Exercises for chapter 1

I. Prove in natural deduction for minimal logic (that is, with introduction and elimination rules for the connectives except $\bot E$):

1. $(A \supset B) \supset ((B \supset C) \supset (A \supset C))$
2. $(A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$
3. $(A \& B \supset C) \supset\subset (A \supset (B \supset C))$
4. $A \& B \supset\subset B \& A$ (commutativity)
   $A \vee B \supset\subset B \vee A$
5. $A \& (B \& C) \supset\subset (A \& B) \& C$ (associativity)
   $A \vee (B \vee C) \supset\subset (A \vee B) \vee C$
6. $A \supset\subset A \& A$ (idempotence)
   $A \supset\subset A \vee A$
7. $(A \& (A \vee B) \supset\subset A$ (absorption)
   $(A \vee (A \& B) \supset\subset A$
8. $A \& (B \vee C) \supset\subset (A \& B) \vee (A \& C)$ (distributivity)
   $A \vee (B \& C) \supset\subset (A \vee B) \& (A \vee C)$
9. $(A \supset B) \supset (\sim B \supset \sim A)$ (contraposition)
10. $\sim (A \& \sim A)$
11. $A \supset \sim\sim A$
12. $\sim\sim\sim A \supset\subset \sim A$
13. $\sim\sim (A \vee \sim A)$
14. $\sim (A \vee B) \supset\subset (\sim A \& \sim B)$ (de Morgan's laws)
    $\sim A \vee \sim B \supset \sim (A \& B)$
15. $A \vee B \supset \sim (\sim A \& \sim B)$
16. $\sim (A \& B) \supset\subset (A \supset \sim B)$

II. Prove in natural deduction for intuitionistic logic (that is, without excluded middle or its special case *reductio ad absurdum*):

1. $(A\& \sim A) \supset B$
2. $\sim(A \supset B) \supset \sim\sim A\& \sim B$
3. $\sim A \vee B \supset (A \supset B)$
4. $A \vee B \supset (\sim A \supset B)$

III. Prove in natural deduction for classical logic:

1. $\sim(\sim A\& \sim B) \supset A \vee B$
2. $\sim(A\&B) \supset \sim A\vee \sim B$
3. $A\&B \supset\subset \sim(\sim A\vee \sim B)$
4. $\sim(A\& \sim B) \supset\subset (A \supset B)$
5. $(\sim B \supset\sim A) \supset (A \supset B)$
6. $(A \supset B) \supset\subset (\sim A \vee B)$
7. $\sim\sim A \supset A$
8. $\sim(A \supset B) \supset A\& \sim B$
9. $((A \supset B) \supset A) \supset A$ (Peirce's law)
10. $(A \supset B \vee C) \supset (A \supset B) \vee (A \supset C)$ (disjunction property under hypothesis)

IV. (Gödel brain test) Prove in natural deduction for minimal logic, in less than 2 minutes for each direction

$\sim\sim(A\&B) \supset\subset \sim\sim A\& \sim\sim B$