

# Time-stamped and free photons – 3D optical remote sensing

I: 3D geometry and sensor models – from the world to pixel/photon detector

II: 3D geometry, from pixels to the world

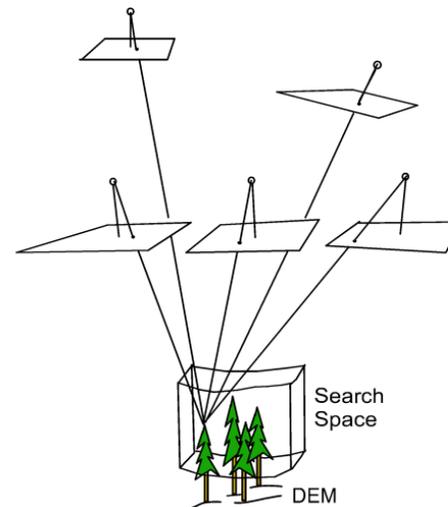
III: Radiometry of passive imaging

IV: Functioning of a LiDAR sensor – 3D geometry and radiometry

RS101, 2016

Ilkka Korpela

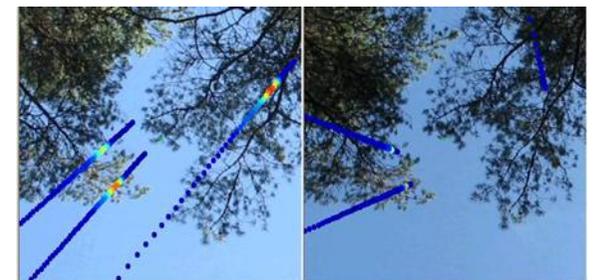
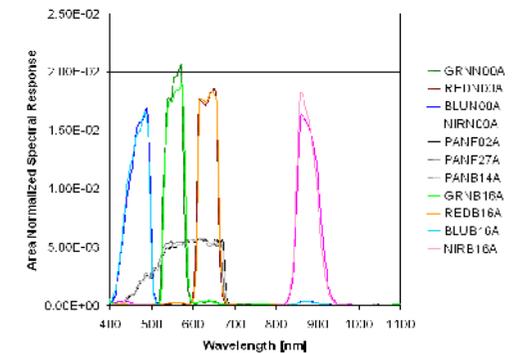
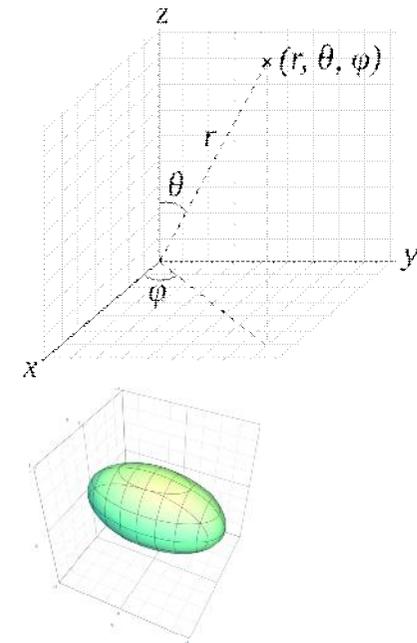
Academy Research Fellow, UH, Dept Forest Sciences



I: 3D geometry and sensor models – From the world to pixel/photon detector

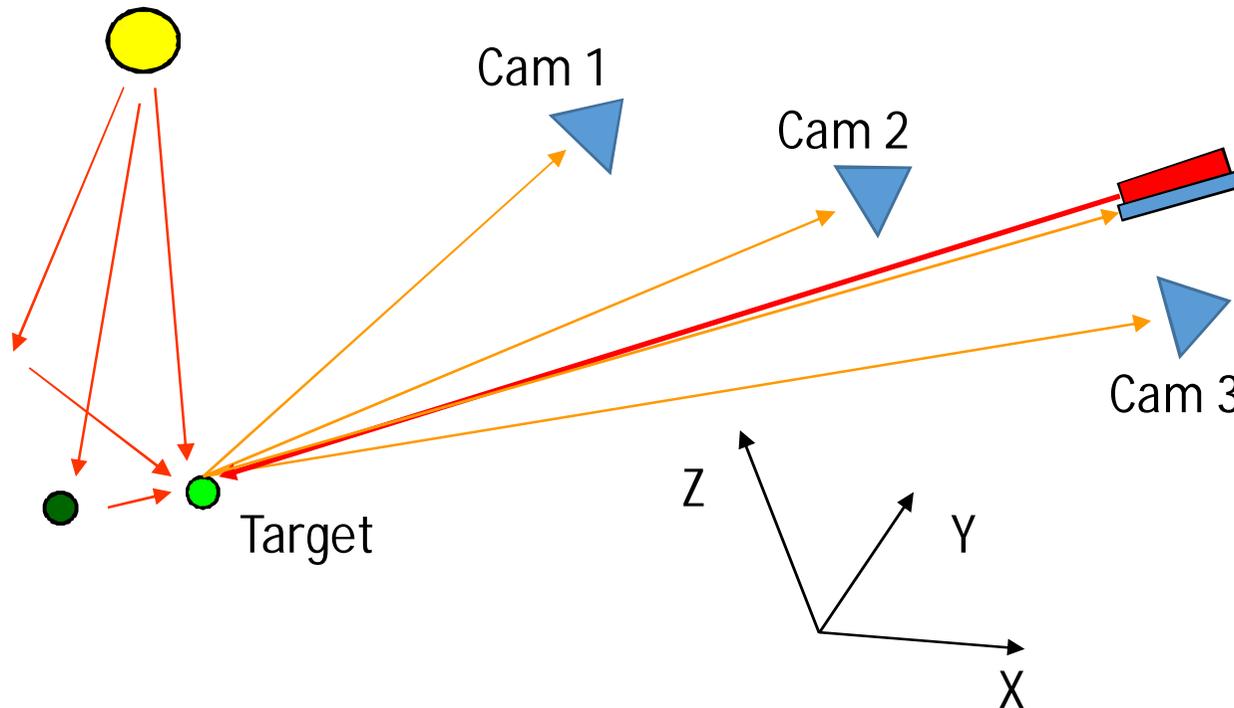
Objectives include gaining basic understanding (awareness) in

- \* How 3D measurements are made with light using
  - passive &
  - active light sources
- \* How the 3D measurements are influenced by sensor, imaging and scene properties and how to derive error estimates
  - \* The radiometry of frame and line-cameras, the main factors influencing the pixel DN-values
- \* The intertwined nature of radiometry and geometry in LiDAR sensors



