Demonstrations supporting physical concept formation – the inertial mass[1]

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The definition of quantities is discussed as a process of concept formation based on perception and measurement. Four components are distinguished in the definition: characterization, experimental definition, defining law and theoretical significance. They correspond to the four successive stages of the experimental approach in instruction where concept formation proceeds as a Gestalt perception process gradually from the observation towards higher hierarchical conceptual levels. The observational basis of this line of thought is discussed in general. It is noted that the physical system of quantities is a hierarchical net, in which the degree of the theoretical depth of the quantities gradually increases and which determines the natural order of the acquisition of the quantities. A series of demonstrations supporting the study of the inertial mass on the basic physics course at the university is presented.

1. Definition of quantities in physics

Quantities form the basis of concept formation in physics. The quantitative representation of natural phenomena is based on quantities. Experimental knowledge is expressed in terms of them. Theories deal with their interrelations. For the learning of physics it is therefore crucial that the meaning of the quantities is learned thoroughly right from the beginning.

The way in which the quantities are introduced or defined in the instruction is not at all immaterial. In the traditional formula-centred instruction the pupil gets easily the impression that quantities are introduced as almost arbitrary algebraic expressions of known quantities, which just "will prove to be useful". In this way, the rather common misconception arises that the quantities can be divided into two classes, the experimental and the theoretical ones.

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[1] In J. Laurén (ed.) Science education research in Finland. Yearbook 1987–1988. University of Jyväskylä. Institute for Educational Research. Publication series B. Theory into Practice **36**, 27–40.

The "purely experimental" quantities would actually include only the basic quantities of the system of units, which have an experimental definition. The others would be "theoretical quantities" defined algebraically. The pupil fails to realize the basic fact that all physical concepts are both experimental and theoretical. All quantities have an experimental basis. Their adoption is always based on a need to represent accurately some observed or experimentally verified properties of natural objects and phenomena.

The introduction and definition of every quantity involves the following necessary components:

1. The *characterization* tells which systems and phenomena the quantity is connected with and what kind of properties it describes.

2. The *experimental definition* indicates how the quantity is measured or what measurements are needed for its determination.

3. The *defining law* is the experimental law (invariance) which motivates the adoption of a new quantity, and expresses the property represented by the quantity in an accurate measurable form.

4. The *theoretical significance* indicates the position of the quantity in the cognitive structure of physics.

The definitions of quantities are always open. With the advancement of physics also the quantities develop. Their area of use expands and their meaning becomes more general. Each extension and generalization means some amendment to all four components of the definition, (see also Kurki-Suonio et al. 1985, 13–17).

In theoretical instruction it is possible to clarify this train of thought and this development, but the approach can be genuine and have concrete contents only, if the experimental basis is demonstrated or discussed in laboratory exercises.

2. Demonstration of the experimental basis of quantities

The four components of the definition of a quantity correspond to the four hierarchical levels of concept formation in physics (Kurki-Suonio & Kurki-Suonio 1989). Their order indicates the natural experimental approach in the introduction and treatment of a quantity in the lectures. Demonstrations and laboratory exercises supporting concept formation are connected with the first three stages.

The characterization is an operation on the hierarchical level of phenomena or qualitative knowledge. At this stage also the demonstrations have a qualitative role, they draw attention to those features and properties of the phenomenon, which require the adoption of the quantity for their representation.

The experimental definition and the defining law are coupled together on the quantitative level of the study of the phenomenon, where also the demonstrations or the pupils' laboratory exercises should be, at least, semiquantitative. The experimental work then includes, in principle, the following line of thought (Kurki-Suonio & Kurki-Suonio 1987a, 20):

1. The phenomenon is studied in a simple (idealized) situation, where the occurrence of the property considered is as clear and pure as possible.

2. The experimental law, obeyed in this situation (approximately) by the known varying quantities characteristic of the phenomenon is verified and expressed in the form of an invariance:

an invariant combination (expression) of the quantities = constant.

Here the invariance (constancy) means some definite, clearly verifiable independence of some parametric quantities, circumstances or the system.

3. On the basis of the law it is natural to introduce this invariant combination as a new quantity, characteristic of the phenomenon. The law becomes thus the defining law of the quantity, which motivates the adoption of the quantity and

gives it a *defining expression* as a derived quantity in terms of the known quantities. Thereafter the law can be expressed in a simple form:

the quantity = constant.

