Three Lectures on Concept Formation

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The first title of my lectures *The Meanings"* is the main heading, the two others, *"Quantification"* and *"The Hierarchies"* are two central specific sub-themes. I have been speaking about the meanings of the concepts of physics since long, at least for a quarter of century[1]. Those who have listened to my instruction will certainly recognise many of the same ideas in these lectures. Still I hope, that they will notice also some development to have taken place in my thinking.

INTRODUCTION. THE INTUITIVE FOUNDATION OF KNOWLEDGE

It is customary to think, that *man asks and science replies.* We wish to know the truth about how the things are: what is matter, electricity, gravity or life, consciousness, death *etc.*, what is their basic essence or meaning, and we insist that the science should give the answer. When something has been *"scientifically proved"*, it is *"the objective truth"*, and there is nothing further to be said.

Then, however, it is said – as I have read in a textbook in the beginning of the chapter on electricity, *"No one knows, what electricity is"*. When this is said about a subject which is one of the best known things scientifically, it is actually claimed that *no one knows anything, and that it is even impossible to know anything.* Well, this is true, because in empirical science there are no intrinsic truths, although this is perhaps not a very motivating start for teaching. It is at most the propositions resulting from the deductive structure of *mathematics and logics*, having the form *"if A then B",* that can possibly regarded as intrinsic or *a priori* truths.

Asking for final answers, absolutely certain knowledge includes the demand of objectivity. *Objectivity is a good target*, a direction to be aimed at, but reaching it is *impossible*. We are, however, fairly convinced that *the tremendous progress of science is a historical fact*. *"By exploring you will find out"* has also been the second motto of my own teaching! Something has been found out, which we regard as *certain* scientific knowledge and *worth teaching*.

The subject of these lectures lies in the middle of this problem of knowing. Concept formation is one of the very core problems of the philosophy of science. It couples together the ontology and the epistemology or the questions, *"what exists"* and *"what one can know about it"*. In physics, knowledge and understanding mean, in the first place, conceptual mastery of nature, which is based on the meanings of concepts and conceptual structures. Thus, the concept formation, or the birth, conceptualization, structurisation and generalisation of meanings, is an important problem of physics teaching. It is also a central problem of the history of science, since the gradual build-up of meanings and of the concepts based on them is a development which has taken place in the history. At the same time it connects the history of science to teaching, because the build-up of meanings taken place in history ought to get realized in learning.

The basic problem of concept formation is the relation of observation and mental pictures or empiry and theory, which leads deep into the endless questions of subjectivity and objectivity – or intuition and logics. We have, thus, a most fascinating topic. Only unsolvable problems can be genuinely interesting. The solution makes the problem trivial.

Here *science* and *intuition.* are opposite to each other. For science, which aims at absolute truth, intuition is an abomination. *"Metaphysics out!"* was the war cry of the positivistic physics of the early 1900. Only knowledge which is empirically confirmed will do. Intuitive mental pictures and associated statements like "a force is acting", and atoms, of which, by then, no direct empirical evidence existed, belonged to metaphysics, illegal in science. However, *the questions of man are intuitive*. Well, science tends to answer "as well as it can", on the basis of what it is possible to "find out by exploring". But by exploring one cannot even try to answer any other than *operational questions i.e.* questions which one can "ask the nature". This kind of asking means realization of an operation, which forces the nature to reply.

For exploration science therefore has to operationalise the questions of man. Operationalisation is a *reduction.* But when a question is reduced, also its meaning gets reduced. And on a reduced question one will get a reduced answer. So, science "changes the subject". It does not answer the questions of man but its own questions. It

[1] See *eg.* Kaarle & Riitta Kurki-Suonio: *Meanings and Structures of Physics.* (In Finnish). Limes ry., Helsinki, 1994.

produces reduced answers to reduced questions. The deepest intuitive meaning of the original question is lost, it is reduced away. Intuitive questions can have only intuitive answers.

Elimination of intuition from science is impossible. There is no science without intuition. Behind every reduced question there lies an intuitive basic question. Also the operationalisation is based on intuition. An operation is sought, the realisation of which would, according to an intuitive judgement, correspond to the meaning of the original question as well as possible. And the reduced answer resulting from the operation requires an intuitive interpretation. The meaning of the answer obtained from the nature must be interpreted intuitively from the point of view of the intuitive meaning of the original question.

The objectivity of the operationalism itself is only apparent, since the operations, investigations for setting questions to the nature, are based on our mental pictures about nature. The mental pictures are intuitive. And the answers obtained from the nature, are also bound to our mental images. This is what the philosopher means when speaking about the *ladenness of observations by theory* – what else could the theory be but accurate representations of mental images. When, however, all concept formation in science is based on observations obtained in experiments aiming at operationalism, it is equally justified to speak of *ladenness of theory by observation*. In this way even the goal of objectivity is hidden deep inside the idea of operationalism, into the inseparable dualism of observations and metal images – or of empiry and theory.

The only possible escape of science is "*inter-subjectivity*". When objective truths are not achieved by exploring, the truths are agreed upon. Answers of science, scientific knowledge, scientific interpretations, scientific experiments and observations consist of everything which can commonly be considered scientific. Then, the requirement of inter-subjective co-understanding refers to everything, to the meaning of intuitive questions, to their operationalisation, to the realisation of the operational investigation, to the resulting observations, their conceptualisation and intuitive interpretation in light of the original question. Only after all this we have a *scientific concept*, the core of which is the commonly understood empirical meaning. But in order to understand properly its meaning and significance and its degree of certainty or reliability it is necessary to be aware of the central position of intuition in this process.

Scrutiny of the activities of the "scientific society" and of the history of science leads easily to the conviction, that also the milder requirement of inter-subjectivity is a rather hopeless goal to reach. Still, we have something, which we consider sufficiently well founded, even certain, worth knowing and being taught. The achievements of science, more generally, the whole human culture, our shared language, the translatability of languages *etc.*, almost compel us to think that there is some factor behind, some *"inter-subjective intuition"* guiding people in different times and on different sides of world into understanding of meanings in a way which is in essential respects similar. We recognise in the history of science and even more generally in the history of culture the peculiar feature that same ideas, similar trends seem to be awakened simultaneously on different places, "when time is ripe".

What could be the nature and origin of the inter-subjective intuition? Could we perhaps speak even about shared social consciousness of people? Is it some innate species-specific humanity, or is it something more general, an intuitive connection of our consciousness with the unattainable "reality beyond a veil" reflecting the genuine nature of this reality, of which an individual can share only little by little and gradually through one's own efforts? Then, why does it often seem that science, in its search for truth, has to "*proceed upstream"* people striving with all their strength to prevent the development? We remember the discouraged sigh by Planck: *"The only way for a new idea to break through is that the old generation dies away."*

It would be tempting to continue this thought, ponder upon its meaning and implications. Does it possibly mean, that there is, in fact, no alternative to the scientific concepts based on perception of empirical meanings? This would, however, lead us still further astray from the ordinary subject of my lectures.

1. EMPIRICAL AND MATHEMATICAL MEANINGS OF CONCEPTS

1.1 The empirical meanings

The origin of these thoughts lies far in the past in the time of my own studies. I have been working them in my mind through my whole academic career when I again and again have met two different views on the role of formulae[2].

1. Most seemed to consider the formulae mainly as starting points of calculation, as working instructions and tools. For them, understanding of a formula meant just knowing how to use it in calculations. The emphasis in their studies was, then, in training of solving computational problems and in development of calculatory skills.

2. I myself learned to regard the formulae in the first place as representations of empirical meanings. In understanding of a formula, it was most important to find out how it was formed on the basis of its empirical meaning.

For me, with just the short mathematics syllabus of secondary school as background, the differential and integral calculus caused great difficulties in the beginning. I still remember the great experience in the middle of my first spring semester, when, quite suddenly, the enormous freedom offered by mathematics for representation of most different kinds of empirical meanings was opened to me. It is notable that the idea was born specifically in the context of *mathematics*. This experience carried me through all my studies and through the whole of my academic career both as a researcher and as a teacher. Later on, it was crystallised into the philosophical starting point of my teaching: *"Meanings are first",* which many of you know as my motto characterising learning of *physics* and as the carrying principle of the *perceptional approach* of teaching. According to it, the main thread through the teaching should be *an observation-based and systematically proceeding creation and conceptualization of empirical meanings.*

The keyword is *"meaning"*. The purpose of the introduction was to emphasize that meanings are intuitive. They are born through perception. This means that they arise from a cooperative action of observation and mind, where the empirical and theoretical elements are completely inseparable. *Perception* is at the same time observing based on and making use of mental images and forming and constructing and developing mental images on the basis of observations. In perception both *theory-ladenness of observations* and *observation-ladenness of theory* is realised simultaneously.

The mental images, thus formed, are *empirical meanings*, identified Gestalts. In this sense the meanings are born first. *Conceptualisation* is naming of them. It builds language and terminology making possible discussion about observations and about their interpretations, "the negotiation about meanings" in order to attain inter-subjectivity required by the principle of science. Perception and conceptualization of meanings are, thus, submitted to *the social process.*

Perception of meanings is governed by the boundedness of human mind to *space* and *time*. We perceive our environment in space and time. This largely determines the whole mental-image structure, according to which we perceive the surrounding world of phenomena. As a consequence, the natural ontological interpretation of our observations consists of *entities* or subjects[3] of nature, existing in some *position* of space in some instant or interval of time, and of phenomena, which are events or processes taking place in time and somewhere in space, *i.e.* that what happens to the entities, what they are doing and how they are behaving.

Identification of entities and phenomena is based on their *properties*. The entities possess permanent characteristic properties, which make their identification possible, and properties, which are changing in phenomena, while identification of phenomena becomes possible on the basis of the nature of the changes of properties of the participating entities. Correlations between the changing and permanent properties are perceived as *dependences*. The dependences indicate conformity to law – the phenomena are proceeding each according to its own laws. The laws of phenomena are their identifying properties

Inherent in the image of conformity to law there is a Gestalt of *affecting*. The laws of phenomena are understood to reflect effects of the entities to each other. But affecting of an entity upon another is a *causal relation*; there is an effector and a target of the affecting. The phenomena become ways of affecting. We end up with mental *causal models*, giving us the experience of understanding the phenomena. This is close to what Planck said: *"We understand a phenomenon, when we have a mechanistic model of it."*

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[2] Cf. K. Kurki-Suonio: *Kaavatauti - oireet, hoito ja ehkäisy.* (*Formula disease – symptoms, care and prevention*.) Matemaattisten Aineiden Aikakauskirja **44**, 1980.

[3] The word *object* is avoided because it indicates the relation to an observer.

The conceptual understanding, thus building, is based on the intuitive ontological conviction, that the observations and the concepts created to represent them are standing genuinely for the entities and phenomena of nature, their properties, dependences between the properties and the causal relations of the phenomena, such as they exist and take place in space and time.[4]

Perception and the succeeding conceptualization mean, thus, modelling of reality in terms of concepts and conceptual structures, the core of which is their empirical meaning.

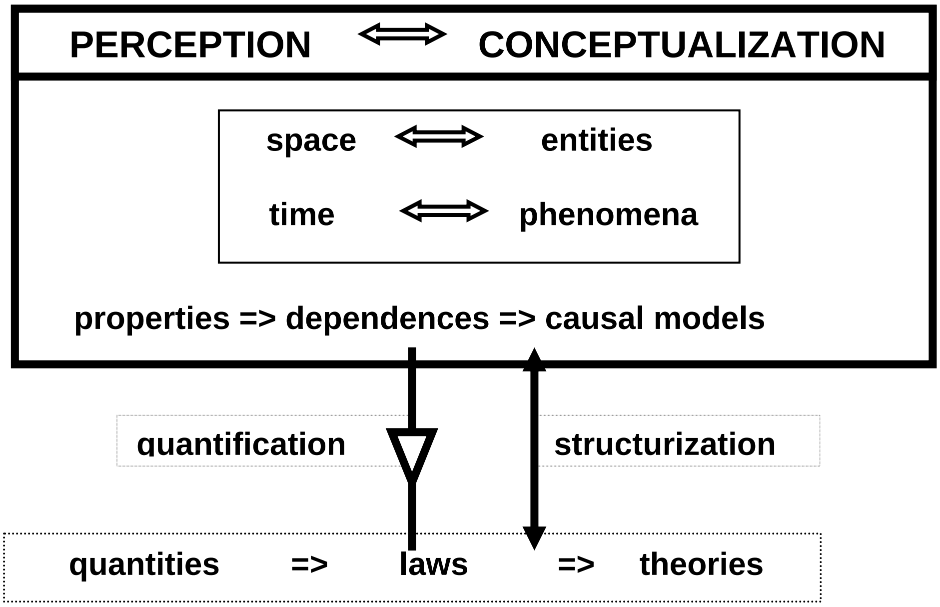

Figure 1.1 Creation of empirical meanings.