# *Photogrammetry is about using light to measure coordinates*

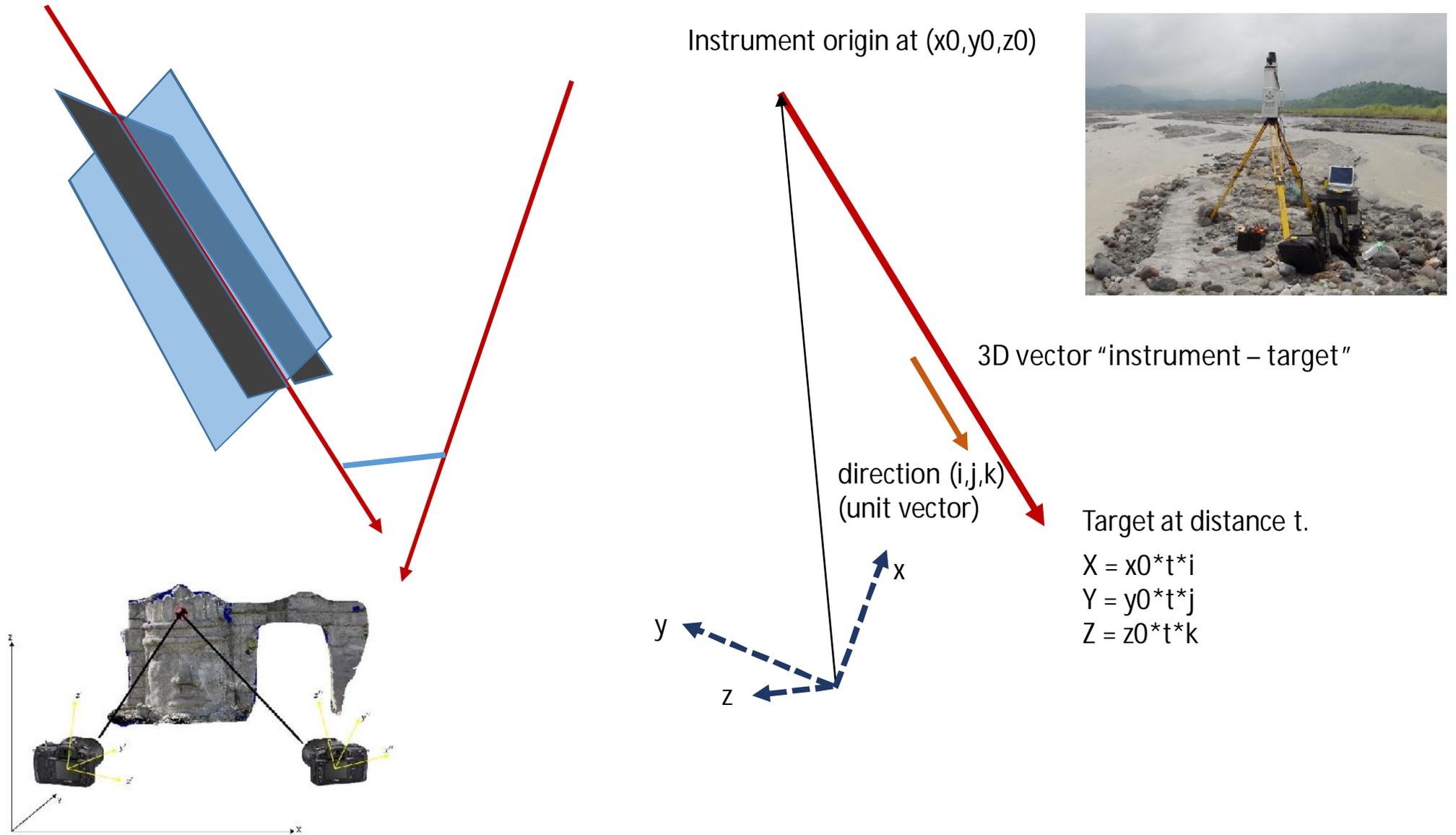
Source of free photons



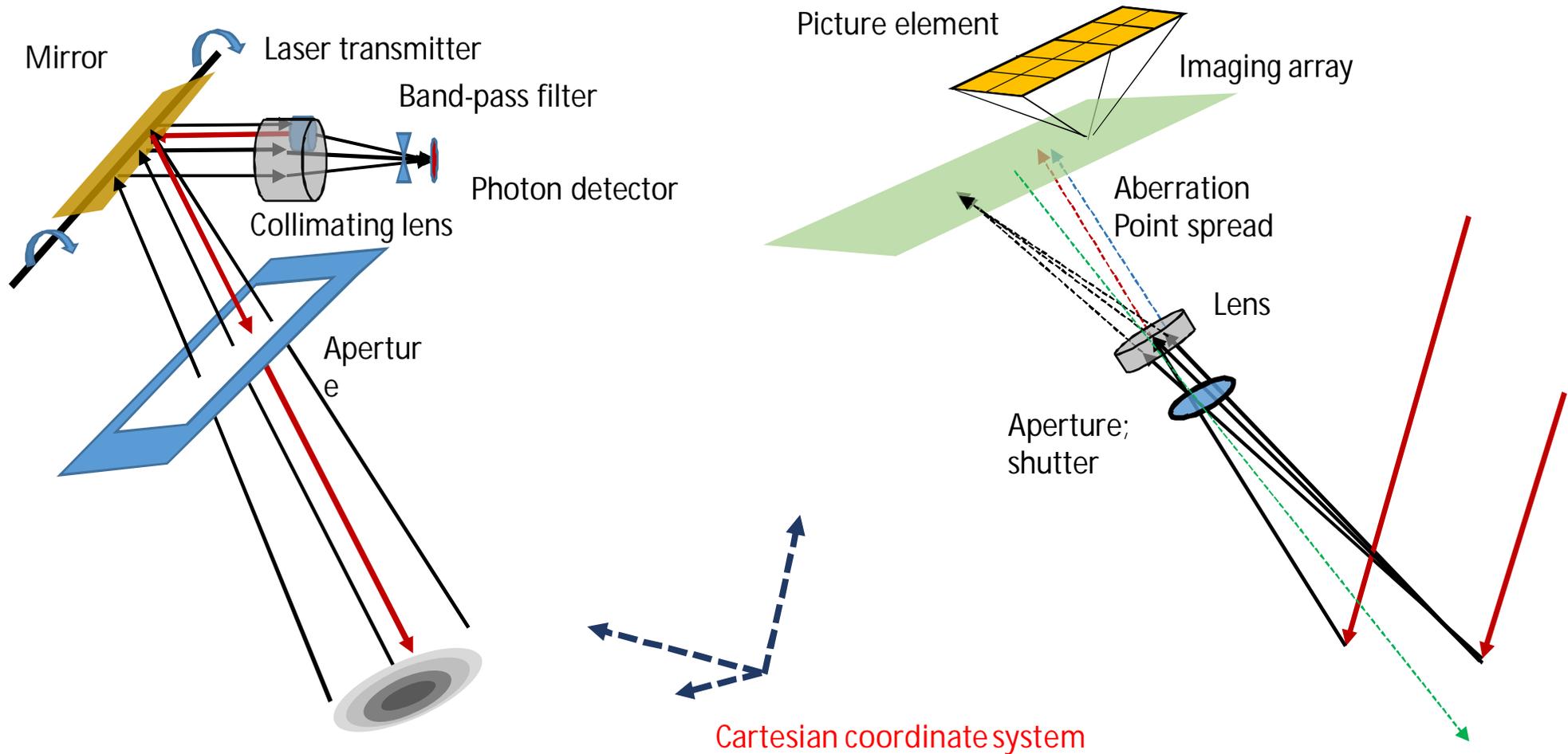
Source of time-stamped photons

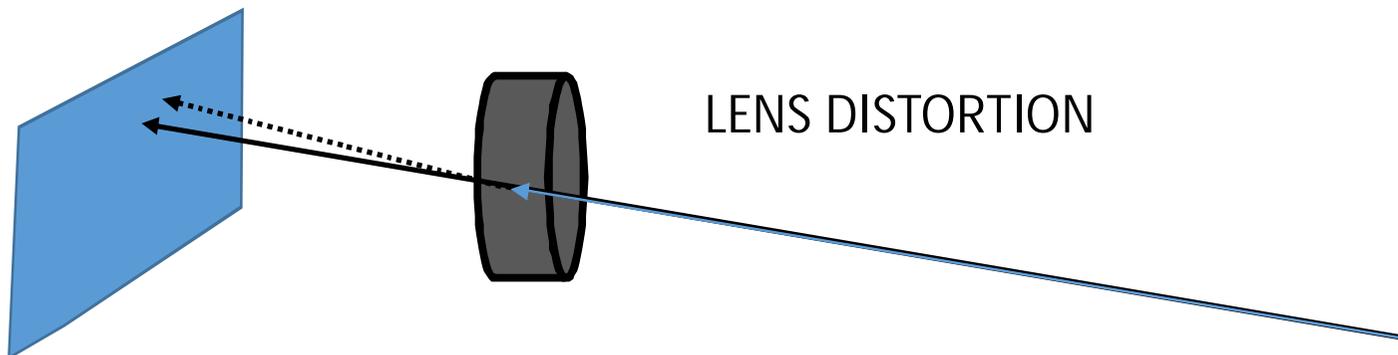
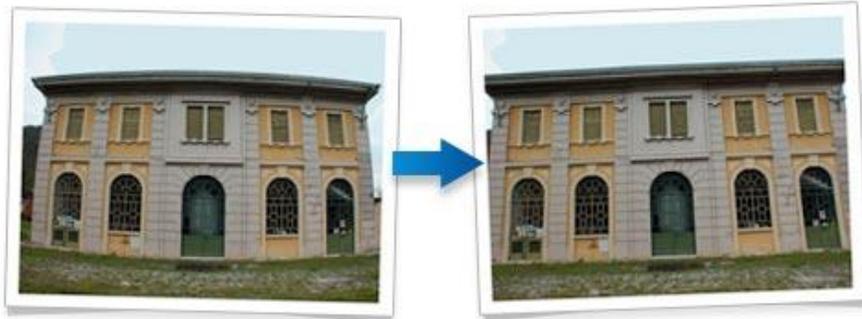
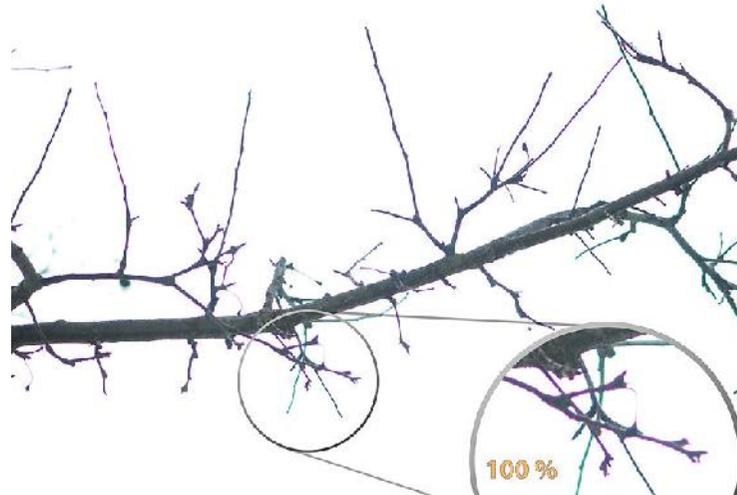
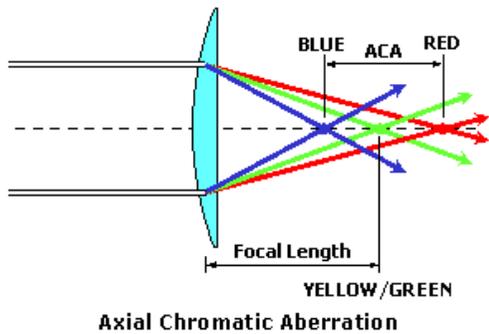
*"It is all about 3D light rays" and  