4. The resulting expression can then be used as the definition of the quantity in more general situations, where it is no more constant. In this way, the area of use of the quantity expands, the quantity becomes detached from its defining law, and it becomes possible to describe the phenomenon in more general terms with the aid of the quantity. The defining law becomes a simple algebraic model, representing the phenomenon in the validity area of the law.

This line of thought can most easily be recognized in a situation where some quantities are already known and the definition of the new quantity can be based on them.

For example, the definition of *resistance* is based on the already known electric current and voltage. When the current I in a resistor and the voltage U between its ends in a DC circuit are measured, the (approximate) proportionality $U \sim I$ can be verified. This is Ohm's law, which is valid for certain kinds of resistors and for sufficiently small currents at a constant temperature. Consequently, the component's ability to resist the flow of electricity can be represented by the invariant U/I, which is independent of the current. This defines a new quantity, the resistance R of the resistor. Ohm's law can now be expressed in the form R = constant. The expression U/I, or $\mathrm{d}U/\mathrm{d}I$, which is its equivalent in the validity area of Ohm's law, can be used as the definition of the resistance for resistors which do not obey Ohm's law, cf. Kurki-Suonio & Kurki-Suonio (l987b, 280–282, 286, 309–313).

As a second example we consider *velocity*. It is such a familiar quantity that one is not used to recognizing the law on which its adoption is based in the same way as in the case of resistance. The train of thought is, however, simple and elucidates clearly the principles involved in the definition of quantities.

It is noted that in certain simple motional phenomena the displacement of a body is (approximately) proportional to the time interval, $\Delta x \sim \Delta t$. As far as this law is valid, the ratio $\Delta x/\Delta t$ is invariant, independent of the time interval Δt. It is thus logical to introduce it as a new quantity, the velocity of the motion. The expressions $\Delta x/\Delta t$ and $\mathrm{d}x/\mathrm{d}t$, which are equivalent in the validity area of the defining law, are then generalized, in a well-known manner, into the average velocity and the instantaneous velocity representing motion more generally. The model, which this law defines, is the uniform motion.

3. The hierarchy of quantities and the order of their adoption

When the experimental approach is followed systematically (Kurki-Suonio & Kurki-Suonio 1987a) a picture of the *hierarchy of physical quantities* takes shape gradually. The introduction of a new quantity is based on certain former quantities and is possible only if these are already known. It is, however, not a question of an algebraic definition but of an experimental law, which can be verified by measurements of the known quantities and which determines the defining expression of the quantity. In this way chains and nets of quantities are formed possessing a definite direction and defining thus the order which should be followed in the adoption of quantities. Quantities which follow later in order are higher in the hierarchy, they have a higher degree of theoretical depth and a more structural nature.

Resistance lies rather high up in the hierarchy. There is a long chain of preceding quantities. It is based on the electric current and the voltage. The definition of the former is based on the force, and the latter on the potential energy, and both force and energy are already very theoretical structural quantities.

Velocity is down in the hierarchy. For its definition only the distance and the time interval are needed, which are among the very first basic quantities of physics.

Normally, little attention is paid to the definitions of the distance and the time interval as physical quantities, at least in the primary stage of the instruction. Therefore the pupils are easily left with the impression that just these two quantities should be regarded as so called experimental quantities. However, even they are based on laws. At the same time there is a deep theoretical idea involved, which is basically an assumption of the homogeneity and isotropy of the space-time.

The defining laws of the distance and the time interval belong among the basic invariances of physics:

1. The ratio resulting from a measurement of the length of a body with the aid of another body is invariant, independent of the position and orientation of the bodies as well as of the instant of time of the measurement.

2. The ratio of the numbers (of periods) of two periodic phenomena is invariant, independent of the time interval studied as well as of the location of the measurement.

Both of these laws deal with numbers, or ratios of numbers, which are determined experimentally. Thus, the "basic quantities" distance and time interval are based on an even more basic quantity, the number, which evidently is the "progenitor" of all quantities and is resting at the bottom of the hierarchy.

The *units* of length and time can be defined by choosing, in principle, quite arbitrarily one body and one periodic phenomenon as the reference body and phenomenon, the length and the period of which are the units.

