YLE13: Duopoly

Marko Lindroos

Research questions

- Market power and the formation of cartels (OPEC)
- Dynaamic game theoery models
- Nash equilibrium
- Stackelberg equilibrium(cartel leader, others followers)
- Stability? Individual countries in OPEC have an incentive to produce more

Duopoly

- Two identical firms, linear demand
- Max $\int_0^T e^{-rt}\left[p^b - b(q_1(t)+q_2(t))\right]q_i(t)dt$
- St $\dot{x}_i(t) = -q_i(t)$

Hamiltonian for firm 1

$$H_1 = \left[p^b - b(q_1(t) + q_2(t))\right] q_1(t) - m_1(t) q_1(t)$$

Maximum principle

$$\frac{\P H_1}{\P q_1} = 0 \text{ Þ}$$

$$p^b - 2bq_1(t) - bq_2(t) = m_1(t)$$

$$- \frac{\P H_1}{\P x_1} = \dot{m}_1(t) - rm_1(t)$$

$$m_1(t) = m_0 e^{rt}$$

Timepath of costate variable

- Backstop-price at T is equal to shadow price (see previous conditions)

$$p^b = \mu_1(T)$$

à $$\mu_1(t) = p^b e^{r(t-T)}$$

Reaction function

$$\Rightarrow \quad p^b - 2bq_1(t) - q_2(t) = p^b e^{r(t-T)}$$

$$q_1(t) = -\frac{p^b e^{r(t-T)} + bq_2(t) - p^b}{2b}$$

Extraction paths

$$q_1(t) = - \frac{p^b e^{r(t-T)} + b q_1(t) - p^b}{2b}$$

$$\frac{3}{2} q_1(t) = - \frac{p^b e^{r(t-T)} - p^b}{2b}$$

$$q_1(t) = \frac{p^b}{3b}(1 - e^{r(t-T)}) = q_2(t)$$

Time of exhaustion

$$p^b T_d - \frac{p^b}{r} + \frac{p^b e^{-rT_d}}{r} = \frac{3}{2} b x_0$$

Price path

$$p(t) = p^b - b(q_1(t) + q_2(t)) = \frac{p^b}{3}(1 + 2e^{r(t-T)})$$

N firms

$$q_1(t) = - \frac{p^b e^{r(t - T)} + (N - 1) b q_1(t) - p^b}{2b}$$

$$(\frac{N - 1}{2} + 1) q_1 = - \frac{p^b e^{- r(T - t)} - p^b}{2b}$$

$$q_1 = \frac{p^b}{(N + 1) b} (1 - e^{r(t - T)}) = q_2 = \ldots = q_N$$

$$p^b T_n - \frac{p^b}{r} + \frac{p^b e^{- r T_n}}{r} = \frac{1 + N}{N} b x_0$$