

Exercises for Chapter 1

I Prove in natural deduction for minimal logic:

1. $(A \supset B) \supset ((B \supset C) \supset (A \supset C))$

$$\frac{\frac{\frac{[A]^1 \quad [A \supset B]^3}{B} \supset E \quad [B \supset C]^2}{\frac{C}{A \supset C} \supset I, 1} \supset E}{\frac{(B \supset C) \supset (A \supset C)}{(A \supset B) \supset ((B \supset C) \supset (A \supset C))} \supset I, 2} \supset I, 3}$$

8. $A \& (B \vee C) \supset (A \& B) \vee (A \& C)$

$$\frac{\frac{\frac{[A \& (B \vee C)]^2}{B \vee C} \& E \quad \frac{\frac{[A \& (B \vee C)]^2}{A} \& E \quad [B]^1}{\frac{A \& B}{(A \& B) \vee (A \& C)} \vee I} \& I \quad \frac{\frac{[A \& (B \vee C)]^2}{A} \& E \quad [C]^1}{\frac{A \& C}{(A \& B) \vee (A \& C)} \vee I} \& I}{\frac{(A \& B) \vee (A \& C)}{A \& (B \vee C) \supset (A \& B) \vee (A \& C)} \supset I, 2} \vee E, 1}$$

$(A \& B) \vee (A \& C) \supset A \& (B \vee C)$

$$\frac{\frac{[A \& B]^1}{A} \& E \quad \frac{[A \& B]^1}{B} \& E \quad \frac{[A \& C]^1}{A} \& E \quad \frac{[A \& C]^1}{C} \& E}{\frac{\frac{[A \& B] \vee [A \& C]}{A \& (B \vee C)} \& I \quad \frac{[A \& B] \vee [A \& C]}{A \& (B \vee C)} \& I}{\frac{A \& (B \vee C)}{(A \& B) \vee (A \& C) \supset A \& (B \vee C)} \supset I, 2} \vee E, 1}$$

16. $((A \& B) \supset \perp) \supset (A \supset (B \supset \perp))$

$$\frac{\frac{\frac{[(A \& B) \supset \perp]^3}{\perp} \supset I, 1 \quad \frac{[A]^2 \quad [B]^1}{A \& B} \& I}{\frac{B \supset \perp}{A \supset (B \supset \perp)} \supset I, 2} \supset E}{\frac{(A \& B) \supset \perp \supset (A \supset (B \supset \perp))}{(A \& B) \supset \perp} \supset I, 3}$$

$(A \supset (B \supset \perp)) \supset ((A \& B) \supset \perp)$

$$\frac{\frac{\frac{[A \supset (B \supset \perp)]^2}{B \supset \perp} \supset E \quad \frac{[A \& B]^1}{A} \& E}{\frac{\perp}{(A \& B) \supset \perp} \supset I, 1} \& E}{\frac{(A \supset (B \supset \perp)) \supset ((A \& B) \supset \perp)}{(A \supset (B \supset \perp)) \supset ((A \& B) \supset \perp)} \supset I, 2}$$

Exercises for Chapter 5

1. Prove in **G0ip**:

$$\text{I } 2. \Rightarrow (A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$$

$$\frac{\frac{\frac{\frac{\frac{B \Rightarrow B \quad C \Rightarrow C}{L\supset} \quad A \Rightarrow A}{B \supset C, B \Rightarrow C} \quad L\supset}{A \Rightarrow A \quad B \supset C, A \supset B, A \Rightarrow C} \quad L\supset}{(A \supset (B \supset C)), (A \supset B), A, A \Rightarrow C} \quad L\supset}{(A \supset (B \supset C)), (A \supset B), A \Rightarrow C} \quad \text{Contr}}{(A \supset (B \supset C)), (A \supset B) \Rightarrow A \supset C} \quad R\supset}{(A \supset (B \supset C)) \Rightarrow ((A \supset B) \supset (A \supset C))} \quad R\supset}{\Rightarrow (A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))} \quad R\supset$$