This is the perceptual basis, where physics starts from, a kind of *"pre-understanding"* to be realized and definitized when nature is perceived and conceptualized. The notice, that nouns, verbs and adjectives belong to the structure of all languages, making possible our speaking of entities, phenomena and properties, suggests that this is largely *the core of the inter-subjective intuition characteristic to man*.

Awareness of this image structure describing the reality as being intuitive – despite of its apparent certainty and self evidence –, means that, from the scientific point of view, all of its elements must be regarded as working hypotheses. With the progress of physics they also have been questioned, one at a time. The difficulty of adopting the basic ideas of modern physics can be understood to stem from the fact, that the theory of relativity and the quantum mechanics are based just on such questionings, which contradict this "inter-subjective pre-understanding" but have turned out to be justified by the compulsion of empiry. The *empirical compulsion* means, that nature has answered our operational questions in a way we did not expect, but that the boundedness of science to the aim of objectivity compels us to accept the answers and to take them into account.

1.2 Quantification

Time, space, entities, phenomena, properties *etc.* are qualitative concepts, intuitive ontological interpretations of our observations, with the help of which we understand natural phenomena. *The basis of understanding is qualitative.* Among these ontological elements of qualitative understanding the properties are "the weakest link"[5], which allows operationalization of concepts.

The standard states laconically, as if a self-evidence: *Quantity is a property, which can be identified as to its quality and measured as to its amount.*[6] And in another place this is expressed briefly: *Quantities are measurable properties (of natural entities and phenomena).* However, the whole basis of understanding physics is hidden here. The transfer

[4] K. Kurki-Suonio: *Modernin fysiikan perushahmojen synty.* (*The origin of the basic Gestalts of modern physics.*) Dimensio **64,** 2 and 3/2000.
[5] This refers to the title of a sadistic quiz in the Finnish TV.
[6] This is the wording of the Finnish standard SFS 3700: *Metrology. Vocabulary of basic and general terms*. The wording of the newest version of the corresponding international document, ISO VIM (DGUIDE 99999.2) *International vocabulary of basic and general terms in metrology*, is lengthier but contains the same idea.

from 'identification as to quality' into 'measurement as to amount' means quantification, conversion of the qualitative system of concepts into a quantitative one. There the quantities have a key position. Both theoretical and experimental considerations are always made in terms of quantities. It is impossible to avoid them. It is not justified to speak about understanding, if one does not know what properties of what entities or phenomena one is dealing with. *Quantification is a threshold operation, the core of which is transmittance of the perceived meanings of the properties to the quantities.* I shall return to this more closely in the second part of my lectures. Here I shall concentrate briefly on what is important from the point of view of the subject of the present lecture.

When the properties get converted into quantities, they obtain a unit and a numerical value. They become computational elements. At the same time the mutual dependences of properties become laws of quantities, relations, which can be expressed in a mathematical form, sometimes as algebraic equations, and the mental causal models become theories and theoretical models, presented as mathematical structures of laws.

In this process *the empirical meanings are conserved.* Quantification transfers the ontologically interpreted empirical meanings of the qualitative concepts as such to the quantitative concepts. *Quantification does not create new meanings;* it just definitizes the empirical meanings already understood. Quantities, laws and theories are understood – in the same way as their preceding qualitative counterparts – as representations of true properties of entities and phenomena and of real causal relations, for which we just have found an exact form through quantification.

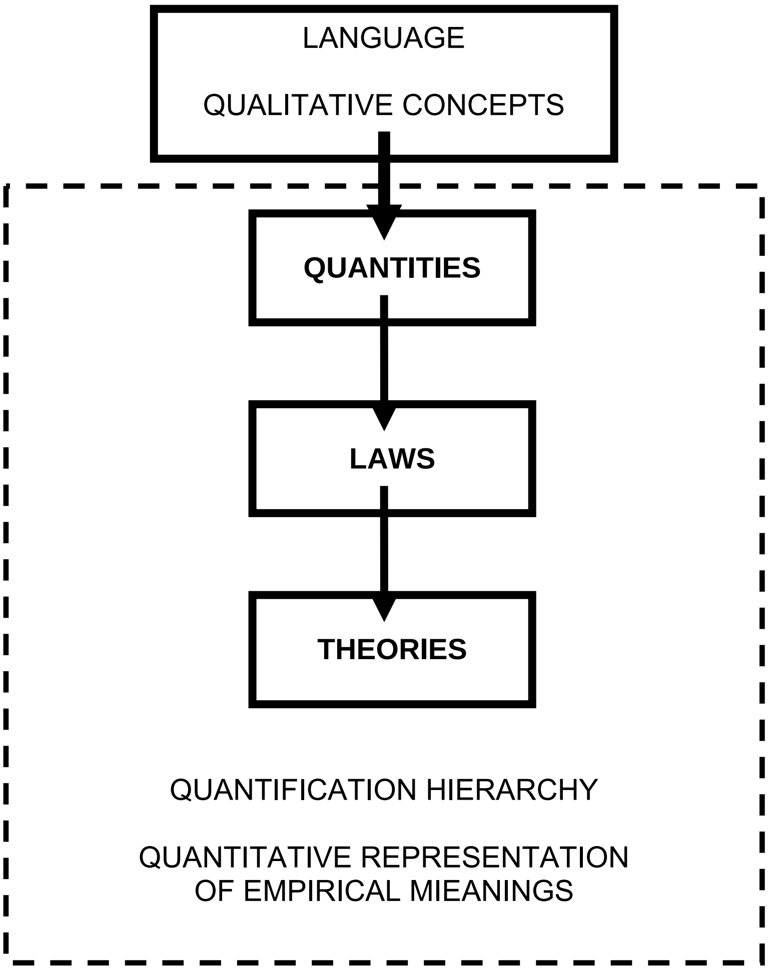

Figure 1.2 The three-level hierarchy of quantitative concepts.

Quantification is the basic operationalisation of physics, the gate, which opens the conversion of properties into quantities. This does not mean that qualitative experiments would not been operational as well and would not offer answers of nature, but the change from qualitative observations into the exact reproducible results of measurement of the quantitative experiments raises the reliability of the results to a new level, where it is also easier to reach mutual understanding about their meaning.

The resulting conceptual structure possesses a clear *three-level hierarchy* of quantitative concepts, quantities – laws – theories, because quantities are the elements of laws and laws are the elements of theories. The laws are relations of quantities and the theories are structural wholes span by certain basic laws.

For each property, quantification into a quantity is its own specific problem. One needs a *quantifying experiment*, where the degrees of the same property of two entities or phenomena can be compared in a way where it is intuitively evident when they are equal or when, for instance, one doubles the other. A quantifying experiment is, however, in itself a quantitative experiment, where something has to be measured. It is, thus, necessary to know already one or some more primary quantities which must have been quantified before. In this way a "locally directional" net of quantities becomes spun, with each knot representing one quantification. This leads to the problem of the first quantification and the first quantity, the solution of which leads us directly to the heading of this lecture.

1.3 The mathematical meanings

The net of quantities starts from the concept of number. *Number* is the very first quantity, the root quantity of all quantitative considerations in physics. It is a quantity because it represents an empirically observable property, the magnitude of a set of identifiable individual entities (fingers, sheep, apples, coins *etc.*). *Natural numbers* are the possible values of this quantity: The *additivity of number* is perceived as an *empirical law* governing the uniting of such sets. Hence, it is justified to regard it also as the source of addition and subtraction and the mathematical laws concerning them[7].

In this way mathematical concepts can be seen to breed from empirical meanings. The first elements of mathematics, the *natural numbers*, are born as representation of the quantity '*number*', and the first mathematical operations, the addition and subtraction are born from the empirical laws of this quantity. This is also the way how teaching of mathematics to children starts, by perception of properties of sets of things seen in the everyday surroundings. Quantification gives birth to the quantitative physics. At the same time it signifies the beginning of mathematics. *Mathematics is born together with the quantitative physics from the structure of empirical meanings.*

The idea of the "ordinary" pure mathematics is the ambitious endeavour to "liberate" the structure of meanings from the *empirical ballast.* The quantities are, then, reduced to mathematical elements and the laws to the structural relations of the elements. Once the quantities have been undressed of their intuitive empirical meanings, only their "mathematical meanings" *i.e.* the pure structural meanings, remain. A mathematical element has no more any intrinsic meaning, but its meaning is created and defined by its structural relationships with other elements, relationships which are submitted to the mathematical laws of the mathematical operations.

The mathematical meanings of concepts are structural meanings insulated from the empirical meanings.

Pure mathematics becomes reduced to exploration of conceptual structures, where "the formulae" really are the starting point and foundation of meanings. In physics, instead, empiry creates the meanings. The inherent intrinsic meanings of elements as properties have been eliminated. The relations, which create their mathematical meanings, and the mathematical operations governing them, have, however, an intuitive origin in the perceived dependences of properties, which constitute the structure of the empirical meanings.

Proceeding on the path of physics is based on perception of empirical meanings. There, one is proceeding again and again onto new phenomenal areas, where new empirical meanings, new quantities and laws are perceived as properties of identifiable entities and phenomena and as causal relationships of properties. Inductive generalisations of laws, which become possible expressly through making use of the mathematical representations and mathematical meanings, lead to generalisation of concepts. This means *generalisation of empirical meanings* onto new areas. At the same time *the structurality of empirical meanings* increases. This is the basis of *the hierarchically proceeding unification development* of physics, where the understanding is gradually deepening when phenomena are perceived as different kinds of realisations of more general phenomena, like, for instance, static electricity, magnetism, electric current, induction and light are understood as manifestations of electromagnetism governed by the same Maxwell laws. With the development of the hierarchical structure of concepts, also the perception process itself gets structurised so, that one starts, more and more clearly, to distinguish sub-processes of dominantly theoretical or empirical nature. Ultimately, this leads to the fallacious illusion of separate fields of theoretical and experimental physics.

The number can be understood as the gate leading from empirical meanings to *the path of mathematics*, where proceeding is based on structural meanings alone, on their inductive generalisations and logical reasoning. Number yields just the natural numbers as mathematical elements. The laws of calculation offer a purely mathematical possibility for creation of *new elements* and adoption of *new operations*. No empirical perception is needed because the empirical meanings have been eliminated right at the gate. On this path the skills of logical reasoning will develop. In this way the

[7] *Cf.* G. H. Hardy: *A Mathematician's Apology.* Cambridge University Press (1940) Chapter 16.

natural numbers and the laws of addition and subtraction give rise to the *hierarchically proceeding chain or net of generalisations of the concept of number and of new calculational operations,* the early stages of which involve the negative numbers, multiplication and division, fractions and decimal numbers, and further on, powers, roots, logarithms *etc.*, until the structure becomes closed. A new higher level of hierarchy of mathematical concepts is reached, *the level of theory*.

In this view the conceptual structure of mathematics is parallel to the three-level hierarchical structure of the quantitative concepts of physics. This comparison makes also evident their fundamental differences due to the different nature and origin of the meanings of concepts.

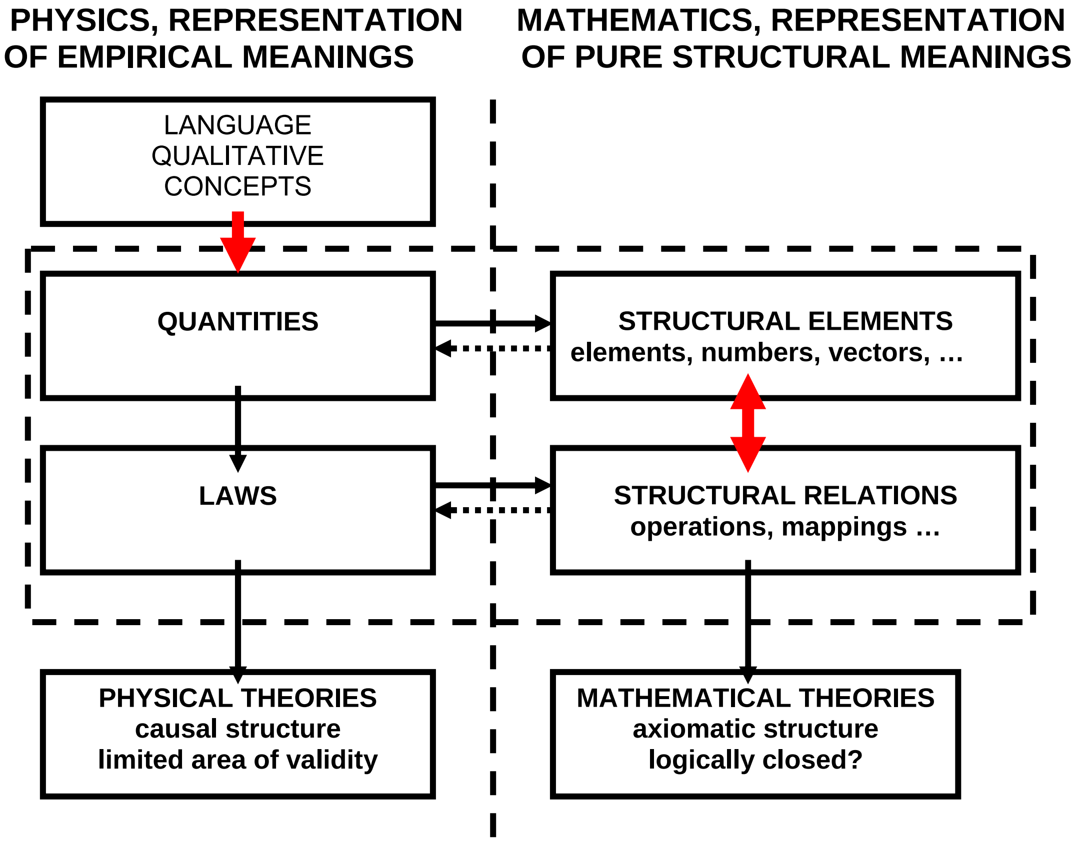

Figure 1.3 Comparison of the conceptual structures of physics and mathematics.