Introduction of the distance, time and velocity need perhaps not be supported by thorough demonstrations. *Mass* and *force* are the first quantities encountered, the meanings of which are strange to the pupils. The words are familiar but their physical meaning differs from what one expects on the basis of their uses in the standard language. That is why they have become such eternal stumbling blocks in the instruction of physics. The whole conceptual system of physics, however, rests on them. The experimental basis of the adoption of force has been discussed extensively also in Finland and special demonstrations have been proposed to support it. (See e.g. Seinelä 1989).

4. The experimental foundation of the inertial mass

The term mass has two different meanings in classical mechanics: the inertial mass and the gravitational mass. They represent two different basic properties of a body, and their definitions are based on different laws, they are different quantities with regard to their experimental foundation.

The inertial mass represents the inertia or the difficulty of changing the state of motion of a body. The basic observation is that in interactions e.g. in the collision of different bodies the velocities of the bodies change differently. This difference is the clearer the greater the difference in the "size" of the bodies. "Larger" bodies have a larger inertia.

In order to find a quantity for the representation of the inertia one can study more closely the velocity changes of colliding bodies. The experiments may be arranged according to the following line of thought (Kurki-Suonio & Kurki-Suonio 1987b, 97–100):

1. The velocity changes of the colliding bodies P and Q are measured and the proportionality $\Delta \boldsymbol{v}_P \sim -\Delta \boldsymbol{v}_Q$ is verified. It is essential that, in addition to the initial velocities of the bodies, also the nature of their interaction (soft ... hard surfaces, inelastic ... exoenergetic collisions, rotating ... non-rotating bodies) is varied in the experiment.

According to results, the ratio $|\Delta \boldsymbol{v}_P|/|\Delta \boldsymbol{v}_Q| = k_{PQ}$ is an invariant, independent of the velocities of the bodies and of the nature of their interaction. It can, thus, be introduced as a new quantity, which is a positive constant characteristic of the pair of bodies (P, Q) and which expresses quantitatively the inertial ratio of the bodies. If $k_{PQ} = 1$, the bodies have equal inertia, if $k_{PQ} > 1$, P has greater inertia than Q. From the definition of the quantity it follows that $k_{QP} = 1/k_{PQ}$.

2. When the inertial ratios of different pairs of bodies are studied, it can be verified experimentally, that the "connective law" $k_{PQ}/k_{RQ} = k_{PR}$ holds for any three bodies P, Q and R.

On the basis of this law it is possible to interpret the inertial ratio k_{PQ} of a pair of bodies as the ratio of the inertias of the bodies. By making bodies collide with the same chosen reference body O, one can measure the body-specific constants $k_P = k_{PO}$, $k_Q = k_{QO}$, the ratio of which expresses the pair-specific inertial ratio: $k_P/k_Q = k_{PQ}$. The constant k_P is, however, the smaller the larger the inertia of the body P. A proper quantity for representing the inertia would be the inverse $m_P = 1/k_P = 1/k_{PO}$, yielding the expression $k_{PQ} = m_Q/m_P$ for the inertial ratio.

The quantity thus obtained is the inertial mass of a body. The reference body O acts as a definition of the unit of the inertial mass. On the basis of these considerations one can write

$$m_P = (1/k_{PO})\, m_O \,,$$

where the ratio $1/k_{PO} = |\Delta \boldsymbol{v}_O|/|\Delta \boldsymbol{v}_P|$ can be understood as the numerical value of the inertial mass and m_O as the unit, which carries the (possible) dimension attached to the quantity.

3. The study can be completed by joining the bodies P and Q and by determining, through collision experiments, the inertial mass of the combined body (P+Q), $m_{(P+Q)} = 1/k_{(P+Q)O}$.

In this way one can verify the additivity of the inertial mass

$$1/k_P + 1/k_Q \;=\; 1/k_{P+Q} \quad \text{or} \quad m_P + m_Q \;=\; m_{P+Q}$$

as an experimental law.

These considerations show that only the kinematic quantities based on the concepts of distance and time interval and, of course, the inertial frame of reference, based on the law of inertia, are needed as the basis of the definition of the inertial mass. Therefore the position of the inertial mass in the hierarchy of quantities is immediately after these quantities. Especially, it precedes the impulse and the force, which become possible to define only when the inertial mass is known.

With the aid of the inertial mass, thus defined, the initial experimental law can be written in the form $m_Q\Delta\mathbf{v}_Q = -m_P\Delta\mathbf{v}_P$, which offers the opportunity to introduce the impulse as a quantity describing the strength of the collision. Changing the collision into a "softer" interaction and using very short time intervals in the determination of the velocity changes leads correspondingly to the instantaneous form of the law $m_Q\mathbf{a}_Q = -m_P\mathbf{a}_P$, where the velocity changes have been replaced by accelerations.