"How accurately can we define their path/intersection and/or length"*

# Straight, collinear lines. Spherical observations or spatial intersection



# 3D geometry and sensor models – From the world to pixel/photon detector





## Assumption on collinearity

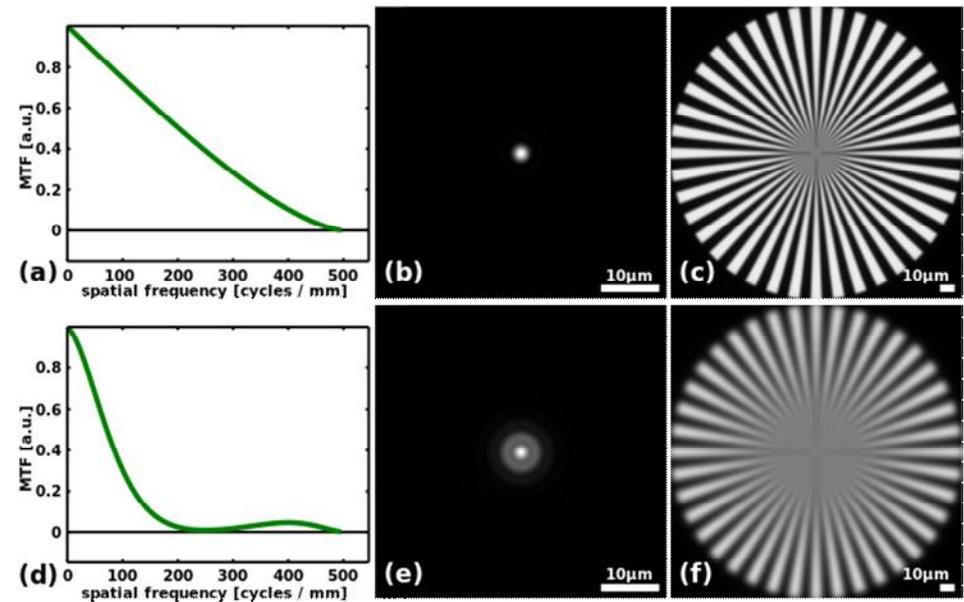
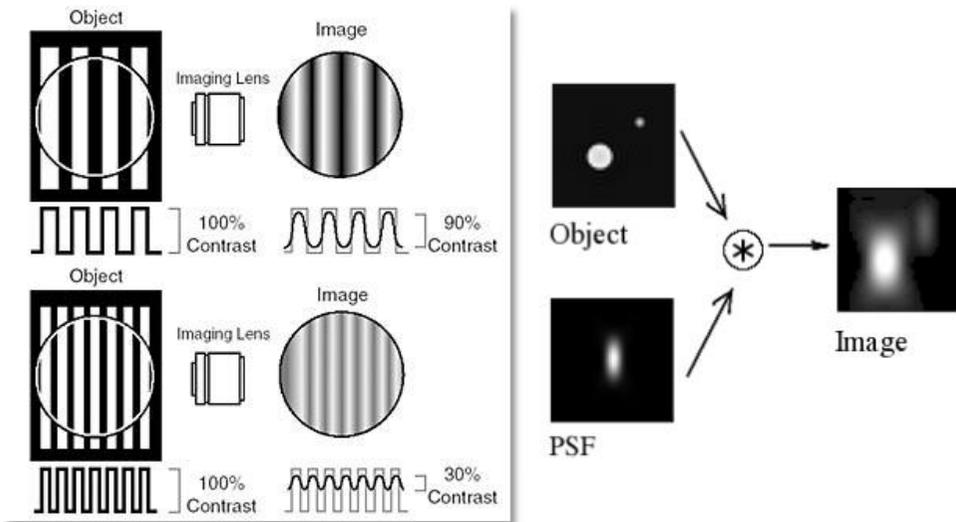
(straight 3D lines or image rays thru a single principal point in the lens) in a real camera is never realized

