NWHV ratio at age J, and the average net worth-to-income ratio will now be substantially lower. In addition, the share of borrowing constrained households increases from 7/60 to 13/60. We then create a comparison case without the borrowing constraint by choosing the endogenously calibrated preference parameters so that the three aggregate ratios that were used as targets in the benchmark calibrations are the same in the two economies.

Figure 8 shows the result. The solid lines are same as those reported in the top-left panel of Figure 6. The two new lines show the price dynamics in a model economy where households are much more leveraged.
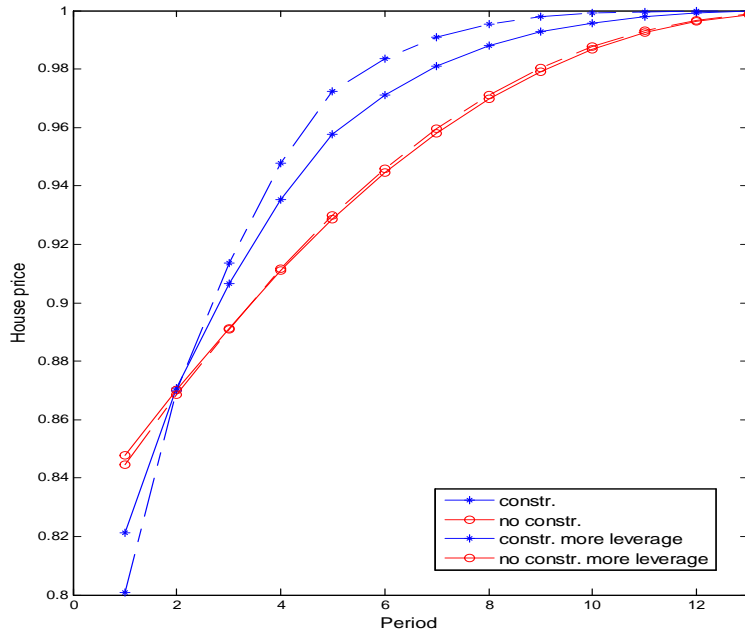

Figure 8: Price dynamics in more leveraged economies and in benchmark economies.

The figure suggests that household leverage only matters for house price dynamics via the borrowing constraint. Without the borrowing constraint, there is virtually no difference in the

dynamics with different initial levels of household leverage. With the borrowing constraint, the economy with more leveraged households features a somewhat bigger impact effect of the shock. This is consistent, at least in a qualitative sense, with the empirical results in Lamont and Stein (1999) and Benito (2006). These studies find that a higher level of household leverage magnifies house price effects of income shocks. The figure also reveals that together with the borrowing constraint, a higher level of household leverage makes house prices converge more rapidly towards the steady state price. Hence, a higher level of household leverage magnifies the effects of the borrowing constraint on house price dynamics.

4.4   Effects of a marginal house price change

The multiplier mechanism highlighted in Stein (1995) relates to the fact that a fall in house prices may reduce buyer liquidity through the down payment requirement. The reduced liquidity may imply that over some price range, a fall in the house price induces borrowing constrained households to demand less housing. If this effect dominates in the aggregate demand response, a small change in house prices may lead to a jump in the aggregate demand. That could result in multiple equilibria. That is, there may be several house price levels that clear the housing market.

We analyze the possibility of multiple equilibria in the following way. First, we compute the equilibrium house price sequence following the temporary negative income shock studied in subsection 4.2. The house price dynamics related to this shock are shown in the top-left panel of Figure 6. Starting from the equilibrium price sequence, we lower period 1 house price by 1% while leaving all other prices unchanged, and solve again for households' housing demand. Of course, with this new house price sequence, the demand for housing will no longer equal supply in every period. We then compute how much the additional 1% reduction in period 1 house price changes the housing demand for different household types and age groups. This gives us a measure of the elasticity of housing demand for different household types and ages around the equilibrium price path.

Figure 9 shows the results for type 1 households of different ages in periods 1 and 2. We focus on type 1 households because they are the most likely to be borrowing constrained. Following the negative income shock, all type 1 households from model age 1 to 5 are borrowing constrained in period 1. We display the change in demand for periods 1 and 2 to highlight the delayed demand effect for borrowing constrained households.

