PROCEDURES OF EMPIRICAL CONCEPT FORMATION IN PHYSICS INSTRUCTION

Ari Hämäläinen[*], Kaarle Kurki-Suonio[*], Jukka Väisänen[*], Jari Lavonen[**]

[*]Department of Physics, [**]Department of Teacher Education, University of Helsinki, Finland

ABSTRACT

The problem of learning physics can be identified with the problems of introducing physical concepts, particularly teaching and studying physical quantities, because understanding physics is mainly based on understanding the meanings of its concepts. This study presents examples of the actual procedures of physics teaching based on empirical concept formation. Emphasising the importance of empirical concept formation in physics at procedural level of teaching, leads to certain guidelines for experimental instruction: Introduction of a new concept must be well motivated, the hierarchy of concepts defines the order in which the topics can be learned, and quantities are introduced via quantifying experiments. Our first example is about mechanics. It starts with qualitative experiments, in order to recognise and classify basic entities, phenomena and their properties, and to find dependencies between inertia, magnitude of change in motion and magnitude of interaction. These findings motivate the quantifying experiments, which are first presented for *velocity* and *mass*. *Linear momentum* and *impulse* are defined via structurisation. Finally, quantifying experiments are presented for *acceleration* and *force*. Our second example about electricity is available via Internet.

1. INTRODUCTION

This paper provides practical examples of implementing the processes of teaching that are presented in a contribution by Koponen et al [1]. Therefore, the processes are introduced here only briefly.

Perception: Concept formation starts from the level of qualitative information, *by perception*, and builds up basic meaning schemata by recognition and classification of phenomena and their relationships. In this process the *meaning is created first* through experimentality in form of sensory experiences, observation and qualitative experiments. A *class of meaning schemata* is thus formed, which include not only entities (objects), but also phenomena, and their properties (qualities).

Prequantification. Properties have magnitudes. This makes it possible to *compare* the degrees of the same property of different objects or phenomena, and to observe which properties stay constant, which change, and how they change, in a specific physical phenomenon.

Quantification creates a quantity from a property (quality). Quantification is based on experiments, which verify the defining law of the quantity, and also tell how the quantity can be measured. These are called *quantifying experiments*. Their design is based on prequantification.

Quantification leads to formation of quantities, which are *quantitative representations* for properties. In this process quantities also acquire numerical values and units.

The mutual dependency of the properties becomes represented as relations between quantities, and these representations are *laws*. Therefore, the definition of every quantity is tied on laws and on the process where the laws and quantities are created. The defining law is always a conservation law, i.e. a well-defined invariance between some factors that affect the phenomenon. These factors, and the conditions in which the property that is to be quantified stays constant, are recognised by the preceding preqantification. Invariances may exist in different forms, but every defining law can be expressed as invariance. Therefore, *quantities are essentially invariants*.

Structurisation includes the formation of theories from qualitative meaning schemata.

We use the term perceptional experimentality for the use of experiments in physics instruction in a way that supports the processes described above.

2. GUIDELINES OF EXPERIMENTALITY

Implementing perceptional experimentality at procedural level leads to certain guidelines for experimental instruction.

- Introduction of a new concept must be well motivated.
- The hierarchy of concepts defines the order in which the topics can be studied. This offers students a possibility to join new concepts to earlier ones. The hierarchy is flexible for the qualitative concepts, but quite rigid for the quantities, laws, and theories.
- Quantities are defined via quantifying experiments. Every quantity is introduced as a quantitative representation of a property of either an entity or a phenomenon.

To learn the practice of perceptional experimentality is the major goal of the laboratory courses offered for the pre-service and in-service teachers at the University of Helsinki, Department of Physics. Feedback from the schoolteachers indicates that these ideas really work for them, and help their students to learn physics better.

3. EXAMPLE 1: MECHANICS

3.1. Curriculum

Applying the guidelines to mechanics leads into a curriculum that is different from a traditional one. Kinematics and dynamics are not discussed separately. This comes from the fact that there is no motivation for, say, studying acceleration before the concept of uniform interaction has been discussed. From the requirements of the hierarchy of concepts follows that inertial mass is defined by the 'Machian' approach with collision experiments [2, 3], and that linear momentum and impulse are studied before force.