- chromatic aberration
- lens **distortions**

# Image sharpness - Measuring the other end of the "3D target-sensor image ray"

## Optical resolution

- Best at image center
- Depends on contrast (defined for a contrast ratio)
- Pixel views the world 'thru a Gaussian lens'
- Decreasing the pixel size not necessarily always improves





Hasselblad H4D-50 images from 750 m. RGB Bayer-filter.  
August 2015, May 28 2013, November 2011.

# Instantaneous field of view – “pixel”

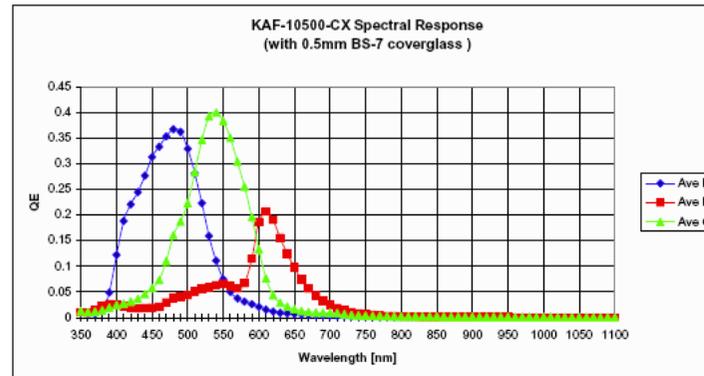
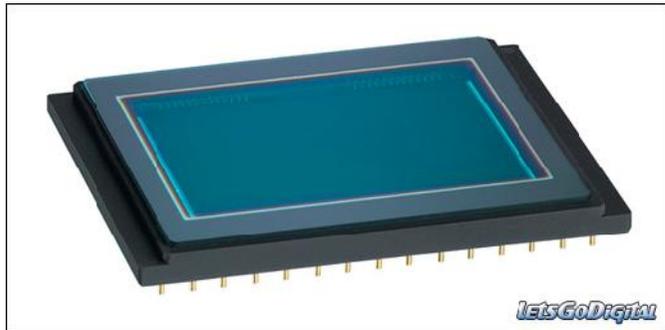
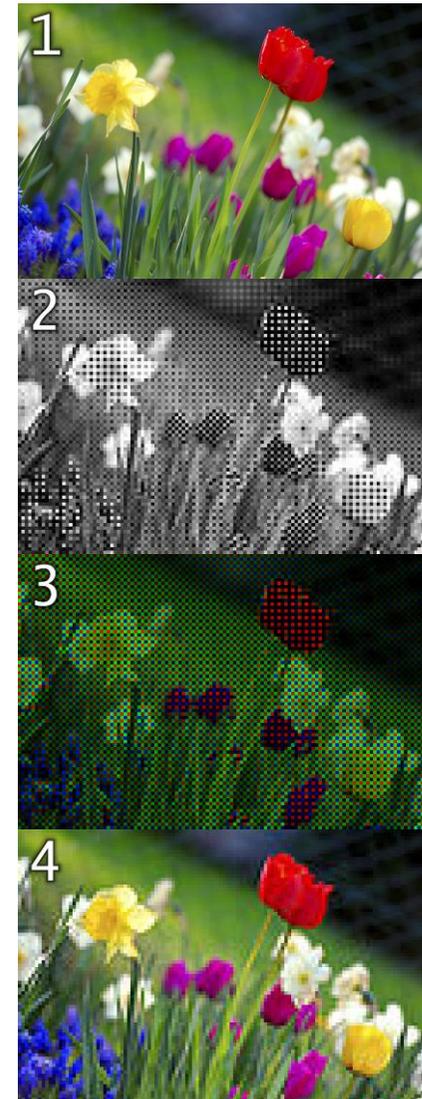
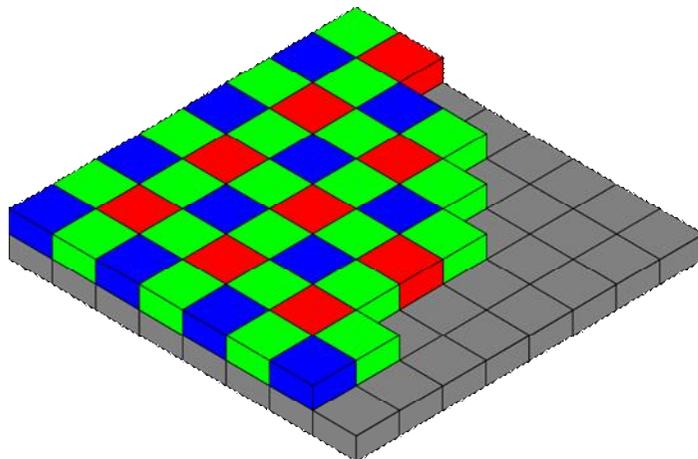
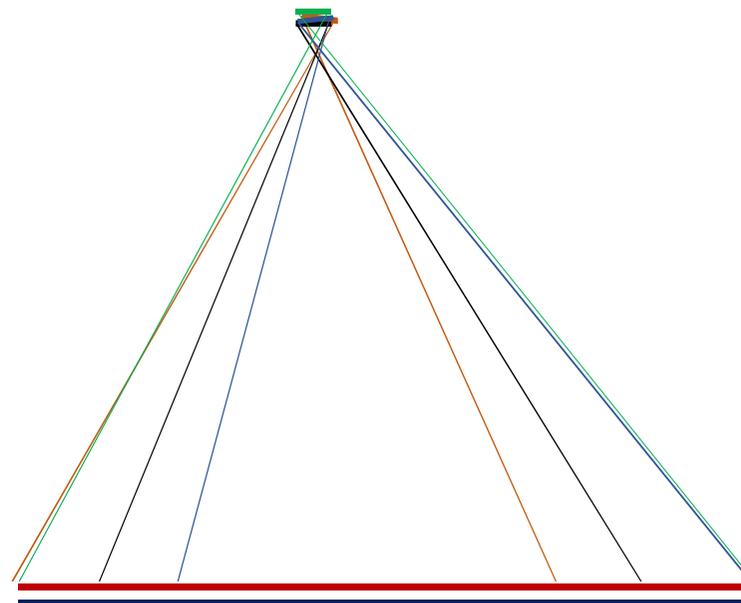


