Exercises for Chapter 8

1. Glivenko's theorem states that if a negated formula of propositional logic is derivable classically, it is derivable intuitionistically. It was proved in Chapter 5 for the classical single succedent calculus through an explicit proof transformation (theorem 5.4.9). Work out the proof for classical natural deduction.

2. Give a derivation of Ekman's formula $\sim(A \supset\subset \sim A)$ in classical natural deduction. Apply the transformation of exercise 1 to obtain an intuitionistic derivation.

3. Work out, similarly to exercise 1, the proof of the subformula property of Chapter 5 (theorem 5.4.12) for classical natural deduction.

4. It was noted in the end of Section 5.4.(b) that Peirce's law $((A \supset B) \supset A) \supset A$ is derivable with the rule of excluded middle restricted to A. Give a translation of the derivation to natural deduction. Can you prove directly that there can be no derivation of Peirce's law in intuitionistic natural deduction?