The meanings of the quantitative concepts of physics are empirical. They originate from outside this structure, from the preceding qualitative level. They follow from the perception-born qualitative understanding based on our intuitive ontological interpretation. In quantification, which primarily converts properties, 'identifiable as to quality', into measurable quantities, the meanings get transferred as such from the qualitative concepts to the corresponding quantitative ones. Laws obtain their empirical meanings from the meanings of the quantities, which are their elements, and the theories further from the meanings of their constituent laws.

In mathematics, instead, the meanings are internal to this hierarchical structure. They are based on the relationships of the middle level. The concepts of the first level, the elements, have no intrinsic meanings. Their meanings are purely structural, defined by the relations of the middle level of concepts. Also, the only meaning of a mathematical theory is structural. Just as the only meanings of the elements derive from their mutual relations, *the only meaning of a mathematical theory is its structure, the net of relations.*

The ideal goal is axiomatisation of the theory, reduction of the structure into axioms, the least possible set of basic relations, which span the whole structure. Axioms are the reduced core of the meaning of a mathematical theory. The whole theory, all of its possible statements concerning relations of the elements, is logical consequence of the axioms. (Here we are sliding over Gödel's theorem according to which complete closed wholes cannot exist even in mathematics.) Algebraic theory of numbers, axiomatic geometry, theory of linear spaces, group theory etc. give a good idea of this objective.

A physical theory, on the contrary, is *a causal structure of quantified empirical meanings.* The basic elements and relations of a theory, the quantities and the laws, are bound to their empirical meanings. Causality itself is an empirical meaning. A theory is spun by its *basic laws* presenting the basic idea of the causality built in the theory. The whole empirical meaning of the theory is concentrated in them, and they form the basis of the "modelling capacity" of the theory. In the structural sense they correspond to the axioms of a mathematical theory, but it is not genuinely justified to regard them as axioms, because they are representations of intuitive empirical meanings, and intuition cannot be axiomatised.

A mathematical theory is *closed*. It represents "aprioric truths" of the nature: if A (the axioms) then B (statements of the theory). A physical theory is *open*. It represents the empirical reality perceived intuitively. Its basic laws are an idealized basic model of a certain phenomenal area, making possible specific modelling of phenomena of this area. It has an area of validity, where it holds with certain accuracy, expanding with the progress of physics.

1.4 Gates of empiry to mathematics

In the history of science, mathematics and physics have developed and are developing in a continual tight interaction. They need each other, but their different relation to the empiry is separating them.

Quantification transforms properties into quantities making, thus, possible mathematical treatment of empirical meanings in terms of graphical representations, equations *etc.* Theoretical physics becomes mathematics of empirical meanings. Starting from the modelling possibilities offered by the theories and by making use of the mathematical meanings, i.e. by calculation, it is possible to derive empirical predictions, testing of which opens the path to generalization of empirical meanings and to further development of the conceptual structure of physics.

The first quantification, *the number*, was the opening of game, *the first gate* of empiry to mathematics. When progressing on the path of perception in the chain or net of subsequent quantifications, one meets situations, where new kinds of mathematical means are needed for representation of empirical meanings. Empiry motivates formation and adoption of new kinds of mathematical elements and operations giving them an intuitive perceptual basis. Again and again new gates of empiry are opened.

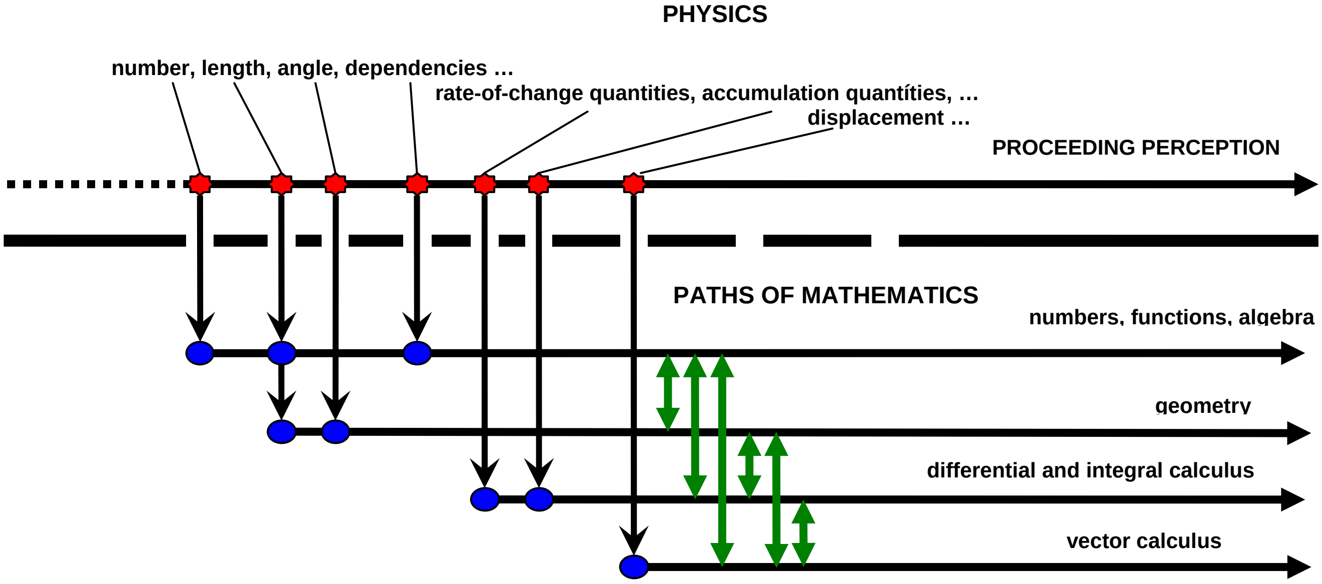

Figure 1.4 Gates of empiry

For representation of the number and of its laws, natural numbers, addition and subtraction were needed. From this one could continue on the path of mathematics. Generalisation of the concept of number and adoption of new operations can, however, be considered also by starting from that observable property, which the quantity *number* is representing. By viewing ordered sets, their grouping and partitioning, for instance with the aid of cross-ruled paper or bricks, one can perceive empirical laws of the number, the representation of which leads to multiplication and division.

It is interesting to ponder this further, and in teaching of mathematics this is also done. It has been realised that mathematical motivation can be evoked through empirical meanings. They offer support for the mathematically proceeding concept formation, without any need of speaking condescendingly about empirical mathematics. I shall continue a bit further to clarify what I mean.

The number makes possible quantification of the *length* of a body and the *period duration* of phenomena perceived periodical. This is done simply by determining how many lengths of a shorter body fit in the longer one and by comparing the numbers of periods of two phenomena in the same time interval. This is the way, how the first dimensional quantities are introduced, also in the traditional teaching – of mathematics (!). They offer the empirical perceptual basis for the positive real numbers.

Examination of the area of a rectangle and the volume of a rectangular parallelepiped in the traditional way of teaching is the primary quantification of the quantities *area* and *volume*, offering an empirical meaning for the *multiplication of real numbers*. And, for instance, examination of uniform motion and homogeneous matter, for quantification of *velocity* and *density*, offers empirical meanings for the foundation of the *division of real numbers.*

Thereafter, quantification of the directional difference into the quantity *angle* opens the way to the conceptual mastery of shapes and sizes, and offers, thus, the perceptual basis for geometry in the way, which *eg.* Jaakko Joki has made use of in his research aiming at development of teaching of geometry [8].

In quantitative examination of motion, *displacement* offers an empirical meaning for the starting point of *vectors an their addition.* Projection and the area of a parallelogram offer primary Gestalts for the scalar product and the vector product, respectively.

Quantitative examination of empirical laws, where dependence of one scalar quantity of another one is considered, form the natural perceptual basis for the concept of *function* in general and also specifically for *many simple algebraic functions* like the *proportionality*. Also the *derivative* and the *integral* can be perceived as representations of the empirical meanings of speed or rate of change and of accumulation of additive properties. One could continue this as far as you wish, to gradient, divergence, rotor, tensors, matrices, vector spaces, Fourier series, group theory *etc. etc.* .

These all can be regarded as gates of empiry to mathematics, opening on the path of perception. Physics means proceeding on the path of perception, while mathematics is proceeding on the path of mathematical deduction. But in studying physics it is necessary to study also mathematics. The same mathematical concepts are met on both paths. The gates can work both ways. Mathematical concepts can be introduced in physics as known tools of representation, or they can be adopted as abstractions of structural empirical meanings. Once the need for a new kind of mathematical representation is realized, there may be a solution waiting behind the gate on the side of mathematics. But it is also possible to use the empirical meaning as the starting point for formation of the required new mathematical concept or operation. It is possible, that the way on the path of mathematics to the concept is long, but the way on the path of perception to its empirical meaning is short. In such cases the teacher or student of mathematics finds the concept difficult, but the teacher or student of physics finds it easy. Personally I believe that this is the case quite often, not only in the case of the natural numbers, addition and subtraction, but also, for instance, in the case of vectors, derivative and integral.

There are plenty of occasions in the history of mathematics, where, through a gate of empiry, a new untrodden path has opened and lead towards a new goal of an axiomatic theory. Many a mathematical theory has received its initial impact from empirical meanings and from the need of representing them. And it is not at all obvious how, for instance, the axiomatic geometry or the theory of vectors could follow by imperative necessity of mere logics by proceeding on the path of pure mathematics, starting from natural numbers. Afterwards, construction of such a route is, of course, possible. Mathematician's alternative for perceivable empirical meanings is *mathematical intuition*, intuitive generalisation of elements and operations. The mathematicians' boast about theories, created purely mathematically without support by any empirical starting points and having ultimately no possible practical applications, is understandable[9].

These two paths have a relationship which looks very similar to that of the empirical and the theoretical approach. In the practice of teaching and studying they could be at their best in the similar way inseparably intertwined into a two-way dynamics of progress as the empirical and the theoretical approach are in the perceptional approach.

It is often stated emphatically: "*mathematics is not an empirical science*", implying that in teaching of mathematics one should proceed "mathematically", basing exclusively on the laws of calculation and deduction. But this noble idea of pure mathematics, operating only with structural meanings released from the burden of empirical meanings, is neither the way how mathematics was born in the scale of history, nor is it fruitful as the guideline of teaching mathematics to children. The power of the idea becomes evident only towards the end of the path, where abstract mathematical structures, purified from empirical meanings, can be projected to the meanings of different fields of empirical research. Then, mathematics takes the position as a quantitative meta-theory of empirical science.

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[8] Jaakko Joki: *Ulkoluvusta hahmottavaan geometriaan.* Väitöskirja. (*From rote learning to perceptual geometry. Dissertation.*) University of Joensuu. Series of Didactical Mathematics 1. Joensuu University Press (2002).
Jaakko Joki: *Hahmottavaa geometriaa.* (*Perceptual geometry*). Helsinki: University Press (2004).
[9] *Cf.* G. H. Hardy: *A Mathematician's Apology.* Cambridge University Press (1940) Chapter 21.

2. QUANTIFICATION AS THE THRESHOLD OPERATION OF CONCEPT FORMATION IN PHYSICS

2.1 Quantification

Quantification means literally, 'giving quantity to something previously regarded as having only quality'[10].

The intuitive elements of the basic-perception stage of concept formation, entities, phenomena, properties and their causal relationships are qualitative concepts. They form our interpretation of observations, Gestalts, mental pictures, empirical meanings. Their existence, as elements of 'reality', is an ontological conviction in accordance with the inter-subjective intuition of man. Quantification is an operationalisation of this mental-image structure in a way, which makes possible testing our expectations based on this mental image by measurement. The conceptual category of 'properties' is the 'weakest link', which allows this. Quantification is realised expressly as *quantification of properties into quantities*.

