Exercises with quantifiers

I. Prove in natural deduction for intuitionistic logic:

1. $\forall x \forall y A \supset \forall y \forall x A$
2. $\forall x(A \& B) \supset \forall x A \& \forall x B$
3. $\forall x A \supset \sim\exists \sim A$
4. $\forall x(A \supset B) \supset (\forall x A \supset \forall x B)$
5. $\exists x(A \& B) \supset \exists x A \& \exists x B$
6. $\forall x A \vee \forall x B \supset \forall x(A \vee B)$

II. Assuming that x is not among the free variables of B, prove in natural deduction for intuitionistic logic:

1. $B \supset\subset \forall x B$
2. $B \supset\subset \exists x B$
3. $\forall x(B \supset A) \supset (B \supset \forall x A)$
4. $\forall x(A \supset B) \supset\subset (\exists x A \supset B)$
5. $B \supset \forall x A \supset\subset \forall x(B \supset A)$
6. $\forall x A \& B \supset\subset \forall x(A \& B)$
7. $\exists x(A \supset B) \supset (\forall x A \supset B)$
8. $\exists x(A \& B) \supset\subset \exists x A \& B$
9. $\forall x A \vee B \supset\subset \forall x(A \vee B)$

III. Give examples justifying the variable restrictions in the rules $\forall I$ and $\exists E$.

IV. Prove $\Rightarrow A$ in **GOi** where A is as in Exercises I, II.

V. Show that the following are *not* derivable

1. $\forall x \exists y A \supset \exists y \forall x A$
2. $\exists x A \supset \forall x A$
3. $\exists x A \& \exists x B \supset \exists x (A \& B)$
4. $\forall x (A \vee B) \supset \forall x A \vee \forall x B$

VI. Find derivations of the following, both in natural deduction for classical logic and in the sequent calculus **G0c**

1. $\forall x A \supset\subset \sim\exists x \sim A$
2. $\exists x A \supset\subset \sim\forall x \sim A$
3. If x is not free in B, $\forall x (A \vee B) \supset A \vee \forall x B$
4. If x is not free in B, $(B \supset \exists x A) \supset \exists x (B \supset A)$

Which of the above also holds in intuitionistic logic?