# Enhancing the documentation by leaving useful traces

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# Enhancing the documentation



An appropriate documentation of the scientific work process

1. advocates the **reproducibility** of the research [1]
2. improves the **data quality** [2]
3. in general, helps to avoid reinventing the wheel

but an *enhanced documentation* also

4. supports **backtracking** and following **side tracks**
5. helps to get back on the track.

Instead of formal documentation, we focus on **traces** that are left behind when working.

# *Leaving useful traces*



Leaving **useful traces** [3] enhances the documentation.

Useful traces are, for example:

- **commands**, **expressions** and **work schemes** (created primarily to utilize operations of the software)
- **free-form notes** and **comments** (written down explicitly, preferably nearby the commands etc.)

They reflect the **thought process**, and help to retrace it later (say, when revising a paper after a review process).

# Demonstration with Survo



We will demonstrate these ideas especially with the matrix operations of **Survo** (SURVO MM) software [4].

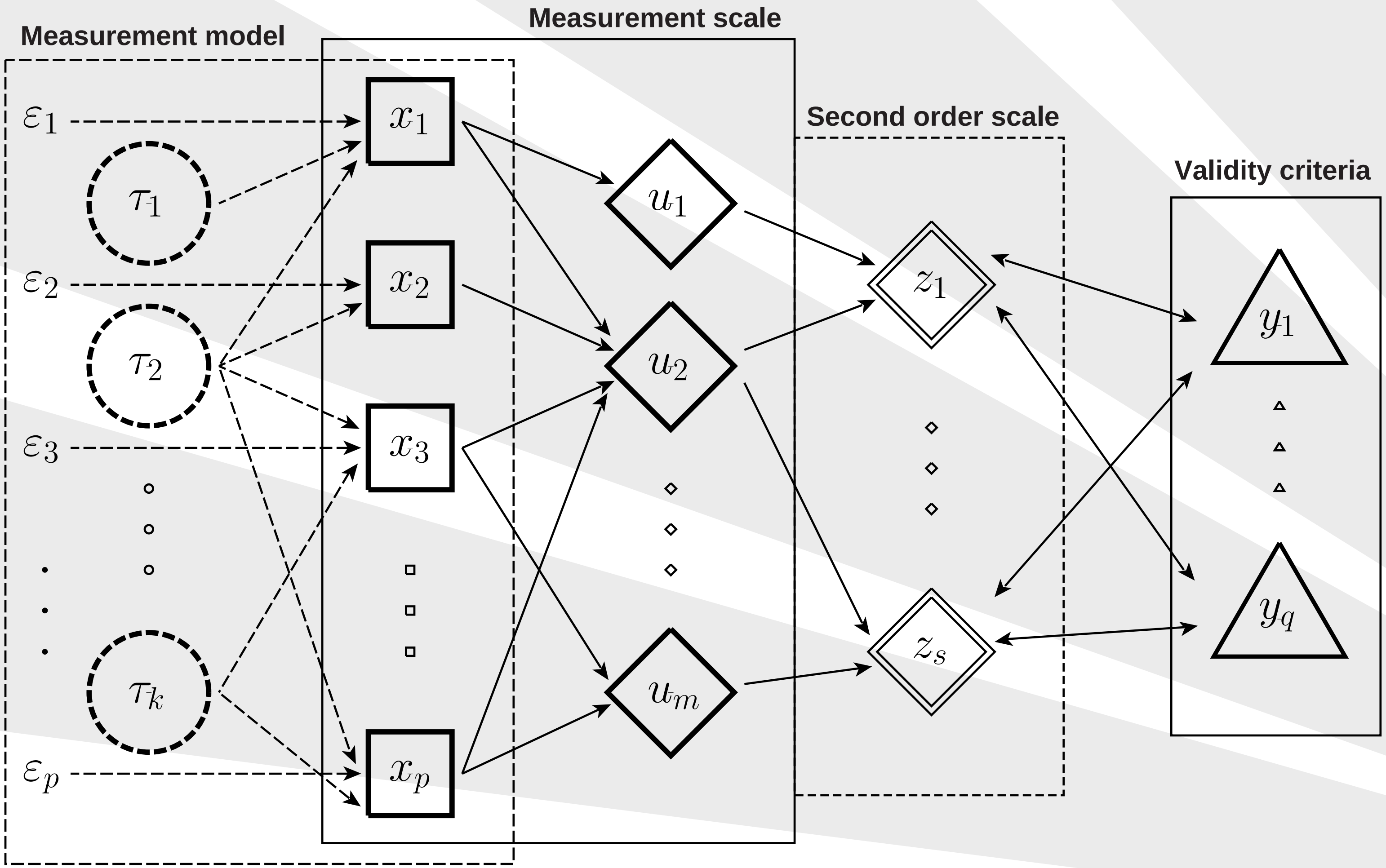
The unique *editorial interface* of **Survo** promotes building and maintaining the work process so that each step is appropriately documented.

Our examples come from certain new developments in the area of multivariate statistical modeling with **measurement errors**. The following slides offer a brief introduction to those issues.