According to the standard "*Quantity is a property, which can be identified as to its quality and measured as to its amount. Quantities are measurable properties of natural phenomena."*

In this definition 'measurement' refers to a physical measurement, which is a demanding specific operation. Any quantitativeness does not mean measurability in this sense, and all 'giving quantity', is not 'making measurable'. Genuine quantification, which transforms properties into quantities, *preserves the perceived empirical meanings* of the qualitative level transferring them to the quantities, laws and theories or theoretical models.

The *empirical meaning of a quantity* is its meaning as a perceived property of the entities or phenomena of some definite class of entities or phenomena. This is the primary and permanent meaning of a quantity. It can get generalized to wider or to completely new classes of entities or phenomena, but it does not change.

The *theoretical meaning of a quantity* is a structural meaning which determines the position of the quantity in a theory. It depends on the theory, since every theory is its own conceptual structure. Particularly, in the quantum mechanics and the theory of relativity the quantities have completely different theoretical meanings from those which they have in the preceding classical physics, but also in different stages of development of the classical physics the positions of quantities vary[11].

The *mathematical meaning of a quantity*, discussed in the previous lecture, is the isolated, reduced structural meaning, naturally also theory-specific.

In order to understand the meaning of a quantity it is necessary, in accordance with the definition of the standard, firstly to *recognise the property*, for representation of which the quantity is needed, and, of course, at the same time to *identify those entities or phenomena* having this property. This is the core of the meaning of a quantity.

Secondly, it is necessary to find out, how this property, first known only as a qualitative one, can be *quantified i.e.* made *measurable* on the basis of this qualitative meaning or 'on the own terms of the property'. Quantification must offer the possibility to *choosing a unit* and to a *numerical representation* of the property based on this unit.

Quantification is necessary in order that measurement of the property may become possible. But it is a different issue. All methods of measurement of a quantity derive from the quantification through some theoretical and/or empirical route.

Identification of properties and quantification ought to be the starting point of physics teaching. They are easily ignored as a self evidence. They are, however, essential in the important first phase of the build up of empirically justified mental images, the *basic perception*, which does not succeed without guidance towards recognition of the 'correct' Gestalts. To my experience, the very basic question concerning any quantity, *which property of what entity or phenomenon*, mostly just puzzles the students. The idea about the meanings of quantities seems to be limited to knowing the standard symbols and some formulae including these symbols, as if hints to the theoretical meanings of the quantities. Recognition of the property represented by a quantity is surprisingly difficult, and there is no feeling about the idea of quantification. The empirical meanings of quantities have remained obscure.

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[10] Webster's Encyclopedic Unabridged Dictionary of the English Language:

[11] Kaarle & Riitta Kurki-Suonio: *Fysiikan merkitykset ja rakenteet.* (*Meanings and Structures of Physics)* Limes ry., Helsinki, 1994. Chapter 3.2.3, passage headed "*Teorioiden näkökulma*" (*The view of theories*.)

The apparent difficulty of quantitative treatment of physics is critically coupled to the fact that one has tried to learn and teach it through "formulae" as a finished product, and its basic nature as a process has been disregarded[12]. The algebraic representation of laws is beyond the quantification. When starting from them one does not understand even to inquire for quantification. But the quantities, laws and theories are then left without empirical meanings, the key to their understanding. Without empirical meanings they are incomprehensible.

2.2 The preconditions of quantification

Every property is a problem of its own. For each property one has to create its own quantity[13], just because the properties are all different. There are both quantifiable and non-quantifiable properties. The degree of quantifiability varies. And even in the genuine quantification one can distinguish different types. The multiformity of quantification and its uniqueness in the context of each area of phenomena and of every single property puzzles and surprises again and again. That is why I can't even try to give a covering presentation but am satisfied with considerations, in light of examples, of principles and distinguishing characteristics of genuine quantification and, for comparison, of other kinds of operations which lead to quantitativeness without fulfilling the characteristics of genuine quantification.

In order to be quantifiable a property must have *different degrees*. Then, already the identification of the property involves *pre-quantifying* observations, *'comparative Gestalts'*. This means observation of different degrees (larger / smaller, stronger / weaker *etc.*) and /or changes (increasing / decreasing, strengthening / weakening *etc.*) and verification of the way in which the differences or variations of the degree manifest themselves.

In general, the properties are understood to have different degrees. This is apparent from the language, where the adjectives denote properties. All adjectives have a comparative form! But, while certain comparisons, like longer, faster, warmer, heavier *etc.* can easily be verified also quantitatively, many others, like better, more beautiful, more precious, more elegant, redder *etc.* depend much more on subjective evaluation. In order to be quantifiable the comparability of a property must, already on this qualitative level, fulfil the *requirement of inter-subjectivity*.

On the qualitative level also the *problem of linkage* is met, *i.e.* the question: *a property of what*? This is connected with the *nature of invariance of the property*, what factors the property is independent of and what it does depend on. We speak, for instance, of the colour of a material and of the weight of a body. These properties are, however, not genuine properties of a material or of a body, respectively. The colour of a material looks different under different illuminations, and the weight of a body changes when it is taken from the Earth to the Moon. So, these linkages are wrong.

Pre-quantifying comparison of the degrees of a property leads inevitably to *quantifying questions*, both relative: how much larger, how much stronger, how much more beautiful *etc*,. and absolute ones: how large, how strong, how beautiful *etc*. Answering requires quantification of the property. There are many different ways to try answering these questions.

Price. In a recent e-mail discussion I received a statement: *"The electric scales in the shops quantify, instead of the weight, another property of a body, namely the price."* At the first glance this looks fine. The price of a thing is, of course, a quantitative property, a quantitative representation of the expensiveness. Price fixing is quantification in the sense of the linguistic meaning of the word. The price has even a unit and a numerical value. However, it is not a quantity in the physical sense. The expensiveness isn't even a genuine property of a thing. The price depends on the pricer, on the pricing policy of the company and on the background expenses of the product, and it may change by bargaining. And, what happens to it when the thing is bought, when it is used and taken to the flea market to be sold further? It would not be possible to reach any inter-subjective unanimity of it by exploring the thing itself. The unit, euro or dollar or what ever, is the basis of price fixing and not vice versa, as it should be in a genuine quantification. The scales don't quantify, that is done by the pricer. The scales don't even measure the price but the weight. They have just been programmed to tell prices of things on the basis of the weight, the code of the product, and the pricing data fed in the electronic system. The price is quantitative but not measurable, hence, it is not a (physical) quantity. .

Evaluations. Different kinds of estimations, evaluations, scorings, tests, examinations, rating discussions *etc.* belong to everyday routine of the school and studies, at present also to the business life, administration *etc.* They are also trials of quantification, some kind of *forced quantification*. There is an administrative compulsion requiring "measurement" of the quality of performance and activity. Their problem lies in the very starting point of the process, in the property, which is difficult to perceive, in fact it is a tremendous tangle of properties. Combining it into one property, which could justifiably be called 'goodness' is, even in principle an impossible task. It is tried to improve the inter-subjectivity of its perception by common instructions and agreements about what should be emphasized. All resulting comparisons and 'measures' are inevitably subjective and open to interpretations, and they are regrettably often tinged with personal preconceptions or even arbitrariness of the measurer. They are 'operational' in the linguistic sense of the word, because they are realised

[12] *Cf.*. K. Kurki-Suonio: *Tuotteet ja prosessit.* (*Products and processes.* Farewell lecture.) Arkhimedes **2**/2005, 21 - 25. Complete version in English translation: http://per.physics.helsinki.fi/~kurkisuo/6.2.PubDidPhys.html

[13] *Cf.* K. Kurki-Suonio: *Massa opetuksen näkökulmasta.* (*Mass from the point of view of teaching*.) Arkhimedes **4**/2005.

by performing a certain operation, but they are not operational in the ordinary sense of the concept, which involves reproducibility, independent of the 'operator', and the requirement of inter-subjectivity. Neither are they genuine quantifications, since they are not based on direct comparison of degrees of the property in a way, which would be based on the perceived meaning of the property offering the possibility to *choose a unit* to *determine corresponding numerical values* for 'goodness'. Thus, the operation is not a quantifying experiment

Competitions. One, in our culture quite common, need for quantification occurs in situations of competition where qualities like *beauty, artistry, elegance etc.* of the participants or of their performances are compared. The problem is then in principle similar to that of the administrative forced quantifications but appears in practice a bit simpler. The tangle of properties to be reduce into one or two comparable qualities, is more limited. On any specific field of competition there prevails also a moderate mutual understanding about the competence or expertise; hence, the quantification can be entrusted to a board or a jury, with confidence in its ability to perceive the degrees of the quality in question. It is justified to call this *quantification by agreement.* The jury agrees upon the order of participants in respect of the quality giving perhaps points finding, thus, an apparent quantitative value for the degree of the quality. The operation is often ad hoc basing on principles negotiated by the jury for that very occasion. However, the commonly acknowledged expertise gives it a stamp of inter-subjectivity. Especially in many sports, like figure skating, gymnastics and ski jumping, there are also certain commonly agreed and registered principles, which are believed to guarantee the reproducibility of the operation.

Intelligence quotient. The *intelligence* is an example of man's personality properties, for measurement of which specific psychological tests have been developed. Also here the basic difficulty of the problem lies in the perception of the quality itself. The nature of intelligence the forms of the manifestations of its different degrees are under continual discussion, and no very great unanimity is in view. The *intelligence quotient* is, however, a moderately inter-subjective measure of 'intelligence. In tests where the intelligence quotient is 'measured', one examines certain types of performances thinking that success in them depends on the intelligence. Such tests have reached a fair degree of inter-subjectivity. Results of the tests are expressed on an agreed scale. However, as in the earlier examples, when the property itself has not been perceived, neither a genuine quantification based on the property is possible. The test itself defines, at the same time, the property which it is measuring. It is completely up to a subjective interpretation, what connection it has to the with-great-difficulty perceivable property of intelligence. This can still be considered *quantification by agreement*. Just that the agreement has now the more general nature of a permanent agreement.

Beaufort and richter. *The Beaufort scale of wind intensity and the Richter magnitude scale of earthquakes* can further be classified as quantifications by agreement. In these cases the property to be quantified, the 'strength' of a phenomenon, and different degrees of it are rather clearly perceivable. Originally these scales were based on observation of the effects of the phenomena and on qualitative, even internationally agreed, criteria about the nature of effects observed to occur at different grades of the scale. At present, these scales are coupled to wind velocity and to the energy or intensity of seismic waves so that it has become possible even to define the units 1 beaufort and 1 richter. Here the question is, however, no more about quantification of a property into a quantity, but about the use of quantities, quantified in the context of the general concept formation, to representation of two specific phenomena.

Phon and magnitude. *The phon scale of the loudness (level) of sound and the magnitude scale of apparent brightness of stars* are originally based on the estimation by sense perception of an assumed 'normal observer', when does the loudness sound or the brightness look double, triple, four-fold *etc.* They differ from the previous example in the respect that one is observing directly the property in question. Therefore, they come clearly closer to the ordinary quantification. But not even here the quantitative comparison of the degrees of the property is realised in a way, which would make possible the choice of a unit and measurement of the property by comparison with it. The possibility to interpret these scales in terms of quantities related to energy or intensity is obvious, but as in the previous case, this is application of ready quantified quantities to sound and light.

2.3 Genuine quantification

Quantification is the threshold process which raises the concept formation from the qualitative level to the quantitative one. *Genuine quantification transforms a quality into a quantity maintaining its perceived empirical meaning.*

As a starting point of genuine quantifications on a phenomenal area an inter-subjective *structural idea* is necessary concerning the nature and relationships of the basic Gestalts, *i.e.* of the entities, phenomena, their properties *etc.* Development of such an idea is a central aim of the preceding basic perception phase. It acts there as the intuitive basis of structurisation of empirical meanings. In the process of quantification it leads to a chain or a net of successive quantifications and quantities, where the nature of quantifications gradually develops. This structural idea is a kind of primary mental image or a basic hypothesis. When the quantification leads to a quantitative conceptual structure which is consistent with it, it is understood as confirmed.

For each property, a *quantifying experiment* is required as a primary operation to initiate the process. It is a series of reproducible quantitative experiments for accurate comparison of different degrees of the property to be quantified.

Using the old metaphor, nature must be forced to answer the quantifying questions *i.e.* to definitise the perceived quality in quantitative terms.

As the starting point for planning a quantifying experiment we need, as a part or product of the structural idea, a mental picture of an *idealized entity or phenomenon* which possesses the perceived quality as a *characteristic invariant property*. This involves a view about the nature of the invariance, *i.e.* both about the factors on which the quality depends and of disturbing factors which violate the invariance. This is a prerequisite for finding or arranging circumstances where the ideal entity or phenomenon can occur or can be realized and where it can be explored. It should be noted, that qualitative perception of invariance implies, by intuitive necessity, quantitative constancy of the value of the corresponding quantity. It is, thus, already a preliminary step from qualitative to quantitative.