Figure 5: Typical Spectral Response with coverglass



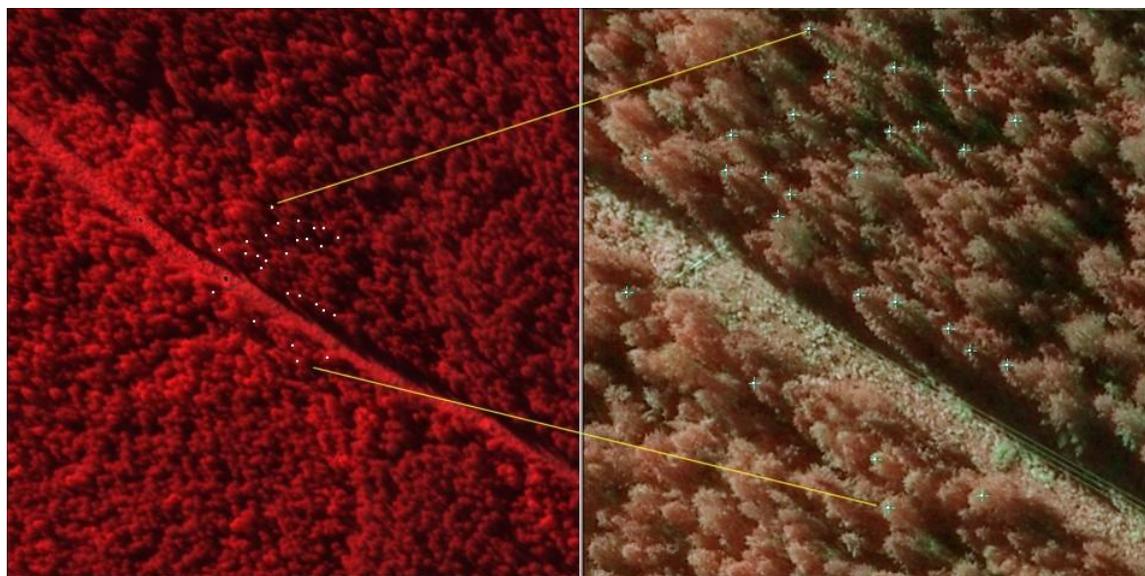


R,G,NIR



Virtual perspective image(s)

R,G,NIR + PAN

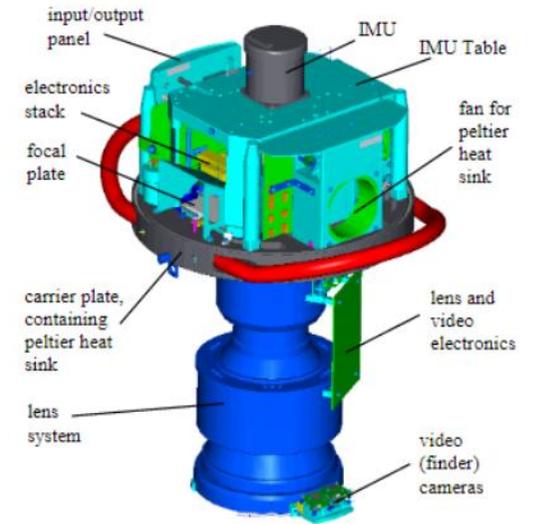
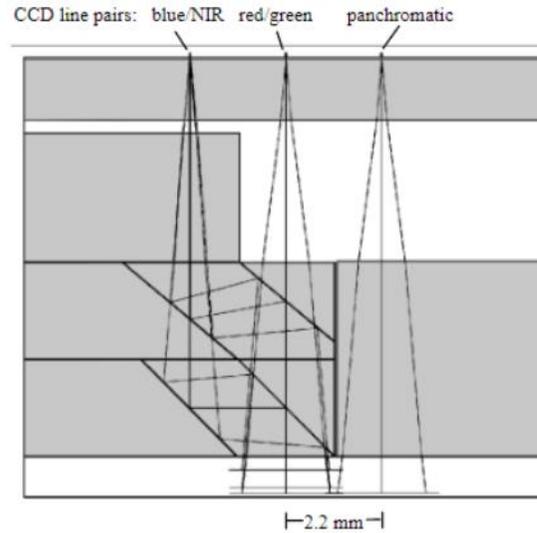
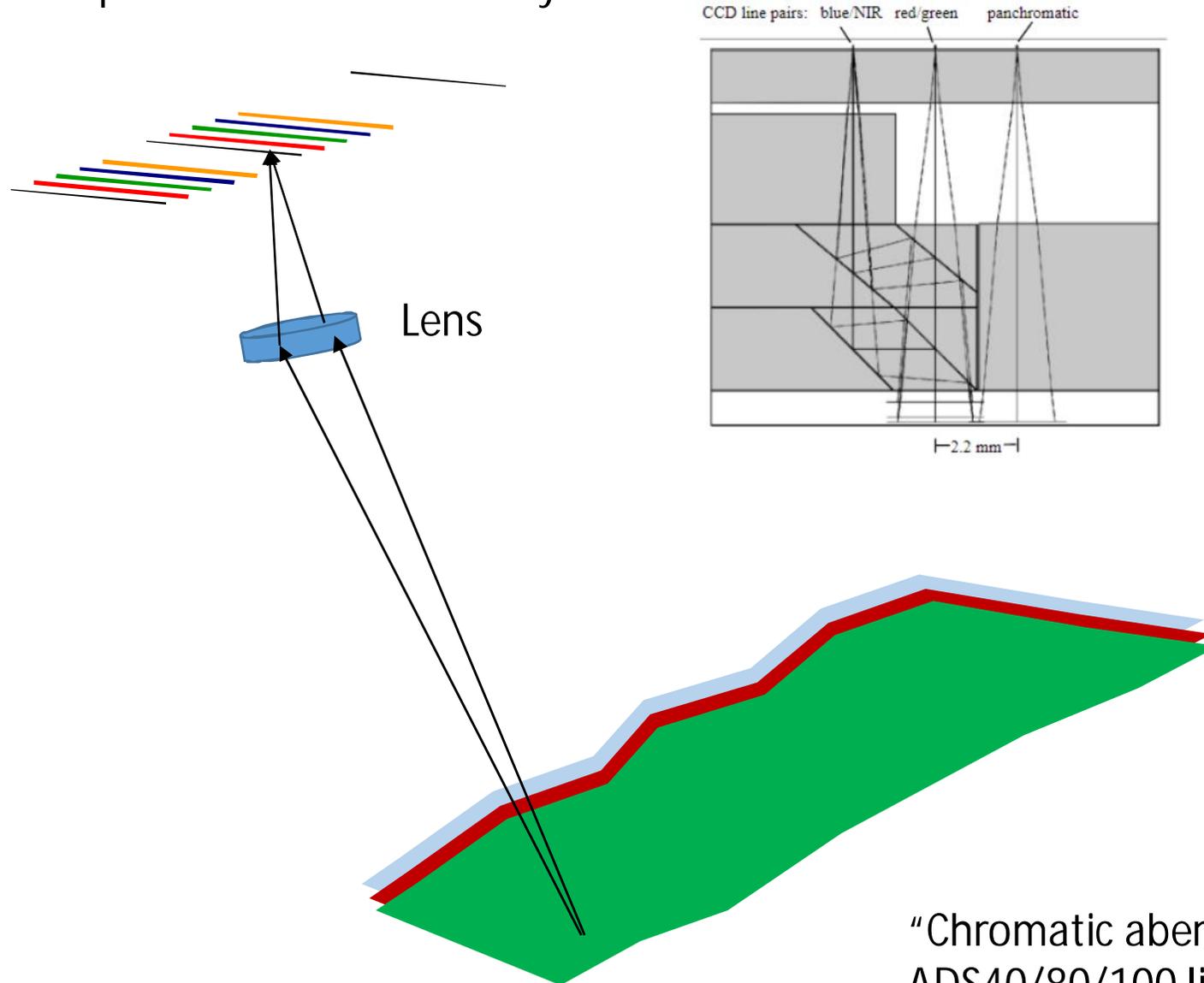