# Measurement framework [5, 6]



- *guidelines of the study from the plans to the analyses*
- basis for a consistent assessment of **measurement quality**



# Measurement model



Formally: Let $\boldsymbol{x} = (x_1, \ldots, x_p)'$ measure $k$ $(< p)$ unobservable **true scores** $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_k)'$ with unobservable **measurement errors** $\boldsymbol{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_p)'$.

Assume $\mathrm{E}(\boldsymbol{\varepsilon}) = \mathbf{0}$, $\mathrm{cov}(\boldsymbol{\tau}, \boldsymbol{\varepsilon}) = \mathbf{0}$. The measurement model is

$$\boldsymbol{x} = \boldsymbol{\mu} + \boldsymbol{B}\boldsymbol{\tau} + \boldsymbol{\varepsilon}, \tag{1}$$

where $\boldsymbol{B} \in \mathbb{R}^{p \times k}$ specifies the relationship between $\boldsymbol{x}$ and $\boldsymbol{\tau}$.

Denoting $\mathrm{cov}(\boldsymbol{\tau}) = \boldsymbol{\Phi}$ and $\mathrm{cov}(\boldsymbol{\varepsilon}) = \boldsymbol{\Psi}$ we have

$$\mathrm{cov}(\boldsymbol{x}) = \boldsymbol{\Sigma} = \boldsymbol{B}\boldsymbol{\Phi}\boldsymbol{B}' + \boldsymbol{\Psi}, \tag{2}$$

where it is assumed that $\boldsymbol{\Sigma} > \mathbf{0}$ and $\boldsymbol{B}$ has full column rank.

# Measurement scale



The variables $\boldsymbol{x}$ are used in further analyses by creating **multivariate measurement scales**

$$\boldsymbol{u} = \boldsymbol{A}'\boldsymbol{x}, \tag{3}$$

where $\boldsymbol{A} \in \mathbb{R}^{p \times m}$ is a matrix of the weights, e.g., factor score coefficients or predetermined values according to a theory.

Using (2) we obtain

$$\operatorname{cov}(\boldsymbol{u}) = \boldsymbol{A}'\boldsymbol{\Sigma}\boldsymbol{A} = \boldsymbol{A}'\boldsymbol{B}\boldsymbol{\Phi}\boldsymbol{B}'\boldsymbol{A} + \boldsymbol{A}'\boldsymbol{\Psi}\boldsymbol{A}, \tag{4}$$

which gives (separately)
the (co)variances generated by the **true scores** and
the (co)variances generated by the **measurement errors**.

# Validity and reliability



In general, the sources of **uncertainty** in statistical research are:

- **sampling** (thoroughly known and handled)
- **measurement** (often neglected in statistics!)

1. **validity:** *are we measuring the right thing?*
   - closely connected to the substantial theory
   - within the measurement framework we can assess:
     (a) structural validity of the measurement model
     (b) predictive validity of the measurement scale
2. **reliability:** *are we measuring accurately enough?*
   - relevant: only if validity acceptable
   - definition: ratio of true variance to total variance
   - required: estimate of measurement error variance

# Reliability estimation



An estimate of reliability depends on the assumptions made about the **measurement model** and the **measurement scale**.

Several estimators suggested:

- most widely used: **Cronbach's alpha** [7]
  - based on Spearman's one-factor model ($>$100 years ago)
  - routinely used for $>$50 years (despite of criticism)
  - problem: underestimation (too strict assumptions)
- new, better alternative: **Tarkkonen's rho** [8, 5, 6]
  - based on measurement framework approach
  - realistic assumptions, well applicable in practice
  - also other research supports multidimensionality [9]

# Tarkkonen's rho & Cronbach's alpha



According to the definition of reliability, Tarkkonen's rho is obtained as a ratio of the variances, i.e., the diagonal elements of the matrices in (4). Hence we have

$$\begin{aligned} \boldsymbol{\rho_u} &= (\boldsymbol{A}'\boldsymbol{B}\boldsymbol{\Phi}\boldsymbol{B}'\boldsymbol{A})_d \times (\boldsymbol{A}'\boldsymbol{\Sigma}\boldsymbol{A})_d && (5) \\ &= \{\boldsymbol{I}_m + (\boldsymbol{A}'\boldsymbol{\Psi}\boldsymbol{A})_d \times [(\boldsymbol{A}'\boldsymbol{B}\boldsymbol{\Phi}\boldsymbol{B}'\boldsymbol{A})_d]^{-1}\}^{-1}. && (6) \end{aligned}$$

Cronbach's alpha is a special case of Tarkkonen's rho under a simple model $\boldsymbol{x} = \mathbf{1}\tau + \boldsymbol{\varepsilon}$ and with a simple scale $u = \mathbf{1}'\boldsymbol{x}$. It is easy to show that in this case, (5) or (6) lead to

$$\alpha = \frac{p}{p-1}\left(1 - \frac{\mathrm{tr}(\boldsymbol{\Sigma})}{\mathbf{1}'\boldsymbol{\Sigma}\mathbf{1}}\right) = \frac{p}{p-1}\left(1 - \frac{\sum_{i=1}^{p}\sigma_{x_i}^2}{\sigma_u^2}\right), \qquad (7)$$

which is the original form of Cronbach's alpha [7].

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