In a quantifying experiment the idealized entity or phenomenon must be realized. The experimental set up must eliminate the disturbing factors and allow different degrees of the property to occur as invariants in each single experiment of the series. The core of the quantifying experiment is the *quantifying idea*, *i.e. an intuitively obvious principle* of quantitative comparison of different degrees of the property, based on the perceived nature of the property itself. As will be elucidated by later examples, it applies, in general, ideas like *similarity, symmetry and additivity*. Once the principle of comparison is invented and realized in the quantifying experiment, it becomes possible to choose, in principle quite arbitrarily, *a unit entity or unit phenomenon*, for which the magnitude or strength of the property is set as the unit. Comparison of the same property of another entity or phenomenon of the kind with that of the unit entity or phenomenon will then yield its numerical value in the units chosen, and completes, thus, the transformation of the quality into a quantity.

The quantifying experiment verifies the invariance of the quantity in ideal circumstances. It becomes the *defining law of the quantity.* It is important to draw attention to the nature of the invariance *i.e.* to the factors of which the quantifying experiment shows the property to be independent. This expresses at the same time the *linkage* of the quantity *i.e.* the class of entities or phenomena which possess the quality represented by the quantity as a characteristic property. This is included, at least implicitly, already in the quantifying idea, and it requires the quantifying experiment to be arranged as a series of experiments, where the invariance is verified by variation of parameters of the experimental arrangement in an appropriate way.

The ideal circumstances required by the quantifying experiment are restrictive. Therefore the primary area of validity of the quantity thus created is narrow. But the threshold from qualitative to quantitative has been crossed. The necessary continuation of the endless process of extensions and generalizations is beyond the scope of this lecture.

In light of this description it is clear, that the earlier examples do not have the nature of genuine quantification. There is perhaps some kind of a perceived structural idea behind, but there is no mental picture about an idealized entity or phenomenon and, first of all, there is no quantifying idea which would allow accurate comparison of degrees of the property. Thus, neither are the operations involved any quantifying experiments.

For every quantity, finding a quantifying principle is its own unique problem. In order to cover exhaustively the subject it would be necessary to go through practically all possible quantities. This is, of course, not possible here. I satisfy myself with a discussion of the first stages of the chain of quantifications with some complementary examples pondering in the light of them upon the multiform nature of the quantification. In accordance with the theme of these lectures, I shall pay special attention to the intuitive ideas on which they are based.

2.4 The starting phases of the chain of quantities

Phase 0. Number. As stated in the previous lecture, the starting point of the chain is the quantification of the magnitudes of sets of entities and series of events into the quantity '*number*', the possible values of which define the natural numbers. It is based on the perception of entities and events as *individual* and *separate* and of the consequent *additivity of the magnitudes* of such sets and series. The intuitive idea of this ***prime quantification*** is, thus, rooted directly in the mental-image structure of our inter-subjective ontological conviction described in the beginning of lecture 1.

Phase 1. Length and time. The first kind. Determination of numbers, *i.e.* counting, can now been used to quantify the *length* of entities and the *time interval* of events or *duration* of phenomena. The basic structural idea is the hypothesis of *homogeneous and isotropic space and time.* It underlies our perceptions that (1) *entities have invariant sizes* and (2) phenomena have definite characteristic *durations* independent of the position and orientation of the entity or system and of the moment of observation, and (3) the *lengths and durations are additive.* They form the intuitive justification of the simple quantifying idea of direct comparison.

In the quantifying experiment of length ideal rods are needed. We can choose a unit rod. We can prepare any number of rods with equal length, set them one after another beside another rod and then count the number of the units required to reach a fit, and define this number with the chosen unit as the length of the rod. The ordinary quantifying experiment should, actually, involve determination of ratios of such length-numbers of rods and verification of the invariance of these ratios and their independence of the choice of unit rod. The quantifying experiment of time is based on periodic phenomena as the ideal phenomena. Periodicity is, of course, an intuitive conclusion from the perception that the phenomenon consists of a long sequence of similar events. The numbers of periods of different phenomena in the same time interval are counted and their ratios are found invariant. Then, any periodic phenomenon can be chosen as the unit phenomenon and its period as the unit of time interval. Since the quantities length and duration, thus defined, are invariant, independent of time, position and orientation, they are characteristic to the entities and (repeatable) phenomena.

The way, how measurement of length and time is normally introduced to children at school, is, thus, completely orthodox quantification, just properly streamlined. However, the following generalization of this procedure, as it is presented in some textbooks, is strange, claiming that any measurement means finding out how many units the quantity to be measured will 'take'. I wonder, what does this wisdom make the pupils think about the measurement of, for instance, velocity, electric current or temperature.

The quantifications of the length of a body and the duration of a phenomenon present a basic type of quantification, which could be called ***primary quantification*** or ***quantification of the first kind***. More generally, it has the characteristics of being on an intuitive principle of comparison. Often this is inferred from symmetry of the situation and, particularly, from the perception that "similarity means equal magnitude". It may also involve the perception of additivity of the property, on the basis of which situations where the property is doubled or halved are intuitively obvious. We shall meet other examples later.

Phase 2. Velocity. The second kind. The next basic structural idea in the chain is the great insight of Newton, the perception that *interaction of two bodies is the common cause-phenomenon of the changes of the states of motion of both bodies.* It leads, at first, to the idea of a *free body*, *i.e.* a body without any interactions, and the *law of inertia*. This is the quantifying idea of the *velocity of motion*.

In the quantifying experiment the idea of free body must be realized. This requires, of course, that it is possible to recognize, on some intuitive grounds, occurrence of interactions and, particularly, their absence. In the circumstances on Earth this involves also the idea of directional specificity: a body can be horizontally free in spite of the unavoidable vertical interactions which are understood to balance. The ideal situation of horizontally free bodies can then be approached by reducing the horizontal contact interactions *i.e.* friction in the first place. In the experiment, motions of such horizontally free bodies are explored. The translation Δs in different time intervals Δt is measured. The proportionality $\Delta s \sim \Delta t$ is verified, the more accurately the more carefully the friction has been eliminated. This is in accordance with the idea of the free body and the law of inertia; hence the experiment supports the justification of the underlying intuition. Invariance of the ratio $\Delta s/\Delta t$ is confirmed to be true in all experiments of the series, independently of the time interval and the speed of the motion, while its value varies from case to case. It is, thus, characteristic to the entire phenomenon, the motion of a free body. Because its values correspond to the comparative Gestalts of the perceived speed-property – they are always larger for the speedier motions –, it can be defined as the quantity representing this quality, the velocity $v = \Delta s/\Delta t$.

In this context, two important general notes should be made:

The first note concerns the nature of the law of inertia, or the first law of Newton. The discussion brings out emphatically, that it is, in the first place, an intuitive idea. It can be considered an empirical law only in the respect, that the results of the quantifying experiment, afterwards, fully support the idea.

The second note concerns the idealisation, necessary in the quantifying experiment and, more generally, in all experimental research. The idealisation must be *empirically justified*. The ideal situation is always unattainable, but it must be possible, in principle, to reduce the deviation from the ideal beyond any preset lower limit. One can become convinced about the realisation of this requirement only on intuitive grounds, and this belongs essentially to the ideation of the experiment. In quantification of the velocity, the idealisation required by the idea of free body can be considered empirically justified because there is always the possibility to reduce the motional resistances by improving the setup of the experiment. This refers to rolling as well as to gliding bodies, which both do well in the free-body experiment. However, exploring collisions of free bodies, which is necessary in the next phase, using rolling bodies (like billiard balls, as is sometimes suggested in textbooks), offers a counterexample. The effect of horizontal external forces cannot be reduced below a definite lower limit.

The *velocity of motion* represents the second basic type of quantification, which can be called ***quantification of second kind*** or ***secondary quantification***. There, already the property is primarily perceived as the rate of dependence of some known quantities. The quantity gets then quantified as an invariant representing the strength of this dependence.

This invariance can be inherent in the idealization required by the quantifying idea. By the quantification of *velocity* it is in the free-body idea. Correspondingly, quantification of the *density of matter* is based on the idea of homogeneous matter referring directly to the invariance of the ratio $\Delta m/\Delta V$ of the mass and volume of samples taken of the same kind of matter, independent of the size of the sample..

It is also possible that the invariance is observed as an empirical result of the quantifying experiment. For instance, in the quantifying experiment of the *stiffness of a spring* it is observed that elongation of the spring is proportional to the stretching force, $\Delta x \sim \Delta F$. The invariant ratio $\Delta F/\Delta x$ can be defined as the *force constant* of the spring, a quantitative measure of its stiffness. Similarly by investigation of electric circuits it is observed that the current in a conducting wire is proportional to the voltage between the ends of the wire, $I \sim U$. The invariants U/I and I/U can then be defined as the *resistance* and *conductance* of the wire, representing the abilities of the wire to resist and to conduct electric current.

In order that the invariant could be understood as a quantity which represents the property, its magnitude in different situations must correspond to the "comparative perception" of the property, at least, it must be systematically larger in experiments where the property is stronger.

Phase 3. The mass. Quantification of velocity as an invariant of free-body motion makes possible the important next quantification in the chain. *Inertia* as a perceived property of a body is "the ability of the body to resist changes of its state of motion". The great structural basic idea of Newton, *interaction as the common cause phenomenon of the changes of the states of motion of bodies,* leads now to the quantifying idea according to which *interaction of two bodies* A and B *compares their inertias*. If the velocity of B changes more, it has smaller inertia. If the change is twice as large as that of A, its inertia is half of the inertia of A. The realization of the idea requires a quantifying experiment, where two free bodies undergo an interaction of short duration.

The ideal situation of the experiment can be approached by collision experiments, where the velocities of the uniform motions of the bodies before and after collision are measured. The results of the experiment confirm the justification of the idea. The ratio $|\Delta \boldsymbol{v}_A|/|\Delta \boldsymbol{v}_B|$ of the absolute values of the velocity changes turns out to be invariant, independent of the initial velocities and of the nature of the interaction, and all such ratios of pairs of bodies can be expressed in terms of body-specific numbers. When the inertia of an arbitrarily chosen unit body is taken as the unit, the inertias of all bodies get, thus, numerical values characteristic to the body. In this way a quantity representing the inertia, the mass, is born. The proportionality $m_A/m_B = |\Delta \boldsymbol{v}_B|/|\Delta \boldsymbol{v}_A|$, valid in all interactions of the same bodies, becomes its defining law. This quantification process has the nature of *primary quantification*, where the interaction acts as the intuitive principle of comparing inertias.

Here the *difference between quantification and measurement* manifests itself clearly. Certainly the masses of the bodies get measured in the quantifying experiment, but the experiment is impracticable for that purpose. In the process of concept formation it, however, leads further to other basic concepts of mechanics, particularly to momentum and force. In this way it becomes possible to verify empirically the proportionality of the gravitational force acting upon a body to the mass of the body $G \sim m$, on which the determination of mass by weighing is founded. It is notable that the empirical verification of this proportionality is one of the most accurate quantitative experiments of physics. Although the quantifying experiment does not measure a quantity very accurately, it defines accurately its meaning and makes, thus, possible development of more accurate methods of measurement.

Phase 4. Momentum and impulse. The third kind. The collision experiment of free bodies acts directly, without any complementary experiments as the quantifying experiment both for the *amount of motion* and for the *strength of the interaction event.*

When, in the context of this experiment, attention is drawn to the opposite directions of the velocity changes, the defining law of mass can be written in the vectorial form $m_A\Delta \boldsymbol{v}_A = -m_B\Delta \boldsymbol{v}_B$. In order that the concepts correspond to the structural basic idea of Newton about interaction as the common cause of the changes of motion, these changes must be expressed in terms of a motional quantity for which the changes are equal. It becomes, thus, obvious that the correct quantity for representation of the state of motion is the *momentum* $\boldsymbol{p} = m\boldsymbol{v}$ of the body.

At the same time, the magnitude of the equal momentum changes indicate the strength of the interaction event and offer a measure for it. It is essential in the structural idea, that the motion of a body and the interaction of two bodies are different phenomena. Therefore both need their own quantity for representation of the magnitude or strength. While the former is represented by the *momentum of the moving body*, the latter is named *impulse of the interaction.* This quantity has actually the nature of a *bivector* $(\boldsymbol{I}, -\boldsymbol{I})$. The equal opposite momentum changes show its magnitude $I = |\Delta \boldsymbol{p}_A| = |\Delta \boldsymbol{p}_B|$. In discussing separately the effects of the interaction upon its two counterparts, it becomes practical to adopt the concept *impulse given by the interaction to a body*, which actually is the other part of the impulse bivector and equals the momentum change of the body caused by the interaction $\boldsymbol{I} = \Delta \boldsymbol{p}$. This is how the traditional mental picture of the teaching of mechanics, impulse and counterimpulse, is born.