5. The demonstration apparatus

In order to demonstrate the above line of thought the apparatus shown in Fig. 1 was constructed. The parts seen in the figure are: air track (1), blower of the track (2), cars (gliders) (3), stands for the reflection sensors (4), specialized measuring computer VELA (Versatile Laboratory Aid, Educational Electronics) (5), coupling unit (6) and microcomputer OSBORNE 5T (7).

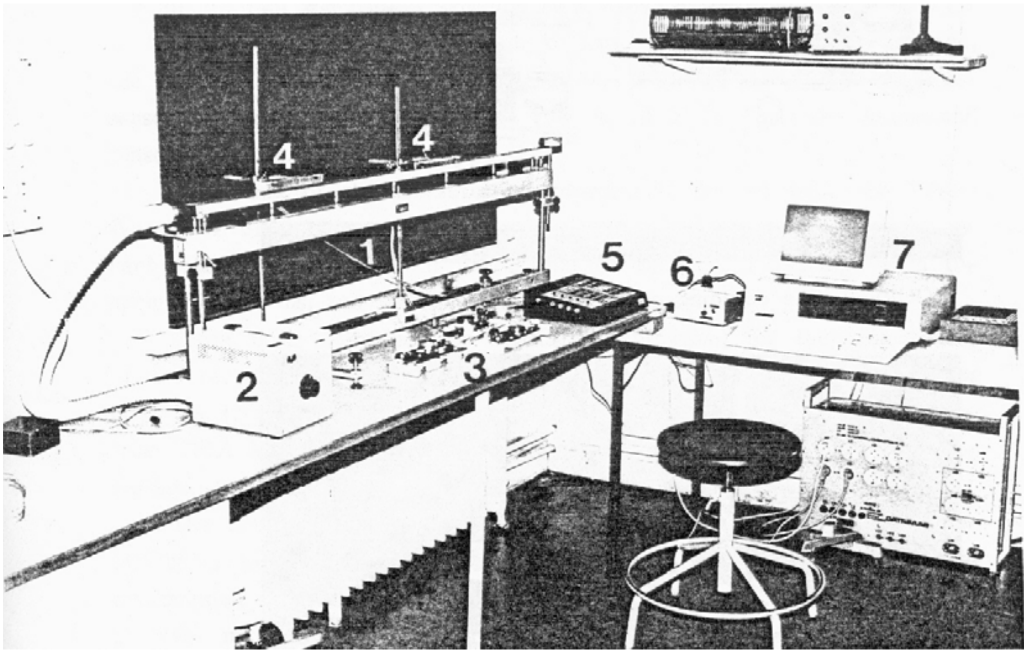

FIGURE 1. The demonstration apparatus

The motion of the cars (Fig. 2) on the air track is observed with the aid of optical reflection sensors. A sensor consists of an infrared LED (Light Emitting Diode) and a phototransistor. They are incased so that infrared light reflecting from a mirror in front of the sensor switches the transistor into the conducting state. When the mirror is removed, the transistor ceases to conduct. Eight sensors in all were attached to two stands at two different levels (Fig. 3). Each car carried a mirror slab attached so that any two cars simultaneously on the track had their slabs at different heights and there were, thus, four sensors observing the motion of each car.