3.2. Perception

Mechanics is started with qualitative experiments, in which the elementary concepts are recognised. These are the basic entities and phenomena (bodies, motions, interactions), and their properties (inertia, magnitude of body's motion, magnitude of change in body's motion, magnitude of interaction). Interaction is recognised as the cause of change of motion. Various types of contact and remote interactions (collision, friction, resistance of medium, gravity, electrostatic, magnetic…) should be introduced. Whenever possible one should be able to see or feel how the interaction affects on both parties. The vector nature of interactions is recognised by studying situations where an interaction makes a body to start moving, accelerate, turn, and slow down.

3.3. Prequantification

Next, the dependencies between the basic properties (inertia, magnitude of change in motion, magnitude of interaction) are studied with prequantitative experiments. Kinesthetics carts are excellent for these experiments. A cart can be loaded with varying number of weights or passengers. The one who pushes the cart going, or stops it, feels the magnitude of interaction in his or her own hands. Using two carts with passengers who push the carts apart, it is found that the interaction always affects on both carts, regardless of whether both or only one passenger is pushing. With deforming bumpers, one finds that the interaction affects both parties equally, regardless of their inertias. By varying the number or size of passengers, one sees that for the body with larger inertia, an equal interaction produces a smaller change in motion than for the body with smaller inertia.

Situations where there are no interactions that would change a body's motion, or the effects of the interactions cancel each other out, are studied. It is found that in these cases the motion of the body does not change.

Next, to turn the study quantitative, one must define quantities that represent the basic properties.

3.4. Quantification of velocity

It has been found that if no interaction affects a body in the direction of its motion, the motion does not change. Since quantities are born as invariants, uniform motion is needed to define the quantity that describes the magnitude of motion. For this, one needs an idealised set-up where the interactions that could change the motion are eliminated as well as possible. An air track or a low-friction wheeled chart is suitable for laboratory experiments, but a bicyclist freewheeling on a level road will also do. The position of the body is recorded as a function of time. From the results, an (t,x) graph is drawn (Figure 1). A computer-based measurement system has the obvious advantage of being able to produce the graph in real time. The (t,x) points are found to lie on a straight line. The experiment is repeated by varying the swiftness of motion. One finds that in every case, the (t,x) points lie on a straight line, whose slope depends on the swiftness of motion. I.e. the ratio $\Delta x/\Delta t$ is an invariant that is characteristic for every uniform motion. A new quantity, *velocity*, can therefore be defined as the slope of the line: $v = \Delta x/\Delta t$.

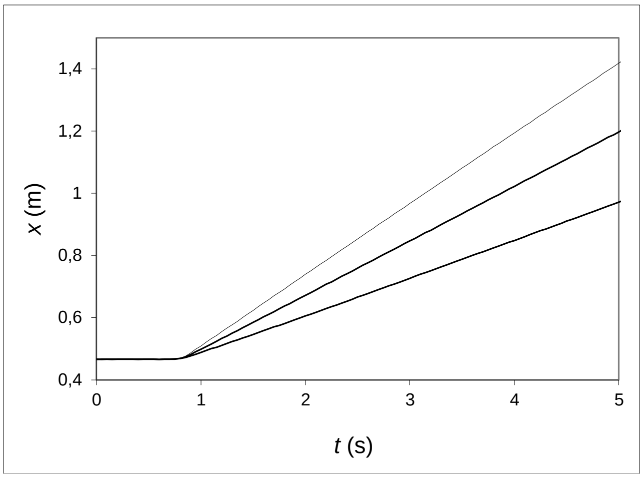

Figure 1. Position versus time graphs of uniform motion.

3.5. Quantification of inertial mass

The prequantitative experiments indicate that in situations where the interaction between two bodies is the only interaction changing their motions (e.g. collision), the magnitudes of changes in motion depend on the inertias of the bodies. Studying the changes of the bodies' velocities should then provide a way of defining a quantity that represents the inertia of body. It is clear that the quantity should depend on the body only, not for example on the interaction. Intuition says that the quantity should also be additive: inertia of two identical bodies attached together is obviously twice the inertia of one of the bodies.

