

## Exercises for chapter 5

1. Give sequent calculus proofs in **G0ip** of  $\Rightarrow A$ , where  $A$  is as in exercises I and II for chapter 1.
2. Prove  $\sim\sim((\sim A \supset B) \supset (A \vee B))$  in **G0ip** without using cut.
3. Prove  $\sim\sim(A \vee \sim A)$  in **G0ip** without using cut.
4. Show that each two-premiss rule of **G0ip** is interderivable with its context-sharing version, that is the one where the same context appears in both premisses and in the conclusion. (For instance, the context sharing version of  $R\&$  is

$$\frac{\Gamma \Rightarrow A \quad \Gamma \Rightarrow B}{\Gamma \Rightarrow A\&B}$$

and similarly for the other rules.)

5. *Context-sharing cut* is the rule

$$\frac{\Gamma \Rightarrow A \quad A, \Gamma \Rightarrow C}{\Gamma \Rightarrow C}$$

Show that in **G0ip** the rule of contraction can be derived by assuming the rule of context-sharing cut.

6. Show that the following inversions (extending those in Lemma 5.1.1) hold in **G0ip**:

(i') If  $\Gamma \Rightarrow A\&B$  is derivable, also  $\Gamma \Rightarrow A$  and  $\Gamma \Rightarrow B$  are derivable.

(iii') If  $\Gamma \Rightarrow A \supset B$  is derivable, also  $A, \Gamma \Rightarrow B$  is derivable.

Show with a counterexample that the corresponding inversion for  $R\vee$  does not hold.