UltraCAM-D sensor

Four lenses (cameras) for high-resolution PANchromatic image. Four cameras for low-resolution R,G,B,NIR images.

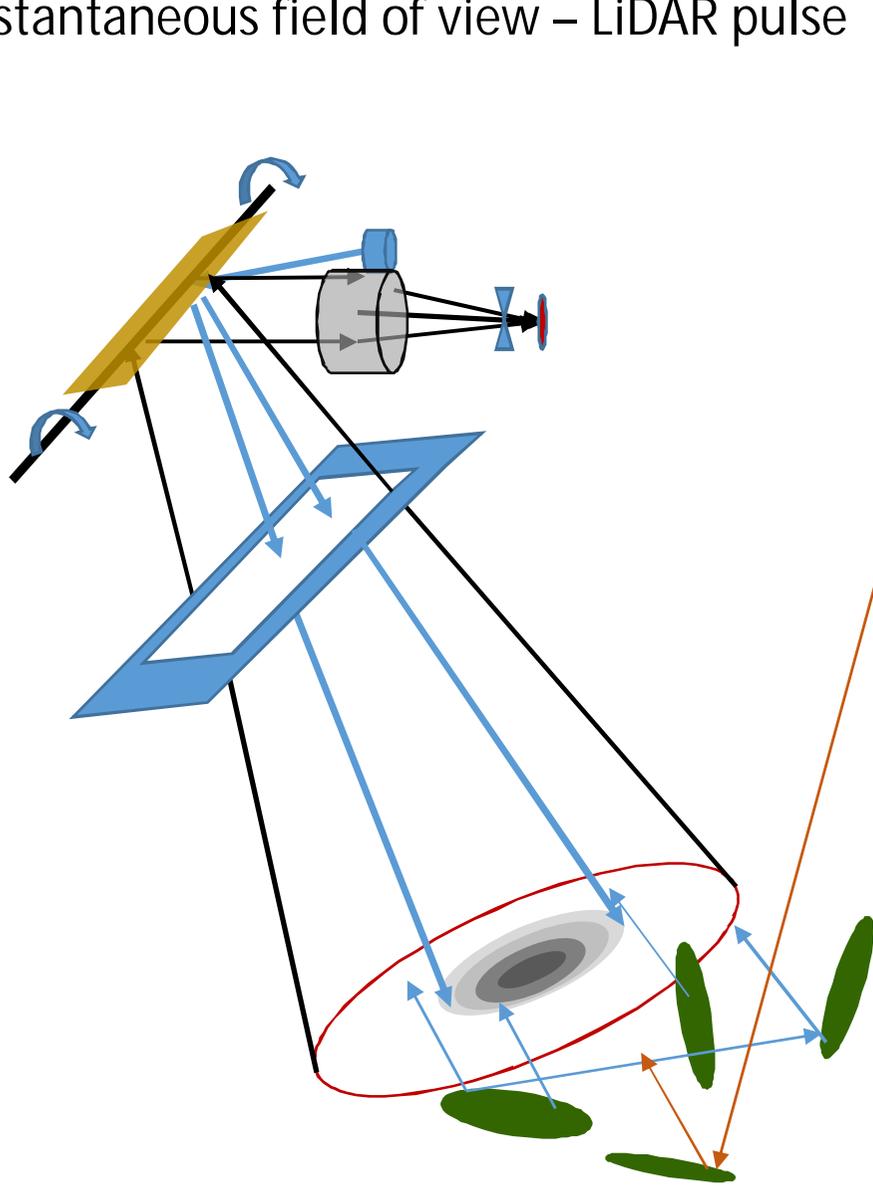
PAN-sharpening.

# Focal plane with CCD line arrays



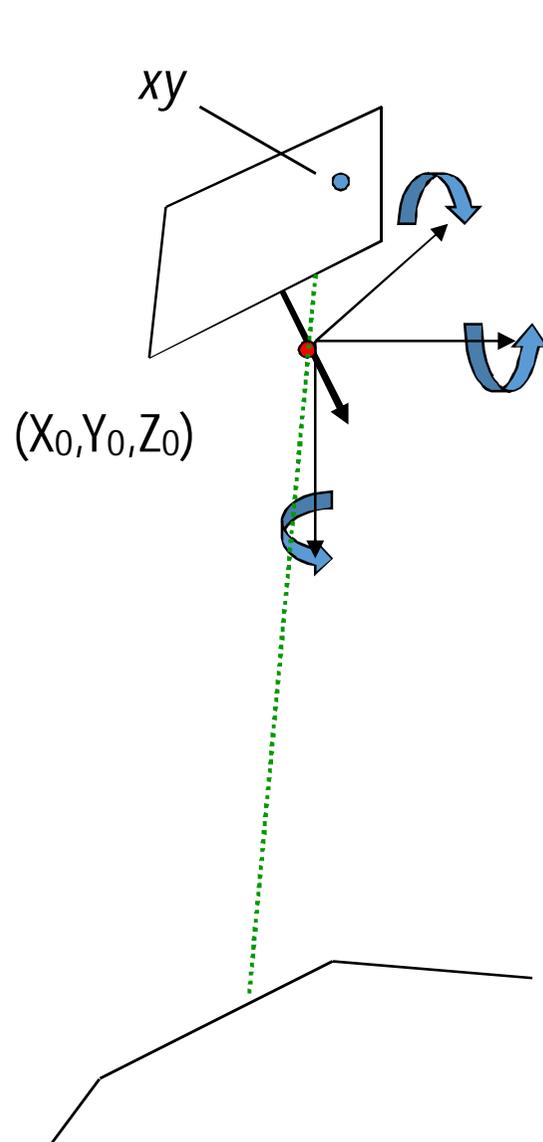
“Chromatic aberration” by purpose  
ADS40/80/100 line camera.