An ideal collision set-up on an air track is used. Two gliders A and B are collided, and their position is recorded as a function of time. The velocities before and after the collision are measured, by varying the initial velocities, and varying also the elasticity of the collision. The changes in velocity Δv_A and Δv_B are found to always have an opposite sign. One also sees that if the inertia of, say, glider A is larger, that $|\Delta v_A|$ is always smaller than $|\Delta v_B|$.

A $(\Delta v_A,-\Delta v_B)$ graph is drawn (Figure 2). One finds that the points lie on a straight line. The slope of the line, $k_{A,B} = -\Delta v_B/\Delta v_A$, is the ratio of the inertias of the bodies, or the inertia of A measured with the inertia of B. One must note that $k_{A,B}$ describes a property of a *pair* of bodies, not a property of a *single* body. To generalise a quantity for single bodies, one must show that its value does not depend on the body that is used for comparisons.

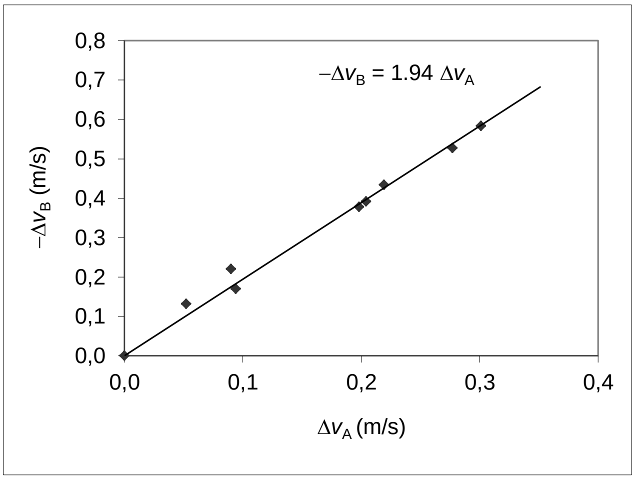

Figure 2. Ratio of changes in velocity of glider pair A,B.

A third glider C is collided with A and B. Ratios $k_{A,C}$ and $k_{B,C}$ are defined. One finds that $k_{A,B} = k_{A,C}/k_{B,C}$, which means that the ratio of the inertias of A and B does not depend on whether it is defined directly by colliding A and B, or indirectly by colliding A with C and B with C. Thus it is possible to choose a reference body (say, O) freely, and define the inertial mass of body A with the inertia ratio $k_{A,O} = k_A$, which now represents the property of a single body.

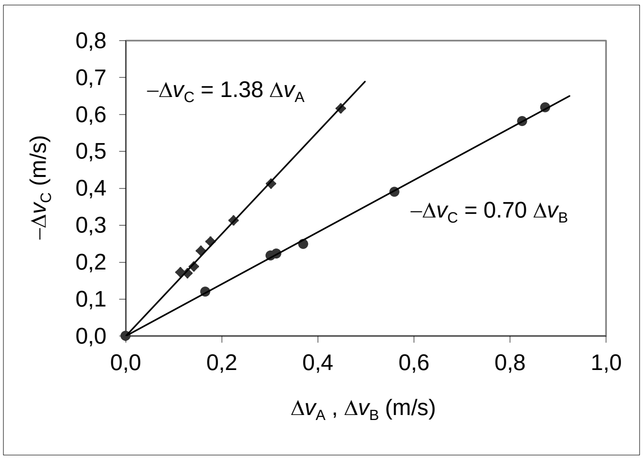

Figure 3. Ratios of changes in velocity of glider pairs A,C and B,C.

The experiment can be continued by connecting A and B together as a single body, and colliding it with C. One finds that $k_{A+B,C} = k_{A,C} + k_{B,C}$, so obviously $k_{A+B} = k_A + k_B$, and the additivity requirement is fulfilled.

