Description of the model of firm dynamics with furloughs referred to in the VN-TEAS report no. 2024:28

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1 Introduction

This document describes the model of firm dynamics that was used in a VN-TEAS -project studying the Finnish furlough scheme. The results are reported in VN-TEAS -report no. 2024:28.

2 Model

We explore the aggregate effects of furloughs with the help of a firm dynamic model in the spirit of Hopenhayn and Rogerson (1993) with an additional element of frictional labor markets. There is an endogenous measure of heterogeneous firms, each producing a homogeneous good using decreasing returns to scale production function in labor. The productivity of the firms is stochastic, which creates a need for labor reallocation between firms. However, firms face adjustment costs if they want to permanently change their employment level. We allow these costs to be asymmetric, depending on whether a firm wants to increase or reduce its size. In addition, firms can also choose to furlough their workers within a period. This allows firms to temporarily reduce their labor inputs without firing costs. However, firms are required to take these workers back at the end of the period or pay the firing costs.

In modelling labor markets and unemployment, we follow Buera et al. (2015) and Buera et al. (2022). There is a centralized labor market from which firms can hire new workers with identical wage contracts. However, unemployed workers entry to this market is frictional and modelled with a reduced form matching function.

2.1 Household

There is a unit mass of infinitely lived workers who each belong to a large family and supply labor inelastically. The workers are heterogeneous in their labor market status as each worker can be either employed, unemployed or furloughed. However, unemployment compensation, paid also to furloughed workers, is equal to the equilibrium wage rate. The ompensation is financed with wage tax. This structure, which is similar to Buera et al (2022), allows us to abstract from the workers' side and focus on how the possibility of furloughs affects firms, aggregate output and total factor productivity.

The behavior of the household sector can be summarized by a representative household that derives utility from consumption. The household faces the following flow budget constraint

$$C_t = (1 - \tau_t) w_t N_t + b_t (U_t + F_t) + \Pi_t \tag{1}$$

where w_t is the wage rate, τ_t is the tax rate, b_t is the unemployment compensation, Π_t are aggregate profits and N_t, U_t and F_t give the amounts of employed, unemployed and furloughed workers. Moreover, aggregate consumption, C_t, is given by

$$C_t = \int_{\Omega_t} c_{t,i}, \tag{2}$$

where Ω_t gives the amount of firms at t. To ease the notation, we drop the time and firm indexes from now on.

2.2 Incumbent firms

There is an endogenous mass of incumbent firms denoted by Ω. Each firm produces a homogeneous good using the following production technology:

$$y = e^z n^\alpha, \tag{3}$$

where $0 < \alpha < 1$, n is the amount of employed workers in a firm, and e^z is the firm-specific productivity. We assume that z follows an AR(1) process:

$$z' = \rho z + \varepsilon, \tag{4}$$

where $0 < \rho < 1$ and the innovation term ε is distributed according to $\varepsilon \sim N(0, \sigma_\varepsilon^2)$.

A firm starts a period with employment level n_- decided in the previous period and it observes its current productivity z. If the firm wants to change its employment level, it needs to pay linear adjustment costs. We allow these costs to be different depending on whether employment is increased, in which case the cost is wc_{lh} per new worker, or decreased, in which case the cost is wc_{ls} per laid-off worker. This allows us to separate between hiring and firing costs in a parsimonious way. In line with Finnish labor market legislation, the firm can also furlough part (or all) of its employees. These employees do not work in the current period and the firm does not have to pay their wages. However, the firm has to take these workers back at the end of the period. The furlough costs are wc_f per worker. In our calibration $c_f < c_{lf}$, thus furloughs offer a cheaper way to temporarily reduce labor inputs. The firm also needs to pay fixed operation costs wc_o. Note that, all costs are scaled with the equilibrium wage, w. Finally, at the end of the period, the firm decides whether it wants to exit or stay in the market.