In this way Newton's structural basic idea of interaction creates the basis for the quantification of the whole of the basic concepts of Newtonian mechanics. It leads to the defining laws of quantities, which are easily recognised as macro representations of the three basic laws of Newtonian mechanics.

The quantifications of momentum and impulse are genuine quantifications of empirical meanings. In the process described, they do not have their own quantifying experiments, but their defining laws follow from the quantifying experiment of mass on the basis of their perceived meanings. This can be named the third kind of quantification, based on earlier quantities and *on the causal structure of the empirical meanings.*

From the viewpoint of principle it is worth while to pay attention to the difference between the quantifications of these two quantities. Momentum gets quantified as the quantitative representation of the state of motion directly on the basis of the perceived structural role of the change of state of motion in an interaction. The quantification of impulse is indirect. It is based on the effects of the interaction phenomenon, the momentum changes, the magnitudes of which are measurable. In this respect it resembles the gauge quantification to be discussed later.

2.5 Complementary examples

Weight. The *weight of a body* is, beside length and time interval, one of the very first physical quantities of mankind. It too is born in a quantification of the first kind. The quantifying experiment is comparison of weights by weighing. The idea is based on symmetry and on the idea of additivity of weights. It is intuitively obvious, that bodies in the pans of a symmetric balance have equal weights when the balance is in equilibrium, and that the common weight of *n* equal bodies is *n* times the weight of one of them. Then, it is possible to prepare unit bodies of equal weight and, further, sets of weights, *i.e.* bodies with weights of multiple units and simple fractions of units, which then can be used for weighing of different kinds of bodies. .

The linkage of weight to a body is an understandable mistake. The dependence of weight on the position of the body on Earth is so subtle, that it is not easy to observe. Weighing as a relative measurement on a fixed position does not reveal it. And its direct determination as a force by simple tools like spring scales is not accurate enough. Behind this linkage there was also haunting the antique mental picture of weight as the bodies' natural striving downwards. A sufficiently accurate measurement of weight on different places on Earth confirms, however, that it is not an invariant property of a body. But it was not until Newton's structural basic idea that its correct linkage to gravitational interaction was revealed. It is a quantity which represents the strength of interaction between the body and the Earth (or some other central body), which as to its nature, is a *bivector* consisting of a pair of forces acting upon the body and the Earth.

Amount of substance. Quantification of the amount of matter to the quantity *amount of substance* with the SI unit *mole* is the starting point of the empirically justified atomic physics. It too is a primary quantification. The *chemical reactions* of different kinds of substance form the intuitive basis for comparison of their amounts. They offered a method by which it was possible to get an intelligible answer to the question, when do we have equal amounts of two different kinds of substances. In this case, the quantifying experiment is the whole set of the quantitative investigations of chemical reactions, which has been needed for determination of the relative atomic and molecular masses characteristic to the different chemical substances. On their basis some, in principle, arbitrary amount of some arbitrary substance can be chosen as the unit, after which the ratios getting determined by chemical reactions expresses how much of any other substance contains the same unit amount of substance.

2.6 Gauge quantification.

Quantification can also be started by taking as the starting point, instead of the property itself, some known quantity, which obviously depends, in some well defined phenomenon, in some simple way on the property examined. Adopting the *length of a column of mercury* as the measure of *temperature* is a good example. This is not genuine quantification, but it is a very general principle of *measurement*. Quantities are measured with gauges, where, for instance, the turning angle of a pointer acts as the measure of the quantity. Turning of the pointer has a causal relation to the quantity to be measured through a well known phenomenon, but the angle as a quantity has nothing to do with the meaning of the quantity as a property. Similarly, through thermal expansion, the length of the column of Mercury has a causal relation to the temperature of Mercury, although length as a quantity does in no way reflect the meaning of temperature as the degree of warmness of matter.

Again, one has to note the difference between quantification and measurement. A quantity does not exist as a concept before it has been quantified, therefore it can neither be measured genuinely before. *Gauge quantification* sets the "gauge quantity" as the measure of the property examined. It is not a genuine quantification, because it does not base on a quantifying experiment comparing degrees of the property, where one could choose a unit degree for the property. The unit of the gauge quantity or some proper multiple of it becomes the unit for the property measured, and it can be given, for eyewash, an own name. Only after quantification it becomes possible to *calibrate* the gauge so, that it can measure the quantity in terms of its own units. The gauge "defines" the quantity to be measured in the same sense as the intelligence is defined by the IQ tests or the wind and earthquake power by the beaufort and richter scales. However, what is different, it is accurate as an operation, because it is based on an exactly defined quantity. Gauge quantification can, thus, be regarded as a definitised quantification by agreement. Because of the arbitrariness of the gauge quantity, in principle, and of the choice of unit I have called it also *ad hoc quantification*.

The gauge quantification of temperature acts also as the starting point for the genuine definition of temperature as a quantity because it has as its basis a supporting structural idea about heat phenomena. The core is the idea of *thermal equilibrium*, according to which contacting bodies take equal temperature. It contains also a preliminary idea about transferring and conservation of heat and of the ability of bodies to store heat, which quite obviously is large for large bodies and small for small bodies. In this way it is understood, that the gauge will show, in addition to its own temperature, also the temperature of a body which is in good enough contact with the gauge, and that the heat-storing ability of the gauge should be negligible as compared wit that of the body. By using the gauge a clear empirical support to this structural idea is obtained.

The investigations of the so called ideal-gas laws act as the basic quantifying experiment of temperature. The quantity pV/n, where p, V and n are the pressure, volume and amount of substance of the gas, respectively, turns out to be invariant having the same value for all gases in the rare-gas limit in the same temperature. The quantities involved have been quantified earlier in the chain, and equality of temperature can be stated on the basis of the preliminary gauge quantification. As to its nature, this quantity corresponds to the intuitive meaning of temperature as a quantity representing the thermal state of the gas. Therefore, it would have been possible to adopt it directly as the definition of temperature. In this way temperature would have obtained the unit of *molar energy* corresponding to its meaning learned later in the context of statistical mechanics. A separate unit of temperature and the molar gas constant as the conversion factor of units would have remained unnecessary. However, in the history of thermal physics the time was not yet ripe for that. Regardless of that, this is an obvious quantification of the second kind, where temperature is born as a physical quantity. It made possible to definitise the originally gauge-based *ad hoc* unit into a genuine physical unit.

In a corresponding way, weighing is originally a gauge quantification both of the fictitious amount of matter and of mass. It is intuitively necessary that the weight increases with growing amount of matter and mass. However, weight of a body cannot, in any sense, be regarded as a property perceived as amount of matter or inertia.

Also the quantitative treatment of electricity has started by gauge quantifications. The *electric force* acting upon a charged body and the *magnetic force* acting upon a conducting wire, or gauges based on them, were taken as the measures of *electric charge* and *electric current*[14]. Force is, however, a property of interaction, while charge and current are instead perceived as properties of a body or a conducting wire. Behind here lies the basic structural idea, according to which the charge and the current as properties of the body and the wire, respectively, are their abilities to electric and magnetic interactions. Also here the basic idea leads forward in tehe conceptual development.

The quantifying experiments are based on the intuitive ideas of conservation, symmetry and additivity. When charge is brought to a body in n lots, which on the basis of the experimental arrangement are plausibly equal, the body obtains obviously an n fold charge. At the same time it is observed that the electric force acting upon the body is n folded. Coulomb made a corresponding experiment sharing the charge of one body among similar conducting bodies by putting them simply in contact. He got, thus, obviously a set of fractions 1, 1/2, 1/3, … of the original charge, and verified the diminishing of the electric force by a factor of $1/n$. Correspondingly, an intuitively obvious n fold current occurs in the "collector wire" of n parallel wires, and the n folding of the magnetic force can be verified. Such experiments are quite inaccurate as measurements, but they are important in principle. They support the original structural idea and complete the gauge-based *ad hoc* measurements into genuine quantifications.

2.7 And so on

In thermal physics the quantifications of *heat* and *specific heat capacity of a substance* follow the temperature as the important next stage. The intuitive basis consists of the ideas of *conservation and additivity of heat* plus the idea that an n fold amount of matter will require n fold amount of heat for equal rise of temperature. In the quantifying experiments the temperature changes ΔT_A and ΔT_B of two one-substance systems A and B set in thermal contact are measured and compared. When different amounts m_A and m_B of the same substance are used, the result $m_A\Delta T_A = -m_B\Delta T_B$ obtained, independently of the masses and the initial temperatures (within reasonable limits, of course), confirms the idea of conservation and transfer of heat, and the quantity $m|\Delta T|$ offers a measure for heat. When, further, there are equal amounts of two different substances, this experiment compares their heat-storage abilities. A unit substance can be chosen, and the experiment becomes a typical primary quantification of *specific heat capacity of the substance*. With water as a natural choice we are then lead to the unit 1 cal/kg.

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[14] The very first definition of electric current was based on the amount of substance carried by the current in electrolysis.

In the study of ***electric circuits*** the quantification of the *strength of a battery* into its *source voltage* is based on symmetry and additivity and on the idea that *n* similar batteries in series form a source with *n* fold strength. This mental picture is parallel with the basic idea of the quantification of electric current, and it is perceived together with it in pre-quantifying examinations of circuits, where equal brightness of similar small bulbs can be understood as indication of equal current.

In optics the *refractive index* is an interesting example. For light refraction in the boundary of two transparent substances, it is easy to verify experimentally the independence of the ratio $\sin\alpha/\sin\beta$ of the angle of incidence. This invariant is characteristic to the pair of media (and the wave length of light). It quantifies the "strength" of the refraction phenomenon in the boundary and is named *refractive ratio of the boundary*. But inventing this expression is a challenge[15]. It requires as its background a rather detailed build up of a mental picture of the behaviour of waves – or of a beam of particles – on a boundary.

In this way, the refractive ratio is born in a second-kind quantification based on angle as a known quantity. Instead, proceeding from this further to the refractive index of one substance can be understood as a quantification of the first kind, where the refraction phenomenon itself acts as the comparing principle of the "refraction abilities" of substances. Vacuum is a natural choice for the unit medium. Linkage of the refractive index to each medium separately can be perceived in an experiment, where light is transmitting several layers of media and the ray direction is found to be always the same in the same medium, independent of its position relative to the other layers in the system..

It would be tempting to continue by pondering upon the basic ideas of different quantifications and their nature in relation to the basic types presented. Particularly, it has appeared in this presentation only allusively, that the process of quantification of a quantity often is a multiphase process. It also continues as successive complementary quantifications generalising and structurising the quantity. They are needed, especially, in progressing from macro quantities to pointwise and momentary quantities and from scalars to vectorial and tensorial quantities, to four vectors etc.

The genuine quantification based on perceived empirical meanings is a special feature of the concept formation of physics. Although quantitative concepts and quantitative research occur on many fields, a corresponding operation, where the quantitative concept is created by comparing degrees of a property on the basis of the perceived special nature of the property, does not occur elsewhere. The system of quantities based on empirical meanings, thus building, is unique in its kind. Insights of the intuitive principles of comparison, which have been needed as the necessary bases of the quantifications, have had a decisive significance for the development of physics as primary sources of the definitions of quantities. Still today, they have the same significance from the point of view of learning to understand the concepts of physics. They also reveal something quite essential about the position of intuition in physics.

[15] The difficulty is well demonstrated by Kepler's numerous unsuccessful attempts to find a proper mathematical expression for representation of the empirical law of refraction.

3. CONCEPTUAL HIERARCHIES AND THEIR SIGNIFICANCE IN PHYSICS TEACHING

3.1 The Hierarchies

Hierarchy is a general type of structural order. The structure of a system is hierarchical when it can be perceived to consist of subsystems of subsequent orders, levels or degrees with respect to some property or principle. Subsystems of one level are mutually comparable; they consist of subsystems of the next-lower order as their elements and are themselves elements of the next-higher-order subsystems. The primitive elements of the system form the lowest hierarchical level and the system as a whole is regarded as the highest level.

We live in the midst of many kinds of hierarchical systems. There are *natural hierarchies, hierarchical functional organisations and cultural hierarchies.*

Natural hierarchies. Matter of the universe is grouped into *a hierarchical chain of entities*[16]. In this all-covering *structural hierarchy* the entities of each level consist of entities of the preceding lower level and they are themselves structural elements of the next higher level. According to our present knowledge, there are the quarks and the leptons at the lower end of the chain, and the groups of galaxies or, perhaps, still larger groups of these galactic groups in the upper end.