To understand the figure, consider a household that is of model age 2 in period 1. The line with stars tells us that the further 1% reduction in period 1 house price increases housing demand of this household by about 0.1% in period 1. The same household is of model age 3 in period 2. The line with circles tells us that the period 1 price change induces this household to demand by about 0.6% more housing in period 2.
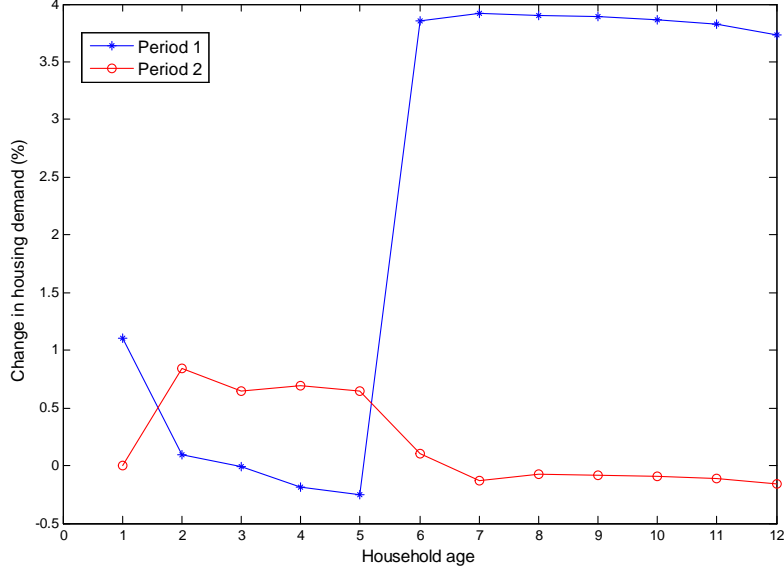
Figure 9: Changes in period 1 and period 2 housing demands following a 1% decrease in period 1 house price.

The figure shows that in period 1 the housing demand of young, borrowing constrained households increases much less than the demand of older, unconstrained households. Hence, the borrowing constraint does substantially reduce the price elasticity of housing demand. In fact, households of model age 4 and 5 demand less housing. For these age groups the liquidity effect indeed dominates. However, the reduction in demand is very small compared to the increase in unconstrained households' housing demand. In addition, consistently with our analytical results in Section 3, the figure also shows that the fall in period 1 house price induces borrowing constrained households to demand more housing in period 2. In equilibrium, this effect drives up the period 2 house price which works to increase also period 1 housing demand because of anticipated capital gains.

5 Conclusions

We have studied the importance of a borrowing constraint for house price dynamics with an OLG model. We found that a down payment requirement creates interesting non-linearites in house price dynamics. These non-linearities stem from changes in the proportion of borrowing constrained households.

We showed that the model can account for a large part of the huge house price fluctuations that were observed in the Finnish housing market from late 1980s to early 2000. More generally, our results are consistent with the observation that credit market liberalizations, which tend to imply a drastic relaxation of household credit constraints, are often associated with a housing boom.

In our model, relaxing the down payment constraint results not just in higher steady state house prices but also leads to substantial overshooting in house prices. In other words, absent other shocks, a relaxation of household borrowing constraints eventually leads to a long decline in house prices. This may explain part of the recent contraction in housing markets observed in countries that were characterized by a relaxation of credit constraints before the financial crisis that started in 2008.

On the other hand, according to our model, a down payment constraint does not substantially amplify the impact effects of adverse income or interest rate shocks on house prices. In other words, a down payment constraint does not help to explain housing busts. Rather, it works to speed up the recovery of house prices after adverse income shocks.

Appendix: Price changes and housing demand

In this appendix, we derive expressions for the marginal effect of an increase in current house price on current housing demand and savings when the household borrowing is unlimited and when the household faces a binding borrowing constraint.

No borrowing constraint

Let us write the household problem in (21)-(24) as

$$\max_{c,h,b} \left\{ u\left(c,h\right) + \beta V\left(b\right) + \lambda \left[y + ph_{-1} + Ra_{-1} - c - \left(p + \kappa p + \eta\right) h - \frac{b - p'h}{R} \right] \right\}.$$

The first-order conditions are

$$u_c - \lambda = 0$$

$$u_h - \lambda \left(p + \kappa p + \eta - \frac{p'}{R} \right) = 0$$

$$\beta V_b - \lambda \frac{1}{R} = 0$$

Combining the first-order conditions and using the budget constraint gives a system of three equations and three unknowns:

$$u_c - \beta R V_b = 0$$

$$u_h - P u_c = 0$$

$$y + ph_{-1} + Ra_{-1} - c - Ph - \frac{b}{R} = 0$$

where $P = p + \kappa p + \eta - \frac{p'}{R}$. Totally differentiating this system with respect to p gives

$$
\begin{pmatrix}
0 & u_{cc} & -\beta R V_{bb} \\
u_{hh} & -P u_{cc} & 0 \\
-P & -1 & -\frac{1}{R}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial h}{\partial p} \\
\frac{\partial c}{\partial p} \\
\frac{\partial b}{\partial p}
\end{pmatrix}
=
\begin{pmatrix}
0 \\
(1 + \kappa) u_c \\
-h_{-1} + (1 + \kappa) h
\end{pmatrix}
$$