The ratio k_A has no dimension, but the inertial mass of the reference body can be defined as (say) m_0 = 1 kg. So the final definition of the *inertial mass* of body A then becomes $m_A = k_A \cdot m_0$.

Table 1 shows some sample results. The weighed masses of the gliders were m_A = 0,408 kg, m_B = 0.209 kg, m_C = 0.299 kg. They are mentioned here only for reference; the concept of measuring an object's mass by weighing can be introduced only after the introduction of force.

Table 1. Sample results of collision experiments.

$k_{A,B}$	$k_{A,C}$	$k_{B,C}$	$k_{A,B}/k_{B,C}$	$k_{A+B,C}$	$k_{A,C}+k_{B,C}$
1.94	1.38	0.70	1.97	2.04	2.09

3.6. Structurisation: linear momentum and impulse.

The next obvious step is to find a quantity that would represent the magnitude of the interaction. We know that the interaction affects similarly on both parties. A reasonable goal for defining the measure for interaction is that a similar result (that is, change of a cart's velocity) must be caused by a similar reason (interaction). By pushing the kinesthetics carts one finds that a similar change in velocity can be achieved by pushing the cart either hard and brief, or by pushing it gently and long. Therefore, observing e.g. the deformation of the glider's bumpers is not an adequate means of measuring the total effect of interaction. One must turn attention to the result, the change of motion.

From the results of the quantification of the inertial mass one gets $m_A/m_B = -\Delta v_B/\Delta v_A \Rightarrow m_A \Delta v_A = -m_B \Delta v_B$. Obviously $m\Delta v$ represents something that changes equal amounts in opposite directions for both gliders. Since the interaction affects both gliders equally but in opposite directions, $m\Delta v$ obviously represents the effect of the interaction. It is a new quantity, which we find also to represent the magnitude of change in motion. We may define the quantity *linear momentum* as $p = mv$, and the collision law becomes $\Delta p_A = -\Delta p_B$, which also tells that the total linear momentum of the system is conserved.

Since the cause must be equal to the effect, we may define a quantity that represents the magnitude of the interaction, *impulse*, as $I = \Delta p$.

3.7. Uniform interaction, acceleration

While impulse is well suited for representing the total effect of an interaction, it is not useful for measuring instantaneous effects of continuous interactions. For example, the interaction between a boulder and the ground can not be described by the impulse of the interaction. We clearly need a new quantity for the instantaneous effect of an interaction. As usual, we start to look for the new quantity in a situation where it should be constant, which means uniform interaction. Gravity is obviously a uniform interaction near the surface of the Earth.

Motion caused by a uniform interaction can be studied on an inclined air track. One finds that the $x(t)$ curve is now not a straight line. In order to study the motion, one needs to generalise the concept of velocity, and define instantaneous velocity as the slope of the tangent of the $x(t)$

curve, $v = \dfrac{dx}{dt}$. By calculating the instantaneous velocity and graphing it, one finds that the $v(t)$ curve is a straight line. I.e. the ratio $\Delta v / \Delta t$ is an invariant that is characteristic for every motion caused by a uniform interaction. Thus one may define the slope of this line as a new quantity *acceleration*, $a = \Delta v / \Delta t$, which describes the rate of change of velocity in a motion caused by a uniform interaction.

What then might be a good candidate for the invariance that is used for defining the new quantity representing the instantaneous effect of an interaction? For example, one may give to a body a similar velocity (and thus a similar impulse) by pushing it briefly but hard, or pushing it longer but gently. We are looking for a quantity that describes "how hard we push", and find that this depends on how fast the impulse is accumulated. So for a uniform interaction, the accumulation rate of impulse, $\Delta I/\Delta t$, seems to represent the instantaneous effect of the interaction.

3.8. Quantification of force

The quantifying experiment must show that for a constant uniform interaction, $\Delta I/\Delta t$ is an invariant, does not depend on the body, and if the interaction is varied, $\Delta I/\Delta t$ is varied too.