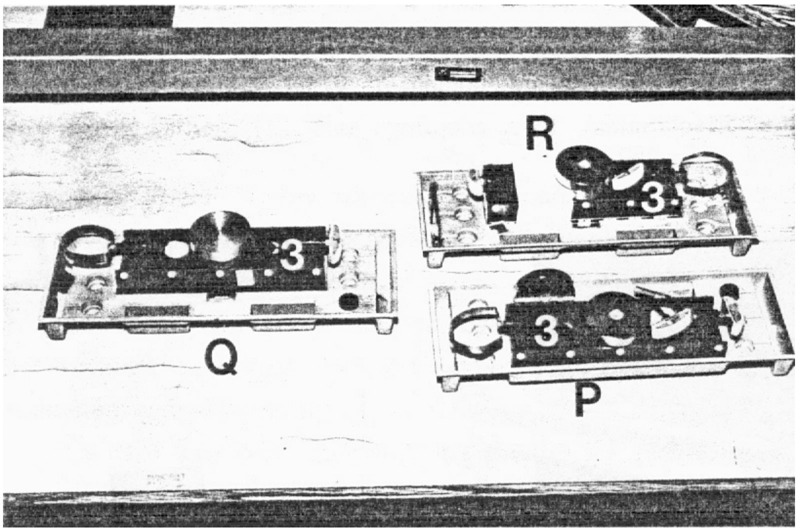

FIGURE 2. The colliding cars

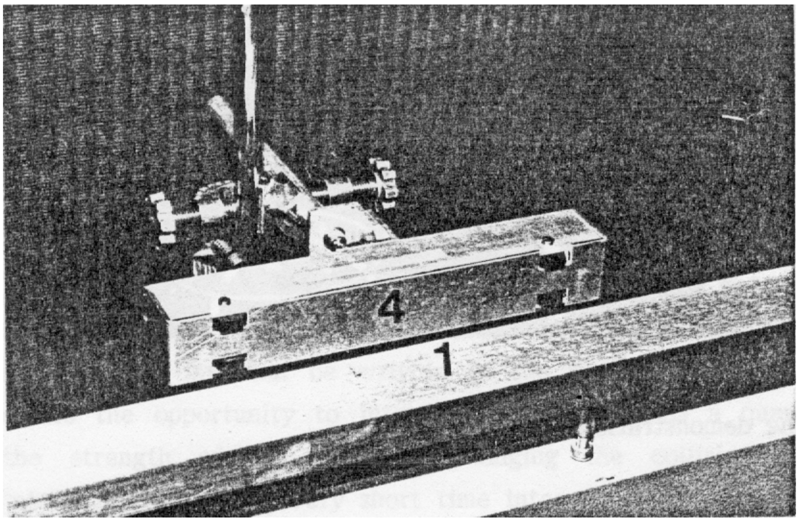

FIGURE 3. A reflection-sensor stand

The coupling unit receives the voltage signals coming from the preamplifiers of the reflection sensors and sends them on to VELA. The signals pass through the 8-channel buffer amplifier, keeping the signals on TTL-level (Transistor-Transistor Logic) and protecting, thus, VELA against possible excess voltages. The coupling unit contains a stabilized 5 V source for the LEDs of the sensor and for the buffer amplifier.

VELA is a microprocessor-controlled versatile measuring and data-recording device, used in this context as a multi-channel timer. There is a multi-channel timer program ready in the ROM (Read Only Memory) of VELA. When the program is switched on, VELA starts to monitor its digital input gate, where the signals are coming from the sensors. VELA is thus reading parallel-form 8-bits bytes with the values of the bits corresponding to the states of the sensors. The first four bits correspond to the car A, the last four bits to the car B. The number 00000000 corresponds to the normal state. When a car comes to a position opposite one of its four monitoring sensors the corresponding bit takes the value 1. When the car has passed the bit returns to the state 0. For instance, the number 00010000 tells, that the car A is opposite the rightmost sensor, the number 00000100, that the car B is in front of the second sensor from the left.

The clock of the measurement program is started from the STAR-switch of VELA. From that instant on, VELA records in its memory, for each change of a bit, both the instant of time of the change with the accuracy of 1 ms, and the resulting value of the bit.

There is in VELA a data-transmission program, for the transmission of the results of the measurements into the microcomputer. The computer must be programmed to receive the data, i.e. to save the bits coming from VELA in its own memory and to produce the necessary hand-shaking signals.

The data were transmitted from VELA to the microcomputer with the aid of a parallel-port card controlled by a (machine language) data-transmission program, saving the information sent by VELA in the memory of the microcomputer. A program in BASIC was written for the calculation of the initial and final velocities of the cars, the velocity changes and the ratios of the changes. In addition, the program records the directions of the motion of the cars, for instance, "the car A came from the left and went to the right".

6. Notes on the accomplishment of the demonstration

The apparatus presented above allows demonstration of the experiments illustrating the experimental basis of the inertial mass in one dimension. For this, three gliders P, Q and R (at the least) are needed, one of which (Q) shall be chosen as the reference (unit) body. The cars are made to collide in pairs.

At least for one of the pairs (P, R) a series of collision experiments are made, where the initial velocities and the buffers of the bodies are varied in order to verify the invariance of the ratio $|\Delta v_P|/|\Delta v_Q|$. For the other pairs a short series of careful experiments will suffice.