# Instantaneous field of view – LiDAR pulse



Backscattering from outside the 'footprint'

## 3D geometry and sensor models – From the world to pixel/photon detector



$$\mathbf{R}^T = \begin{pmatrix} \hat{e} \cos j \cos k & \cos w \sin k + \sin w \sin j \cos k & \sin w \sin k - \cos w \sin j \cos k \\ \hat{e} \cos j \sin k & \cos w \cos k - \sin w \sin j \sin k & \sin w \cos k + \cos w \sin j \sin k \\ \hat{e} \sin j & -\sin w \cos j & \cos w \cos j \end{pmatrix} \begin{pmatrix} \hat{u} \\ \hat{v} \\ \hat{u} \end{pmatrix}$$

$$\begin{aligned} x - x_0 &= -c \frac{r_{11}(X - X_0) + r_{12}(Y - Y_0) + r_{13}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} \\ y - y_0 &= -c \frac{r_{21}(X - X_0) + r_{22}(Y - Y_0) + r_{23}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} \end{aligned}$$

- $(x_0, y_0)$  = offset of the principal point
- \*  $c$  = the calibrated focal length (INTERIOR ORIENTATION)
- \*  $w, f, k$  = camera rotations in  $R$
- \*  $(X_0, Y_0, Z_0)$  = position of principal point (EXTERIOR ORIENTATION)
- \*  $(x, y)$  = the focal plane coords of the 3D point  $(X_a, Y_a, Z_a)$  in meters (cf.  $q$  and  $f$  in theodolite)
- \* if digital image, convert to  $(x', y')$  in the left-handed (row,col) coordinate system of the digital image.



Questions (2-4 students)

What do we need to know (about geometry of the camera - inside and outside) to establish the mathematical 3D ray from a given pixel to the scene?

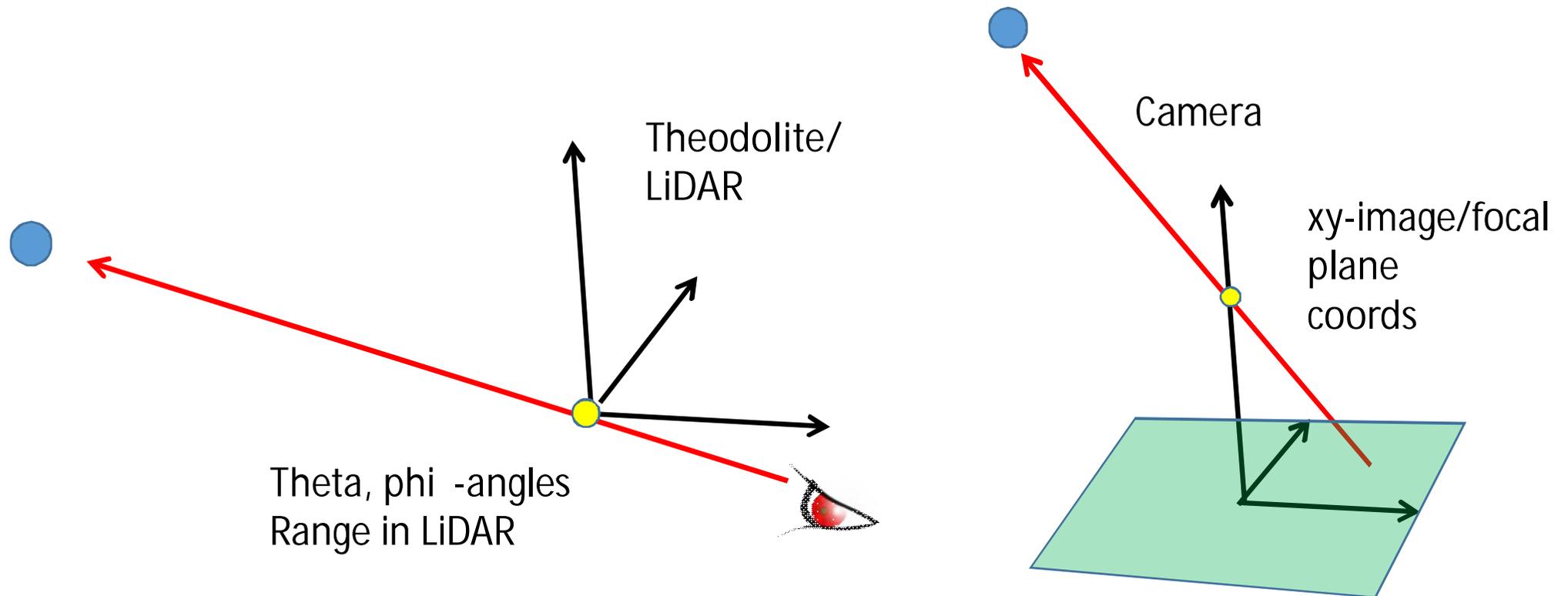
Can we use images and space intersection (of such image rays) to measure 3D coordinates of say large field crops, asphalt surfaces, brick walls, shape of ship's surface, egg surface, glaciers etc.?

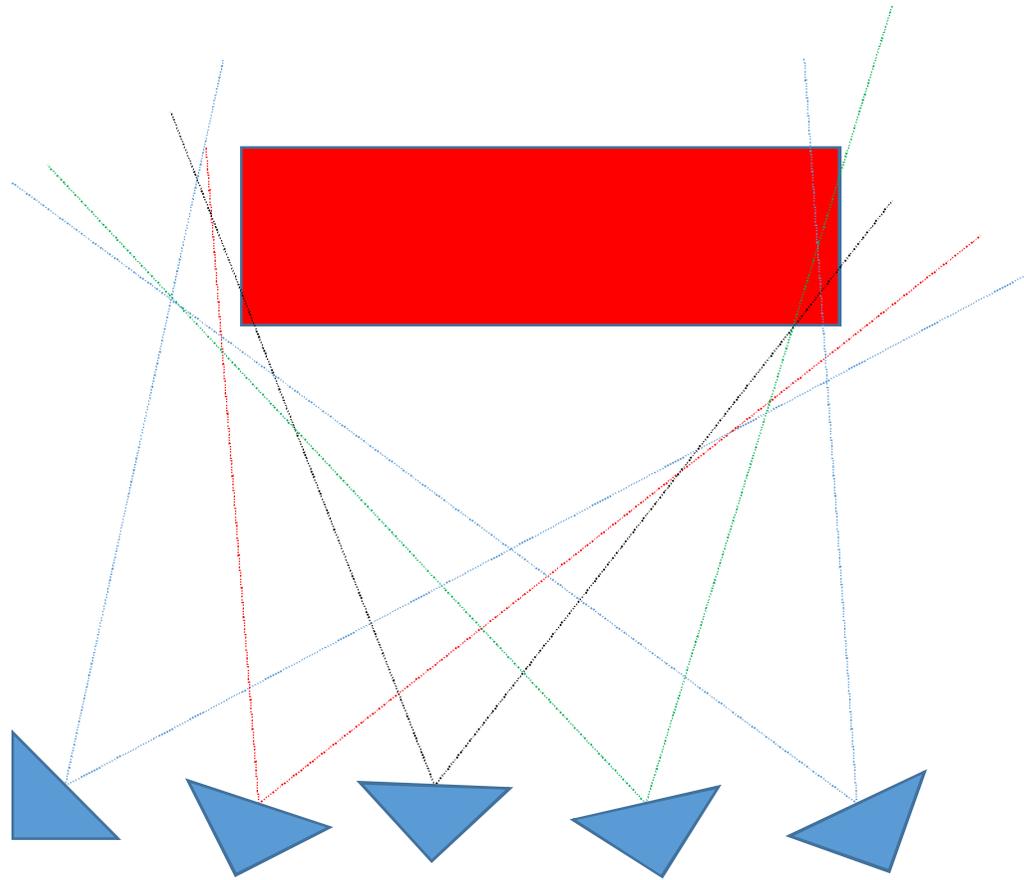
Can we measure the ground under closed forest canopies? With space intersection? With LiDAR?