Since impulse is defined as $I = \Delta p$, the accumulation rate of impulse equals with $\Delta p/\Delta t$. So one must study how the linear momentum varies as a function of time. For this, one needs a set-up where one may vary the interaction and the mass of the body independently.

An air track and a glider are used, in a "modified Atwood machine" set-up. The track is set horizontal. A string is attached to the glider. The string runs over a pulley, and small weights can be hung on the string.

First, the mass of the glider is varied, and the accelerating weight and thus the interaction affecting to the glider+weight system is kept constant. The instantaneous velocity of the glider is recorded, and $p(t)$ graphs are drawn (Figure 4). One finds that every graph is a straight line, and the slopes of the lines are similar. So the mass of the system has no effect on the slope of the $p(t)$ graph, as long as the instantaneous effect of the interaction stays constant.

Then, one keeps the mass of the system constant, but varies the accelerating weight. This is best done by putting small weights first on the glider, and then moving them one by one to the hanger on the string. By drawing $p(t)$ graphs, one finds that again the graphs are straight lines, but now their slopes get larger as the instantaneous effect of the interaction gets stronger. I.e. the ratio $\Delta p / \Delta t$ is an invariant that is characteristic for every uniform interaction's instantaneous effect.

Thus, we may define the new quantity that describes the instantaneous effect of a uniform interaction, *force*, as $F = \Delta I / \Delta t = \Delta p / \Delta t$. Since $\Delta p = m \Delta v$, $F = m(\Delta v / \Delta t) = ma$. This is the defining law of force, which is commonly called as the Newton's Second Law.

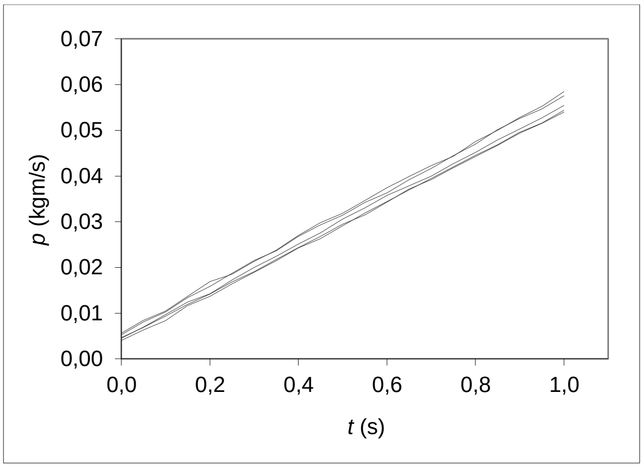

Figure 4. $p(t)$ graphs of the glider-weight system. The weight is kept constant; the mass of the glider is varied.

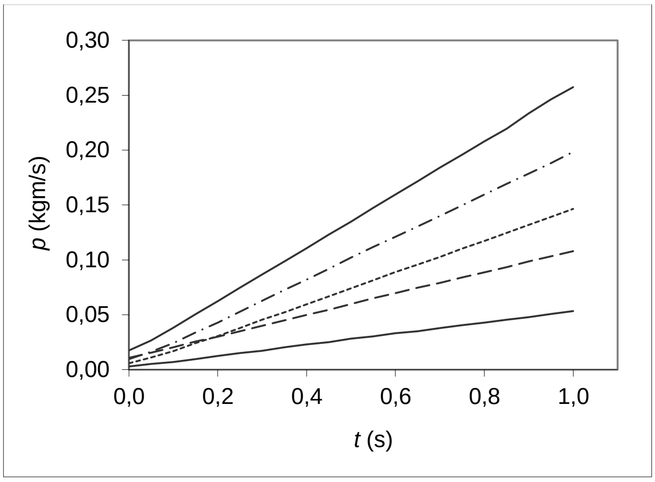

Figure 5. $p(t)$ graphs of the glider-weight system. The weight is varied; total mass of the system is kept constant.

4. EXAMPLE 2: ELECTRICITY

Our second example covering perceptional experimentality for learning of electric current, voltage, and resistance, is available via Internet, at *http://didactical.physics.helsinki.fi/didfys/artikk/PHYTEB2000/* .

5. REFERENCES

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