A NOTE ON A PARTIAL SUMMATION OF GRAPHS IN MANY BODY THEORY

K. KURKI-SUONIO
Department of Physics, University of Helsinki, Finland

Received 18 January 1965

The relation of the Hartree-Fock theory to the graphical perturbation theory of many partial systems was shown by Goldstone [1]. Thouless [2] noted that starting from free particles one can formally do a partial summation of graphs which leads to the replacement of the kinetic energies with the Hartree-Fock energies. In this summation there is a geometrical series and as emphasized several times [2-4] it does not converge in general and the result is therefore only formal.

Attention should be paid, however, to the fact that the rules for graphs are derived by a limiting process and one must always be careful in changing the order of an infinite summation and a limiting process. In this case it proves that doing the partial summation before going to the limit the result will come out quite correctly. The calculation goes then as follows:

Take an arbitrary "naked" graph, fig. 1, and consider especially the contribution in it of a particle line j which starts at the time t_μ and ends at t_ν. The value of the graph can be written

$$\ldots \sum_{\ldots j \ldots} \ldots \exp\{-\mathrm{i}T_j(t_\nu - t_\mu)\} \ldots .$$

Next, "dress" this particular line of the graph as shown in fig. 2 with the additional interaction lines at the times $t_1', \ldots, t_k'$. Now, the sum of all graphs with fixed n_1, n_2, n_3, n_4 wil be

$$\ldots \sum_{\ldots j \ldots} \ldots \frac{\exp\{\mathrm{i}T_j(t_\nu - t_\mu)\}}{n_1! n_2! n_3! n_4!} \{\int_{t_\mu}^{t_\nu} \Sigma_1 \mathrm{d}t'\}^{n_1} \times$$

$$\times \{\int_{t_\mu}^{t_\nu} \Sigma_2 \mathrm{d}t'\}^{n_2} \{\int_{t_\mu}^{t_\nu} \Sigma_3 \mathrm{d}t'\}^{n_3} \{\int_{t_\mu}^{t_\nu} \Sigma_4 \mathrm{d}t'\}^{n_4} \ldots$$

where

$$\Sigma_1 = \Sigma_2 = \sum_{r \le k_F} \tfrac{1}{2} V_{jrjr} ; \quad \Sigma_3 = \Sigma_4 = \sum_{r \le k_F} \tfrac{1}{2} V_{jrrj}$$

yielding

$$\ldots \sum_{\ldots j \ldots} \ldots \frac{\Sigma_1^{n_1} \Sigma_2^{n_2} \Sigma_3^{n_3} \Sigma_4^{n_4} (t_\nu - t_\mu)^k}{n_1!\, n_2!\, n_3!\, n_4!} \times$$

$$\times \exp\{iT_j(t_\nu - t_\mu)\} \ldots .$$

Summing over n_1, n_2, n_3, n_4 we get for the sum of all graphs where the line j is dressed an exponential series

$$\ldots \sum_{\ldots j \ldots} \ldots \exp\{i(T_j + \Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4)(t_\nu - t_\mu)\} \ldots =$$

$$= \ldots \sum_{\ldots j \ldots} \ldots \exp\{i\epsilon_j(t_\nu - t_\mu)\} \ldots ,$$

where ϵ_j is the Hartree-Fock energy of the particle line j. In the same way a partial summation can be performed for each particle- or hole line of the naked graph leading to the replacement of all kinetic energies with the Hartree-Fock energies. There is no difficulties with the convergence since the exponential series converges for all finite Σ_1, Σ_2, Σ_3, Σ_4.

After the summation the limiting process which leads to the rules for calculating the graphs can be performed in the original way. For this process to converge, however, all hole energies in the exponent must be less than the particle energies and therefore the summation must be performed in all lines of the graph in order to get a finite result in the limiting process.

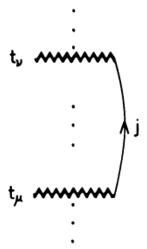

Fig. 1.

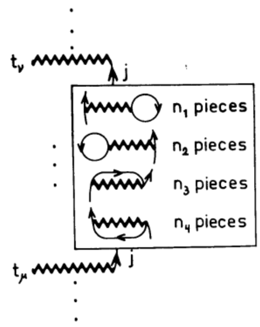

Fig. 2.

References

1. J. Goldstone, Proc. Roy. Soc A239 (1957) 267.
2. D. J. Thouless, The quantum theory of many-body systems IV.6. (Academic Press, New York and London, 1961).
3. G. E. Brown, Lectures on many-body problems (Lecture notes, Nordita) (Copenhagen, 1961).
4. R. D. Mattuck, The many body problem in solid state physics (Lecture notes) (Copenhagen, 1962).

* * * * *