In the organic nature one can recognise a corresponding *structural hierarchy of beings*. Individuals of animals and plants consist of organs, the organs of certain functional parts, these parts of cells. The cells have their own structure. Finally, the structural parts of the cell are built of molecules. At its lower end this hierarchy of beings, thus, joins to the general structural hierarchy of material entities. Ecological and social structures can be understood as a continuation of this hierarchy towards upper levels. Human individuals form nuclear families, the nuclear families form larger wholes of relatives (in English also called families), tribes, nations, groups of kindred nations. The mankind as a whole is comparable with the "communities" formed by the different animal species. Together they form the world fauna, which has a corresponding "social" hierarchical structure *etc.*

The structuring principle of the material world can be seen to be based on a *hierarchical chain of interactions.* In this *hierarchy of phenomena* the strength of interaction acts as the generative principle of hierarchy[17]. The individualization of entities on each level requires that the internal interactions of the entities, *i.e.* the mutual interactions of their structural parts, are essentially stronger than their external interactions, *i.e.* mutual interactions of the entities of the same level, which, further, act as the internal interactions of the next-higher level.

Also the hierarchies of beings can be seen to be based on interactions as the building and maintaining factor of the structure. The ties of the members of a nuclear family are stronger than those between different families *etc.* Having once seen a swarm of waxwings in a Finnish winter day, or swarms of anchovies or bees in a nature program of TV, one cannot avoid thinking that there is some interaction binding the individuals together, whatever its nature.

Hierarchical organizations. There is an immense amount of different kinds of organisations acting in our society. There are the defence establishment, the school system, commercial, industrial and administrative institutions, the church and other religious societies, social organizations, organizations of different cultural areas, an innumerable set of different societies and associations devoted to some specific ideas or targets *etc.* All possible areas of human activities have their own organisations. They in themselves are examples of hierarchical systems formed by people. Often there are also hierarchically higher, international levels where they are elements as members. Their structure and activities are organised hierarchically. The hierarchical relations are based on appropriate principles of relative competence or power as decided, agreed or otherwise assumed within the area in question.

Cultural hierarchies. By culture I mean particularly *science, technology and art*. I consider them and all of their types processes of mankind. Then all the hierarchies of the scientific and artistic *products, procedures, methods and procedural knowledge* created by these processes can be considered cultural hierarchies. Although I am discussing them specifically from the point of view of physics, a great part of my ponderings applies similarly to all areas of science, technology and art *mutatis mutandis.*

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[16] This has been discussed in the Finnish secondary school textbook *Galilei 1. Fysiikka luonnontieteenä.* (*Physics as a natural science.*) Jari Lavonen, K. K.-S. & Harri Hakulinen. Weilin+Göös. 1994. Chapter 3.1 as well as in the university ground-level textbook *Vuorovaikiuttavat kappaleet – mekaniikan perusteet. (Interacting bodies – foundations of mechanics.)* Kaarle & Riitta Kurki-Suonio. Limes. Helsinki 1995. Chapter 5.1.1.

[17] Ibid. Chapters 3.2. and 8.2.5, respectively.

The *cultural processes* which are creating science, technology and art are *common processes of the mankind*. The realisation of them requires a creative process of *individuals*, which, particularly in the starting phase, can be identified with learning. The picture of the cultural hierarchies can be clarified by a model of the processual structure of science and learning consisting of the *scientific*, the *technological* and the *social process*.. (If art is to be included, the model could be complemented by the *esthetical process* which certainly is significant also in science.)

The scientific process aims at *understanding*, conceptual mastery of the world. It considers the world "such as it is" and it is building the *world picture*. Its functioning is *perceptual concept formation.*

The aim of *the technological process* is *utility*, development of the world to fulfil better the human needs and wishes, and adjustment of the human behaviour to the possibilities of nature. It tends to *change the world*. Its action is *purposeful manipulation of the world*.

The social process aims at *mutual understanding*. The essence of it is often said to be *negotiation about meanings.*

The scientific and the technological process are inseparably interwoven. The changes sought by the latter one require conceptual mastery. Arrangements necessary for observations and experiments involve manipulation of nature, by which nature is "compelled" to answer our questions aiming at conceptual mastery of nature. Primarily, the scientific and the technological process get realised in a creative interaction between mind and nature. Because of the requirement of inter-subjectivity they are, however, submitted to the social process (*cf.* the introduction), which combines the individual minds into a common "social mind". In this way, the processes of the individuals get united into a "great process" of mankind[18]. Learning and science can, thus, be considered one and the same process on different levels of the social hierarchy.

Each of these processes is creating its own hierarchy of *products*. The scientific process is building a multiply hierarchical *conceptual structure*, some aspects of which were discussed in the previous chapters. The technological process is producing, particularly, hierarchies of devices and machinery. As an example one can think for instance the computers, from electronic components up to worldwide nets, or any industrial production system.

In the social process *language* is both a primary product and the principal methodical tool. Language, each language separately and the whole of the world languages is a multiply hierarchical system. Its formal structural hierarchy starts from sounds and letters, which form words. The words form sentences *etc.* language contains also the terminology developing along with the hierarchical progress of science and technology. Language follows the development of all cultural processes. Thus, the social process inseparably involved in the build-up of the scientific and technological hierarchies. It acts in and between the levels of the social hierarchy of human communities as both internal and external interactions.

At the same time, the processes are developing the hierarchies of their own *methods*, methods of concept formation, methods of manipulating the world and methods of negotiation about meanings, as well as hierarchies of their *methodical and procedural knowledge*, whereupon they are themselves subject to hierarchical development.

3.2 Processes as generators of hierarchy

Generation of hierarchies seems to be a *general aim of nature as well as of human mind*. This statement seems innocent and obvious, but the word "aim" involves an "inflammable" idea indicating that for generation of hierarchy a purposeful process is required. Processes generating hierarchies are started and they are functioning as driven by their aims. The hierarchical systems are born and developing as realisations of the aims. In this way, this statement opens the great questions about the nature of the organising principles acting in nature and human mind, about the relation of *causality and teleology*, about the relations of natural laws, chance and voluntary action or dispensation, even the problem of free will – and seduces us again into digression far from our ordinary subject. By drawing a parallel between nature and human mind, it refers also to the great question about the relation of reality and mind.

A hierarchical organisation is clearly product of a voluntary purposeful process. It is planned starting from the aim of practical and efficient realisation of certain functions. It is founded consciously in accordance with plans of its founders and it will developed for better efficiency of its function. The starting point is the *organising will* of some one or some group of people – it may, of course, also be against the will of some individuals or communities. Building of the organization is as such a *concrete aim*, and the process of its generation is a *finite project*. – This is not to be mixed up with the aims of the functions for which the organization is founded and the (cultural) processes in which the organization

[18] K. Kurki-Suonio. *Suuren prosessin paradigma*. (*Paradigm of the great process.*) Dimensio **60**. 3/96.

is participating. – Once the organisation has been erected, the aim has been reached. The process of hierarchy generation continues only marginally as the development of the organisation.

The *cultural processes* generating *cultural hierarchies* of concepts, languages, equipment, methods and methodical knowledge are obviously aim-oriented as well. *They can even be identified with their aims*. They are born from their aims and their essence is the endless endeavour towards these aims. The aims can, thus, be understood as definitions of these processes. They are not just aims of individuals or groups but they are spiritual or intellectual cultural aims common to the mankind. They are unattainable aims guiding the endeavours of man, so that also the processes driven by them are endless[19].

There is no doubt that theses aims are conscious, although their nature is to a great extent intuitive. The cultural hierarchies result from an enormous amount of voluntary actions of people – individuals, groups, communities and human organizations. However, the directions of propagation of science, technology and art often warrant wondering, whether these processes really are fully under conscious control of man, or does the development of science, technology and art obey some laws independent of man's will, which we are realising just apparently on our own will. How do we understand the birth of the same insights on different sides of the world simultaneously and independently of each other and occurrence of parallel trends at the same time on different areas of science and art[20]? What causes the impressions about science proceeding "against the current" and about man as a slave of the technology? If the development of the cultural hierarchies is following its own laws, are the laws causal, following from some primary cause, or teleological aiming at some target? If they are teleological, what is the plan, will or guidance they are implementing?

With regard to the *natural hierarchies* science has a definite stand. The paradigm of science sticks fast to the belief that nature is build hierarchical according to its own causal laws. The "principle of science" does not allow any alternatives. The hypothesis of "intelligent design" does not fit into the patterns of science. The whole conceptual development of science is built on the basis of causal relations, causal mental pictures and causal models, in a way discussed briefly in the previous lectures.

Science discusses the development of the universe and its arrangement into material structures of different degrees as a process, the different stages of which we believe to understand gradually better and better. Physics is offering us scientific mental images about those processes which have produced the constituents of matter, as well as of those, which have caused the grouping of matter into galaxies, stellar systems and stars *etc.* Interactions following there causal laws act as the organising principle of these processes. The laws of interactions form the "method of progress" of the natural processes. We can see how these processes are still continuing and producing particles and material, stars, stellar and planetary systems.

According to the scientific conception, the development of a living individual being, also that of a human, is a causal process. The growth process, where cells are dividing and getting specialised producing all the different organs and the whole hierarchical structure of an individual, is conducted by a species-specific DNA molecule – following basically the physical laws of interaction. The *origin of the species* is considered, at least since Darwin, as a natural development process. Its causal progress is, however, guided by two statistical random phenomena, the mutations on the molecular level and natural selection on the macroscopic level, which, thus, would be responsible for the whole of the tremendous hierarchy of species of nature. In recent times, a school which strongly believes to be able to explain also the awakening of consciousness in a "natural process, conducted basically by the causal laws of interactions, has gained a firm footing in science.

On the other hand, it seems natural, in the context of the subject under discussion, to present the question about a possible *hierarchy of consciousnesses*, suggesting that the human individual consciousness would represent one specific level of it. The question about the nature of the different-level interactions, which create the hierarchy of the social communities, together with my earlier allusions to the origin of inter-subjective intuition and to the possibility that the cultural development is obeying its autonomous laws, lead easily to ponderings about the possibility of social consciousnesses of different levels, perhaps even independent of man. In any case, throughout the history it has been considered intuitively obvious that there is a higher directing objective-oriented will behind all development of nature. This mutual contradiction between the paradigm of science and the inter-subjective intuition maintains the "great dialogue", the continuation of which I consider one of the carrying forces of the culture of mankind[21].

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[19] *Cf.*. K. Kurki-Suonio: *Tuotteet ja prosessit.* (*Products and processes.* Farewell lecture.) Arkhimedes **2**/2005, 21 - 25. Complete version in English translation: http://per.physics.helsinki.fi/~kurkisuo/6.2.PubDidPhys.html

[20] *Cf.* Bronowski, J.: *The Ascent of Man*. BBC, London (1973).

[21] Vrt. K. Kurki-Suonio: *Kristinusko ja luonnontieteellinen maailmankuva*. (*Christianity and the scientific world picture)* In "Opettaja tulevaisuuteen kasvattaja" (Teacher, educator for future.) Ed, Esko Kähkönen & Markku Pyysiäinen. Kirjapaja, Helsinki, 1986.

In the total picture discussed, the *conceptual hierarchies* position as a product of the scientific process – not forgetting the necessity of the technological and the social process in their creation. According to our intuitive conviction, the concepts represent genuinely the reality of nature. Therefore we perceive also their hierarchical arrangement as an inherent feature of reality. The genesis process of the concepts warrants asking again and again about the relation of the conceptual hierarchies to the reality, to the hierarchies of entities, phenomena, properties and laws. Do they reflect the nature of the ontological reality, or are they just some order resulting from the perception tendency of the human mind? Perception of meanings and their conceptualisation are organising processes. Understanding of the observable world means perception of order in the observations. Therefore it is just natural that the concepts are forming hierarchical structures. But to what extent this is just some perceived order created by human mind, required by the aim of understanding and dictated by the potentialities of the human mind, and to what extend is it genuine order of nature? Thus, we are again ending up in the unsolvable problem about the relation of human mind and reality.

In this respect, different conceptual hierarchies seem to differ to some extent. Classifications based on different degrees of some property, occurring *e.g.* in the context of operations aiming at quantification (*cf*. previous lecture), are examples of such simple hierarchical sequences which are quite obviously open to interpretation and constructed just because of the need of conceptual analysis. On the other hand, sets of concepts representing the structural hierarchy of natural entities and the related hierarchy of the physical interactions, are easily perceived as genuine descriptions of reality. Crumbling of the concrete world picture, due to the modern physics[22], puts even here a big question mark.

3.3 The two dimensions of conceptual hierarchy

In the aim of understanding, which gives rise to the scientific process and maintains it, one can distinguish two dominating basic components, which act, both in their own ways, as generative principles of conceptual hierarchy. This makes the conceptual structure of physics two-dimensional[23].

