Invisibility cloaking and cosmology

Matti Lassas

University of Helsinki
in collaboration with:

Tracey Balehowsky, Pekka Pankka, Ville Sirviö

Slides available at
www.mv.helsinki.fi/home/lassas/

Invisibility cloaking was used to find a counterexample for the inverse conductivity problem in Greenleaf-L.-Uhlmann 2003. The example was based on a blow-up map $F : B(2)-\{0\} \to B(2)-\overline{B}(1)$.

Using a similar map, Pendry, Schurig and Smith (Science 2006) suggested invisibility cloaking for Maxwell's equations, see also Leonhardt (Science 2006).

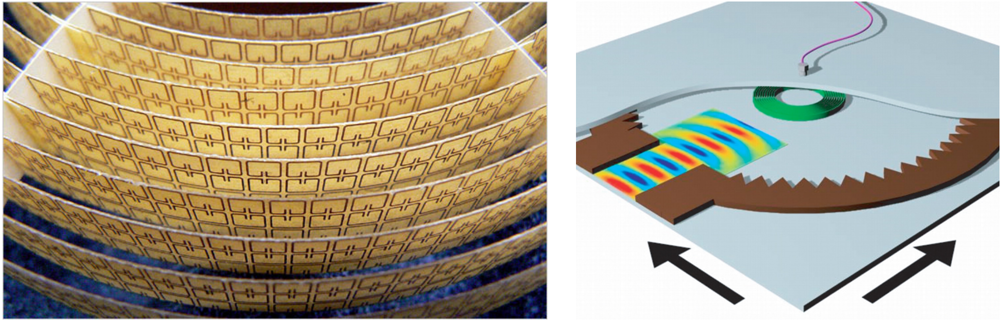
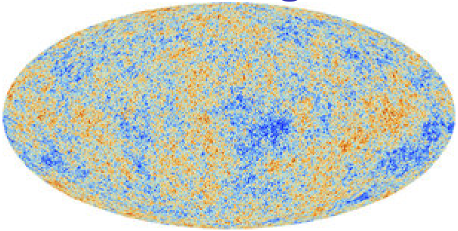
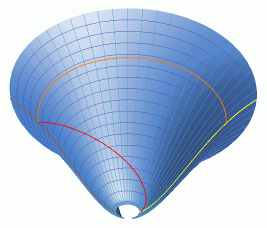


Figure: Implementation of an invisibility cloak for 4 cm waves build using metamaterials by Schurig et al (Science 2006).

Cosmic Microwave Background measurements

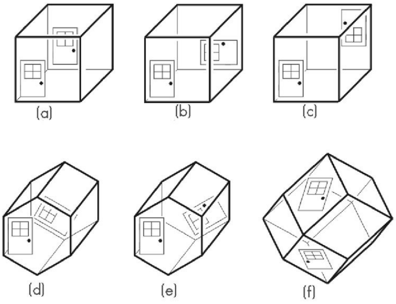




The Friedmann-Robertson-Walker model of the Universe is the space-time $M \times (0, \infty)$ with the metric

$$g = -dt^2 + a(t)\Big(\sum_{j,k=1}^{3} g_{jk}(x)dx^j dx^k\Big), \quad (x,t) \in M \times (0,\infty).$$

Here, M can be a compact or an unbounded manifold, $\dim(M) = 3$.
In 2008, J-P. Luminet suggested that the cosmic microwave background in our Universe (the WMAP data) is compatible with this model, where M is Poincare dodecahedral space $\mathbb{S}^3/\Gamma$. Here, Γ is the binary icosahedral group.



Pictures by Jean-Pierre Luminet.

The right picture shows the Poincare dodecahedral space $\mathbb{S}^3/\Gamma$.

Next we consider a metamaterial device that can simulate waves in the space-time $M \times (0, \infty)$ with a static metric $g = -dt^2 + g_M(x)$,
Here, M is a 3-dimensional manifold with a Riemannian metric g_M.

We proved an extension of the Lickorish–Wallace surgery theorem

Proposition

Let M be a smooth, closed, oriented, and connected Riemannian manifold of dimension n. When $n = 3$, there exists a subset $L \subset M$, having capacity zero and Hausdorff dimension $n - 2$, such that there is a C^∞-smooth embedding $F : M - L \to \mathbb{R}^3$.

When $n = 2$, this result is not true if the genus of M is non-zero.

Theorem (Cosmological cloaking)

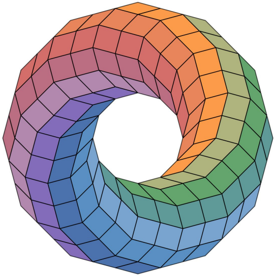
Let (M, g) be a smooth, closed, oriented, and connected Riemannian manifold of dimension 3 and $k^2 \notin \sigma(-\Delta_g)$.
Let $\widetilde{M} = F(M - L) \subset \mathbb{R}^3$ and $\widetilde{g} = F_* g$. Also, let $V \subset M - L$ and $\widetilde{V} = F(V) \subset \widetilde{M}$. Then $\Delta_{\widetilde{g}} : H^2(\widetilde{M}, \widetilde{g}) \to L^2(\widetilde{M}, \widetilde{g})$ is self-adjoint and the source-to-solution map on (M, g),

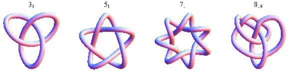
$$L_V(s) = u|_V, \quad (\Delta_g + k^2)u = s, \quad s \in C_0^\infty(V),$$

coincides with the source-to-solution map $L_{\widetilde{V}}$ on $(\widetilde{M}, \widetilde{g})$.

A similar result holds for the approximative cloaking.

A cosmological metamaterial device





Consider the interior of a twisted torus $T \subset \mathbb{R}^3$ and remove several knotted, twisted toruses $K_1, \dots, K_J$. This defines the domain $\widetilde{M} = T - (\bigcup_{j=1}^{J} K_j)$.

Make gratings on the boundary $\partial\widetilde{M}$ and cover it with the Schurig-type metamaterial. This construction gives a metamaterial device that simulates waves on the 3-manifold M.