The firm's problem can be summarized by the following Bellman equation:

$$\begin{aligned} V(z, n_-) = \max_{n \in [0,\infty), f \in [0, n_-]} & e^z \underbrace{(n-f)}_{\equiv e}{}^\alpha - w(n-f) - wc_o - wc_f f \\ & - w\left(\mathbb{I}(n - n_- > 0) c_{lh}(n - n_-)\right) \\ & - w\left(\mathbb{I}(n - n_- < 0) c_{ls}(n_- - n)\right) \\ & + \frac{1}{1+r} \max\{EV(z', n - \theta f), -wc_{ls}(n - \theta f)\} \end{aligned} \tag{5}$$

This gives the optimal decision rules for employment $n(z.n_-)$, furloughs $f(z, n_-)$ and exit $x(z, n_-)$, taking value 1 if the firm decides to exit and value 0 if it decides to continue.

2.3 Entrants

There is a continuum of potential entrants that are ex ante identical. If they want to start producing in the next period, they have to pay the entry cost wc_e. They start the next period as an incumbent firm with $n_- = 0$ and productivity z that is drawn from a stationary

distribution associated with the AR(1) process. The mass of entrants is such that the expected value of entry is smaller or equal to the entry costs. We focus on a stationary equilibrium with positive entry, thus

$$\int V(z,0)G(dz) = wc_e. \tag{6}$$

2.4 Labor market

Firms and workers meet in a centralized labor market where all workers are paid the same equilibrium wage. However, following Buera et al (2015), not all unemployed workers are able to enter this market. The frictional entry is modeled with a reduced form matching function such that only fraction γ of the workers that either were unemployed at the beginning of the period or lost their job during the period are able to enter the labor market. A worker can become unemployed either because a firm decides to destroy jobs or because the worker was furloughed and lost the connection to the firm. We assume that furloughed workers move to unemployment with probability θ.

The amount of workers entering to the labor market, M, is then given by

$$M = \gamma(U + JD) + \theta F, \tag{7}$$

where U gives the amount (mass) of unemployed workers at the beginning of the period, JD gives the amount of newly unemployed workers resulting from firms' job destruction decisions. Moreover, F is the aggregate amount of furloughed workers. Job destruction is given by

$$JD = \int [\mathbb{I}(n(z,n_-) < n_-)|n(z,n_-) - n_-| + \mathbb{I}(\text{exit}(z,n_-))(n_- - \theta f(z,n_-))]\Phi(dz,dn_-), \tag{8}$$

where the first indicator function takes the value of one if a firm does reduce its size and the second indicator function takes the value of one if a firm decides to exit at the end of the period. $\Phi(dz,dn_-)$ gives the measure of firms over different productivity and employment levels at the beginning of a period. Note that in order to avoid double counting, we have subtracted the furloughed workers who lost the connection to their firm during the period. We can also define the aggregate amount of furloughs as

$$F = \int f(z,n_-)\Phi(dz,dn_-). \tag{9}$$

Finally, the steady state unemployment is given by

$$U = \frac{1-\gamma}{\gamma} JD. \tag{10}$$

2.5 Government

The government sets a labor tax so that it can pay the unemployed and furloughed workers an unemployment compensation that is equal to the equilibrium wage rate. Thus,

$$\begin{aligned} \tau w E &= (1-\tau)w(F+U) \\ \frac{\tau}{1-\tau} &= \frac{F+U}{1-U-F} \end{aligned} \tag{11}$$

2.6 Equilibrium

A stationary equilibrium can be defined with employment, $n(z, n_-)$, furlough $f(z, n_-)$ and exit, $x(z, n_-)$, policies; a wage rate, w; tax rate, τ; a mass of entrants, M_e; unemployment, furlough and job destruction rates; and a stationary distribution of firms, $\Psi(dz, dn)$ such that

1. given equilibrium the wage rate, the policy rules $n(z, n_-)$, $f(z, n_-)$ and $x(z, n_-)$ solve the firm problem