The aim of *understanding* leads, primarily, to a ***generalisation hierarchy***. Chapter 2.3.2 of our book (*Meanings and Structures of Physics*) says: "*Understanding means identification of more and more wide and general structural Gestalts of natural phenomena and representation of them in terms of gradually more and more general concepts." … Single events are understood, when they are perceived as different kinds of occurrences of one and the same phenomenon. Phenomena and their empirical laws are understood, when, by investigation of many phenomena, one finds more general phenomena and laws, containing them as special cases. Independent phenomena are recognised as different realisations of more and more general phenomena in different situations. The basic phenomena, entities and laws are very general Gestalts, which make a wide set of different phenomena understood as special cases of them." … "In this way the conceptualisation is serving the continually generalising structural perception." … "Physics does not offer any final explanations, but it leads to a hierarchical chain of more and more general and profound explanations."*

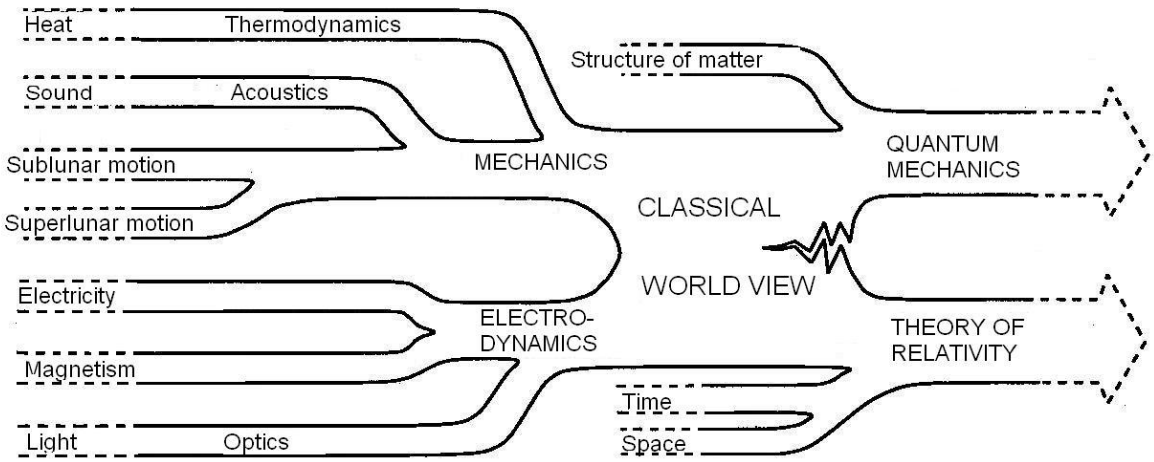

Figure 3.1 The main features of the generalisation hierarchy in the classical period of physics development.

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[22] Kaarle & Riitta Kurki-Suonio: *Meanings and Structures of Physics.* (In Finnish). Ch. 5.2.2. Limes ry., Helsinki, 1994.

[23] *Cf.* Irma Hannula: *Need and possibilities of astronomy teaching in the Finnish comprehensive school.* Dissertation. Report series in physics HU-P-D124. University of Helsinki. 2005. Chapter 5.2.3.

Our book illustrates the unification development, represented by the hierarchically generalising conceptual whole, by three schemes. Figure 3.1 is a reduced version of the first one (Fig. 2.7 of the book), a general view on the formation of the generalisation hierarchy in the classical period of physics. In the last chapter of the book this is taken up anew. The main features of the unification development of electricity, magnetism and light are presented in more detail (Fig. 5.8 of the book) and, finally, the scheme is continued to the area of modern physics (Fig. 5.9 of the book).

The second dimension of the conceptual hierarchy is spanned by *the aim of exactness*. It is creating a ***quantification hierarchy***, the origin and structure of which was discussed in the first lecture (see Figs. 1.1 and 1.2)

These two generative principles of hierarchy are tightly coupled together. The bipartite basic structure of the quantification hierarchy consisting of the qualitative and the quantitative level, as well as their similar three-phase structures, recur in the concept formation of every phenomenal area and of each single phenomenon. Quantities, laws and theories are built on the perceived properties, dependences and causal models. It can be seen in the development of physics how generation of the quantification hierarchy has created necessary foundations for the progress in the generalisation hierarchy. The quantitative concepts of lower-level single phenomena have acted as a part of the qualitative perceptual basis, on which the mental image about the unification of the phenomena into a more general "umbrella phenomenon" has been based. At the same time, this progress gets projected on all levels of the quantification hierarchy so, that the generalisation hierarchy is seen, not only as a phenomenal hierarchy but also as hierarchical generalisations of quantities, laws and theories concerning the more general phenomena. Figure 3.2 is an attempt to describe this coupling of the two generative principles of hierarchy. Actually, the mini-scheme of quantification hierarchy belongs to every detail, to every branch both before and after the junction.

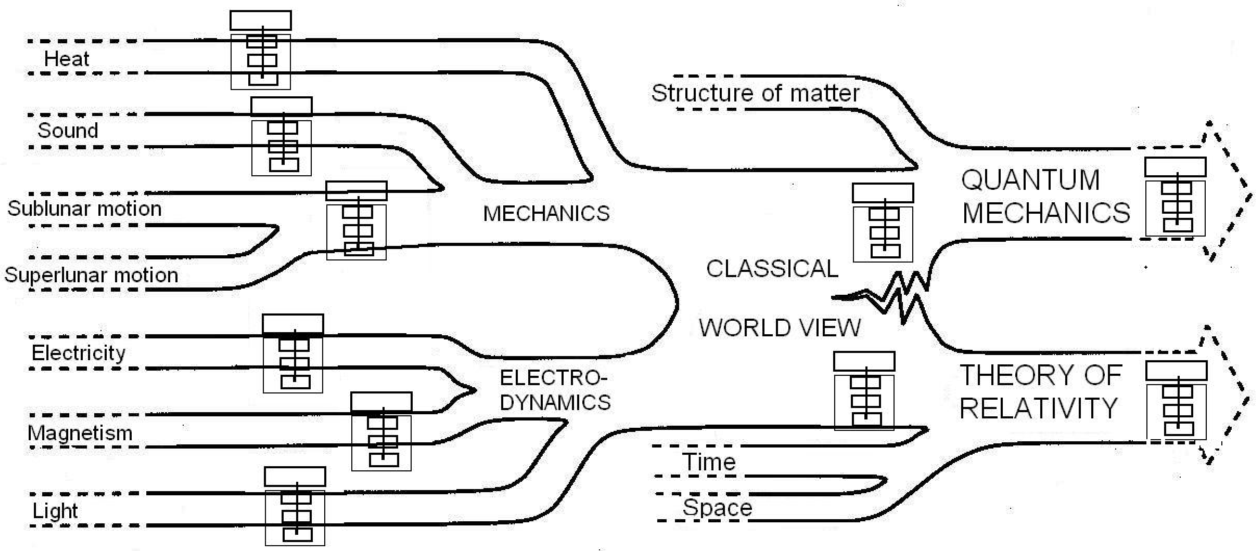

Figure 3.2 Generalisation and quantification are jointly building a two-dimensional conceptual hierarchy.

3.4 The relation of the conceptual hierarchy to the natural hierarchies

Progress in the conceptual hierarchy differs from the progress in the natural hierarchies. The hierarchies of structures and interactions can be perceived as continuous chains from the small to the large, although it is far from clear, where the chains start from and where they are ending, or whether they even have a beginning and an end. However, the analysing concepts have definitely started neither from quarks and leptons nor from groups of galaxies, nor is it plausible to start teaching from these. The processes of concept formation and learning stat from the ¨perceivable world and the environment, from entities and phenomena of ones own order of magnitude, and are expanding gradually. *The development of conceptual hierarchy starts from the middle of the natural hierarchies and proceeds to both directions.*

This is obvious from the schemes of the unification development representing the development of the conceptual mastery of phenomena. The conceptual whole of the classical period in fig. 3.1 refers, in the first place, to phenomena of the human order of magnitude. The revolution of the world picture caused by modern physics divides the progress clearly in two opposite directions in the chain of structural hierarchy, towards the particle structure of matter and towards the cosmic scale. The continuation of the scheme to the period of modern physics (Fig. 5.9 of our book) reveals, that the conceptual hierarchy of phenomena in its further development is reduced to representation of the hierarchy of interactions, which we understand to be the foundation of the structural hierarchy of the material world.

Classification is a fundamental operation, which starts the hierarchical concept formation. It means always perception of hierarchical order and conceptual representation of it. Already there *the principle of starting from the middle* is obvious.

Any classes of entities, phenomena *etc.* consist of entities, phenomena *etc.* which are perceived comparable. Mutually comparable classes are elements of a higher class of hierarchy. One can group the classes further and, thus, build the next-higher level of classification. Whereas within a class one can sharpen the principle of classification and, thus, divide the class to lower-level subclasses.

One can think, for instance, perception of everyday surroundings. Chairs, tables and beds are perceived and learned by the child as chairs, tables and beds before the more general concept of furniture becomes understood and before one learns to classify them more specifically *e.g.* according to structure or use. It takes its time before a child is ready to understand the "body" as a general class of entities, covering all things of home and environment, heavenly bodies, people and animals, dust particles *etc,.* in whatever way they are classified, not to speak about the still more general concept of "entity".

3.5 Learning as a hierarchical process

Learning in itself is a process which is proceeding hierarchically, since all learning of new things is built on the basis of what has been learned earlier and all what is learnt opens new possibilities for the process to proceed further. It is a cultural process of an individual. It is participation of a human to the "great process" of mankind. In learning one can recognise, right from the beginning, the germs of all the cultural processes discussed and of their processual elements. The purpose of teaching is to guide the growth of these germs so, that the pupils would get, each one in his individual process according to his personal abilities, their share of the basic elements of the "great process". It is, thus, guidance to the scientific, technological, aesthetic and social process, as well as to the hierarchies of the inter-subjective products, methods, procedural and methodical knowledge developing in these processes.

In these hierarchies, *an upper level is always based on the lower ones.* The elements of a lower level are structural parts of the elements of the next higher level or in some other way necessary preconditions of production or generation of these. Therefore, *the lower-level concepts,, skills, methodical facilities etc. must be learned or developed before it will become possible to learn upper-level concepts, skills etc.* In learning all hierarchically proceeding cultural processes start from the beginning. Hence, also the teaching must start from the beginning with respect to all generative factors of hierarchy, and in all stages it must be fitted to the hierarchical level reached by the pupil in his own process. I have often pointed out, that teaching of physics cannot start from atoms and formulae. This means, that teaching must start from the beginning with respect to each of the two generative principles of conceptual hierarchy[24]. The atoms and formulae represent high levels in the generalisation hierarchy and in the quantification hierarchy, respectively.

One must know the hierarchical relations of the concepts and methods in order that recognition of the hierarchical level of the pupils would be possible and that the teaching could proceed in a "natural" way adjusting to the respective level of the pupil. This is probably the greatest challenge of teaching physics. The lingual, conceptual and methodical facilities of the teacher are on a hierarchically higher level than those of the pupil. This *hierarchical distance* is so large, that the teacher cannot cope with it without thorough empathy with the pupil's level and without good knowledge of those elements of this level, which are preconditions of the next levels. He easily neglects, as "self-evident details", things, which the pupil ought to work out actively for himself to be able to proceed. One might also think, that one important dimension of talent of the pupil is his *"tolerance of distance"*, the ability to accept teachings of a hierarchically higher level, for the mastery of which he does not yet possess actual preconditions, anticipatively, in view of the future development of his process.

Unravelling of the hierarchical development of concept formation and of its processual structure, and search for related proper ways of teaching in the different phases of the learning process, have been the core of all the ponderings, which I have been marketing with the title of "*perceptional approach*". "Approach" has perhaps not been quite a happy choice of term, because by approach it is often meant methodical receipts designed for specific grades of studies. I have, instead, been searching for a foundation, which would make possible development of teaching which would proceed consistently through all phases of the learning process by taking the processual structure of concept formation for a model and by learning from the hierarchical development of the concepts and methods of physics, as it has been realised in the development of science.

My final figure 3.3 is an elementary attempt to visualize this idea. The beam opening towards the right intends to symbolise the hierarchical development of the cultural processes. The arrow is meant to represent the idea of "perceptional approach" which I have been seeking. The vertical bars describe different levels of development or, say, grades of studies, and the necessary level-specific methodical "approaches".

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[24] Ibid. Chapter 5.4.

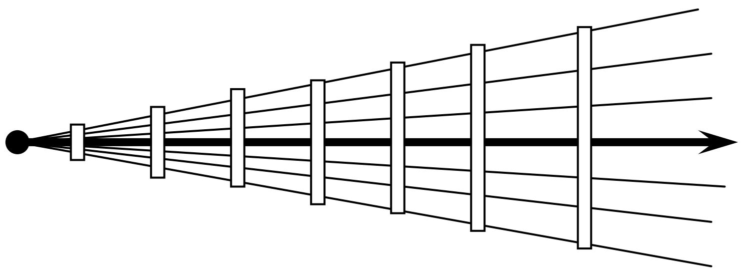

Figure 3.3 The "approaches" in relation to the "perceptional approach".