We then have that

$$
\frac{\partial h}{\partial p} =
\frac{
\begin{vmatrix}
0 & u_{cc} & -\beta R V_{bb} \\
(1 + \kappa) u_c & -P u_{cc} & 0 \\
-(h_{-1} - (1 + \kappa) h) & -1 & -\frac{1}{R}
\end{vmatrix}
}{D}
$$

$$
= \frac{1}{D} \left[\beta R \left(V_{bb} (1 + \kappa) u_c + P V_{bb} u_{cc} (h_{-1} - (1 + \kappa) h) \right) + \frac{(1 + \kappa) u_c u_{cc}}{R} \right]
$$

where

$$
D =
\begin{vmatrix}
0 & u_{cc} & -\beta R V_{bb} \\
u_{hh} & -P u_{cc} & 0 \\
-P & -1 & -\frac{1}{R}
\end{vmatrix}
= \beta R V_{bb} \left(u_{hh} + P^2 u_{cc} \right) + \frac{1}{R} u_{hh} u_{cc} > 0
$$

And

$$
\frac{\partial b}{\partial p} =
\frac{
\begin{vmatrix}
0 & u_{cc} & 0 \\
u_{hh} & -P u_{cc} & (1 + \kappa) u_c \\
-P & -1 & -(h_{-1} - (1 + \kappa) h)
\end{vmatrix}
}{D}
$$

$$
= \frac{-u_{cc}}{D} \left[P (1 + \kappa) u_c - u_{hh} (h_{-1} - (1 + \kappa) h) \right]
$$

Borrowing constraint

Assume now that the household faces a binding borrowing constraint. This means that $a = -\theta p h$ and therefore

$$
b = (p' - R\theta p) h.
$$

The household problem in (21)-(24) can written as

$$
\max_{c,h} \left\{ u (c, h) + \beta V (b) + \lambda \left[y + p h_{-1} + R a_{-1} - c - (p + \kappa p + \eta - \theta p) h \right] \right\}
$$

where

$$
b = (p' - R\theta p) h.
$$

The first-order conditions are

$$
u_c - \lambda = 0
$$

$$
u_h + \beta S V_b - \lambda T = 0
$$

28

where $T = p + \kappa p + \eta - \theta p$ and $S = p' - R\theta p$. Combining the two first-order conditions and using the budget constraint gives two equations with two unknowns:

$$u_h + \beta S V_b - u_c T = 0$$

$$y + p h_{-1} + R a_{-1} - c - T h = 0$$

Totally differentiating the equations gives

$$\begin{pmatrix} u_{hh} + \beta V_{bb} S^2 & -u_{cc} T \\ -T & -1 \end{pmatrix} \begin{pmatrix} \frac{\partial h}{\partial p} \\ \frac{\partial c}{\partial p} \end{pmatrix} = \begin{pmatrix} \beta R\theta (V_b + S V_{bb} h) + u_c (1 + \kappa - \theta) \\ -h_{-1} + (1 + \kappa - \theta) h \end{pmatrix}$$

We have that

$$\frac{\partial h}{\partial p} = \frac{\begin{vmatrix} R\theta \beta V_b + S\beta V_{bb} R\theta h + u_c (1 + \kappa - \theta) & -u_{cc} T \\ (1 + \kappa - \theta) h - h_{-1} & -1 \end{vmatrix}}{D^c}$$

where

$$D^c = \begin{vmatrix} u_{hh} + \beta V_{bb} S^2 & -u_{cc} T \\ -T & -1 \end{vmatrix} = -u_{hh} - \beta V_{bb} S^2 - u_{cc} T^2 > 0.$$

Hence, we can write

$$\frac{\partial h}{\partial p} = \frac{-u_c (1 + \kappa) + u_{cc} T ((1 + \kappa) h - h_{-1})}{D^c}$$
$$- \frac{\theta u_{cc} T h}{D^c} + \frac{\theta u_c}{D^c} - \frac{R\theta \beta V_b}{D^c} - \frac{S\beta V_{bb} R\theta h}{D^c}.$$

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