2. the stationary distribution of firms is given by

$$\Psi(Z', N) = \int_{(z,n)|n(z,n_-)\in N, z'\in Z'} Q(z, Z')(1-x(z, n_-))\Psi(dz, dn_-)+M_e\mathbb{I}(n = 0)\int_{z'\in Z'} G(dz'), \tag{12}$$

   where $Q(z, Z')$ is the transition function giving the probability of moving from z to Z', each (z, n_-) is such that $n(z, n_-) \in N$ and the last term gives the measure of entrants with $z' \in Z'$ given that $0 \in N$

3. unemployment is given by (10), job destruction is pinned down by (8) and the aggregate amount of furloughed is in line with (9)

4. the labor tax is set according to (11)

5. the mass of entrants is such that the labor market clears, that is the unit mass of workers is equal to the amount of furloughed and unemployed workers plus the labor demanded by the firms taking into account that labor is also used to pay adjustment costs, fixed costs and entry costs:

$$\begin{aligned} 1 = \int [&n(z, n_-) + c_o + c_f f(z, n_-) + \mathbb{I}(n(z, n_-) - n_- > 0)c_{lh}(n(z, n_-) - n_-) \\ &+ (\mathbb{I}(n(z, n_-) - n_- < 0)c_{ls}(n_- - n(z, n_-))]\Psi(dz, dn_-) + c_e M_e + U. \end{aligned} \tag{13}$$

3 Calibration

We calibrate parameters related to returns to scale and discounting externally. We set the model period to one year and assume 5% real interest rate. The returns to scale parameter is set to 0.66 to be roughly in line with the labor share. Finally, in Finnish administrative data covering nearly all workers, 10% of furloughed workers do not return to the firm in which they worked before their furlough spell. Based on this, we fix θ to 0.10.

We simplify our analysis by assuming that adjustment costs are symmetric, that is $c_{lh} = c_{ls}$. This leaves us with seven unknown parameters: the persistence of the productivity process, ρ, the variance of TFP shocks, σ^2_ε, entry costs, c_e, fixed costs, c_o, hiring/firing costs per worker $c_{ls} = c_{lh}$, furloughs costs per worker,c_f and the fraction of unemployed workers entering the labor market,γ. We calibrate these parameters internally by targeting the size distribution of firms (excluding the startups), the autocorrelation of employment, exit rate, employment turnover, the furlough rate and the unemployment rate, all calculated from the Finnish register data covering nearly all firms.

Table 1 collects the parameter values for our preferred calibration. The column 3 gives the empirical rationale for each parameter value. For the internally calibrated parameters, the mapping from between the targets and parameters is complicated, thus the table just gives the most important statistics around the preferred parameterization for each parameter. Table 2 shows the model fit relative to a number of empirical moments.

Parameter	Value	Explanation	Rationale
External:			
α	0.66	returns to scale	labor share
r	0.05	interest rate	real interest rate
θ	0.10	no recall after a furlough	Folk data
Internal:			
σ_ϵ	0.228	prod. shock size	targeted to size distribution [0.57, 0.2, 0.12, 0.07, 0.04]
f_e	19.522	entry cost	targeted to size distribution [0.57, 0.2, 0.12, 0.07, 0.04]
f_c	2.117	fixed cost	targeted to entry rate 7%
c_{lf}	0.160	furlough cost linear	targeted to furlough rate 8.5‰
$c_{lh} = c_{ls}$	0.378	firing and hiring cost linear	targeted to turnover of employment 21%
γ	0.466	match parameter	targeted to unemployment 8%
ρ	0.945	persistence of prod. process	targeted to autocorrelation of employment 0.96

Table 1: Simple calibration, match is not perfect, the size distribution fit can be seen from the result tables

Description	Data	Model
1-5 Employees	0.57	0.56
5-10 Employees	0.20	0.19
10-20 Employees	0.12	0.13
20-50 Employees	0.07	0.08
50- Employees	0.04	0.04
Turnover	0.21	0.22
Furloughed	0.0085	0.0091
Unemployment	0.08	0.08
Entry rate	0.07	0.065
Autocorrelation of Employment	0.96	0.926

Table 2: Model fit