In this way the pair-specific inertial ratios k_{PR}, k_{PQ} and k_{RQ} are determined for verification of the law $k_{PR}/k_{QR} = k_{PQ}$, and the inertial masses are determined, with the inertial mass of the car Q as the unit, $m_P = (1/k_{PQ})m_Q$ and $m_R = (1/k_{RQ})m_Q$. In addition, the combination (P+R) is made to collide with the reference car in order to determine the inertial ratio $k_{(P+R)Q}$ and the mass $m_{P+R} = (1/k_{(P+R)Q})m_Q$ and to verify the additivity of the mass, $m_P + m_R = m_{P+R}$.

In a test measurement, the weighed masses of the cars were m_P = 197,6 g, m_Q = 242,6 g, m_R = 303,8 g. Two kinds of buffers were used, some elastic, others inelastic. They were kept as permanent parts of the cars, only turned to the proper position to accomplish the desired kind of collision.

Inertial ratios	k_{PQ}	k_{RQ}	k_{PR}	$k_{(P+R)Q}$
	1,29	0,87	1,53	1,97
	±0,05	±0,06	±0,08	±0,08
Connective law	$k_{PQ}/k_{RQ} : k_{PR} = 0,97$			
Inertial masses/m_Q		m_P	m_R	m_{P+R}
		0,78	1,15	1,97
Additivity	$(m_P+m_R)/m_{P+R} = 0,98$			
Weighed mass ratios		m_P/m_Q	m_R/m_Q	$(m_P+m_R)/m_Q$
		0,81	1,25	2,07

Table 1. Results of a test measurement.

Results of the test measurement are presented in Table 1. The error limits of the inertial ratios are standard deviations of rather short series of 4 to 8 experiments.

The test represented a level of carefulness typical of pupil's laboratory exercises or of a quick demonstration. With the apparatus used it is relatively easy to reach an accuracy of 5 % in a single experiment. Very careful work improves the accuracy to 1 %, which is sufficient to give a fairly convincing picture of the laws.

The technical restrictions and difficulties of the experiment are typical of the use of the air track. The collisions are difficult to carry out so that the cars do not touch the track or start to vibrate. The "harder" the interaction of the cars is, the more difficult this becomes. Particularly, the experiments with the present inelastic buffers were difficult, since the collisions were rather abrupt. Even with the elastic buffers it was difficult to arrange successful collisions if the relative initial velocity of the cars was larger than 0,5 m/s.

The possibilities to vary the masses of the cars are very limited. The absence of friction in the motion and particularly the success of the collision are very sensitive also to changes in the geometry of the cars.

The most critical part of the apparatus, from the point of view of the student laboratory, is the air track itself. To keep it clean, dustless and fully non-distorted (in the circumstances of the student laboratory), which is absolutely necessary in any quantitative experiments, is difficult. The condition of the track must be checked regularly.

So far, there is little experience of performing the presented experiments as lecture demonstrations. The reason is that there is no proper room for the preparation of the demonstrations in the vicinity of the lecture halls and tuning up the apparatus for a quick demonstration on the spot is too time-consuming. In practice conditions, it has been possible to carry through a complete set of measurements in about 20 minutes. For quickness, the automation of the measurements and of the treatment of results is, of course, decisive. The most obvious defect from the point of view of lecture demonstrations is the small size of the microcomputer display.

The experiment was adopted as a laboratory exercise for the first ordinary physics course (Peruskurssi I) in the autumn of 1987. As a studentexercise the experiments take about an hour or slightly more, when well tutored. The introductory presentation of the apparatus takes five to ten minutes.

The first experiences have been encouraging. The feedback of the initial stage has been positive. The exercise has a close connection with thoughts presented in the lectures and in the textbook, and the modern technology applied has made it attractive to the students. So far, this is the only laboratory exercise on the basic physics courses where microprocessors are used.

In principle, the unidimensionality of the experiment is a deficiency. The idea of the experiment can certainly be realized also in two dimensions using an air table. This would increase decisively the possibilities of varying the nature of the collision. For example, one could use more irregular and rotating bodies. The universality of the laws studied would be emphasized while the vector character of the velocity changes would become obvious. However, it is not easy to find a cheap enough solution for the realization of an automatic velocity measurement, which is necessary for these experiments to be done as demonstrations or students' laboratory exercises.

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