LiDAR is based on the principle to carrying out distance measurements by measuring the two-way travel time of light. How far from the sensor is a target that gives the return after 1000 nanoseconds? ( $c = 300000 \text{ km/s}$ ).

How can we know the position of the LiDAR sensor and its attitude?

## II: 3D geometry, from pixels to the world





Intersection



Spherical

# LIDAR and spherical coordinate system

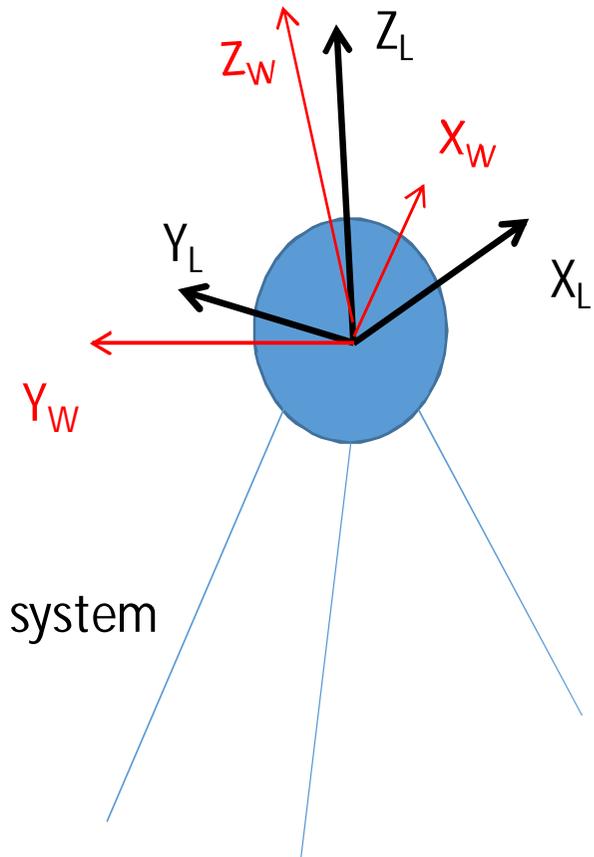
$$[x,y,z]_{\text{Target}} = [x,y,z]_{\text{originL}} + \text{dist}_L \times [\sin(q_L)\cos(f_L), \sin(q_L)\sin(f_L), \cos(q_L)]$$

$f$  = azimuth  $-180..+180^\circ$

$q$  = zenith  $-90..+90^\circ$

Terrestrial LiDAR; leveling with plumb line  $\Rightarrow Z_L \parallel H$   
 $q$  is then rotation about the plumb line  
 $f$  becomes defined on the plane for which plumb line is the normal

Observations  $[x,y,z]_{\text{Target}}$  are 'local' – transformation to another system requires rotations (3) and offsets (DX, DY, DZ).

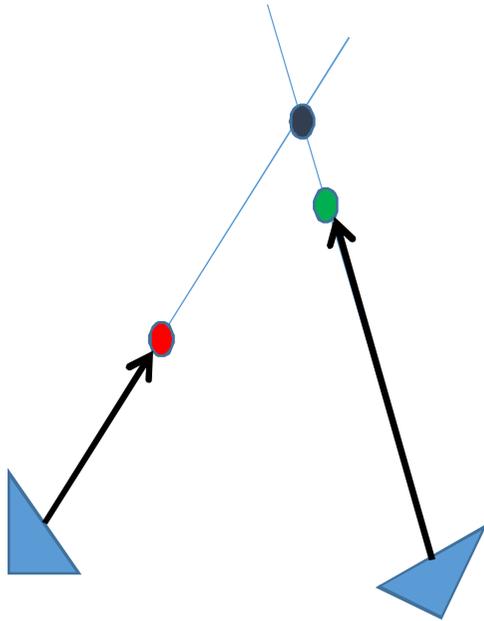


$$[x,y,z]_{\text{other}} = [\mathbf{DX}, \mathbf{DY}, \mathbf{DZ}] + \text{dist}_L \times R(rX, rY, rZ) \times [\sin(q_L)\cos(f_L), \sin(q_L)\sin(f_L), \cos(q_L)]$$

↑  
LiDARin position

↑  
LiDAR attitude

# XYZ measurements using images (2D observations of 3D lines that intersect) Camera @Theodolite



Teodoliitti(1) sij. origoon  $(0, 0, 0)$ .

Se kuylataan  $(Z \parallel z_1)$ .

Ja sen x-akseli suunnataan Teodoliitti(2):een ( $j_1 = 0.00^\circ$ ).

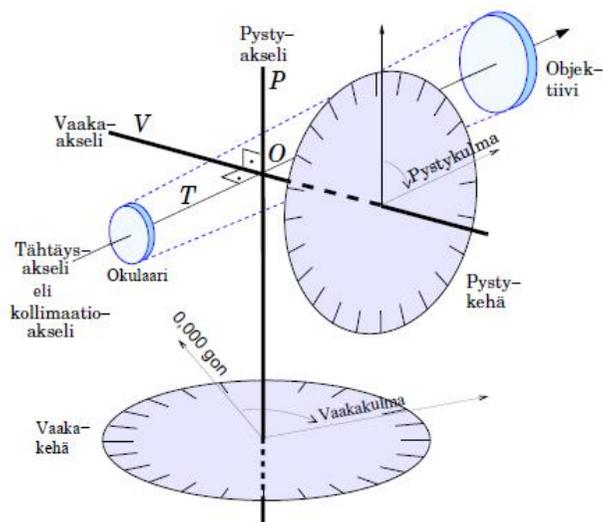
Teodoliitti(2) sijaitsee pisteessä  $(DX, 0, DZ)$ .

Se kuylataan  $(Z \parallel z_2 \parallel z_1)$ .

Sen negatiivinen x-akseli suunnataan Teodoliitti(1):een ( $j_2 = \pm 180.00^\circ$ )

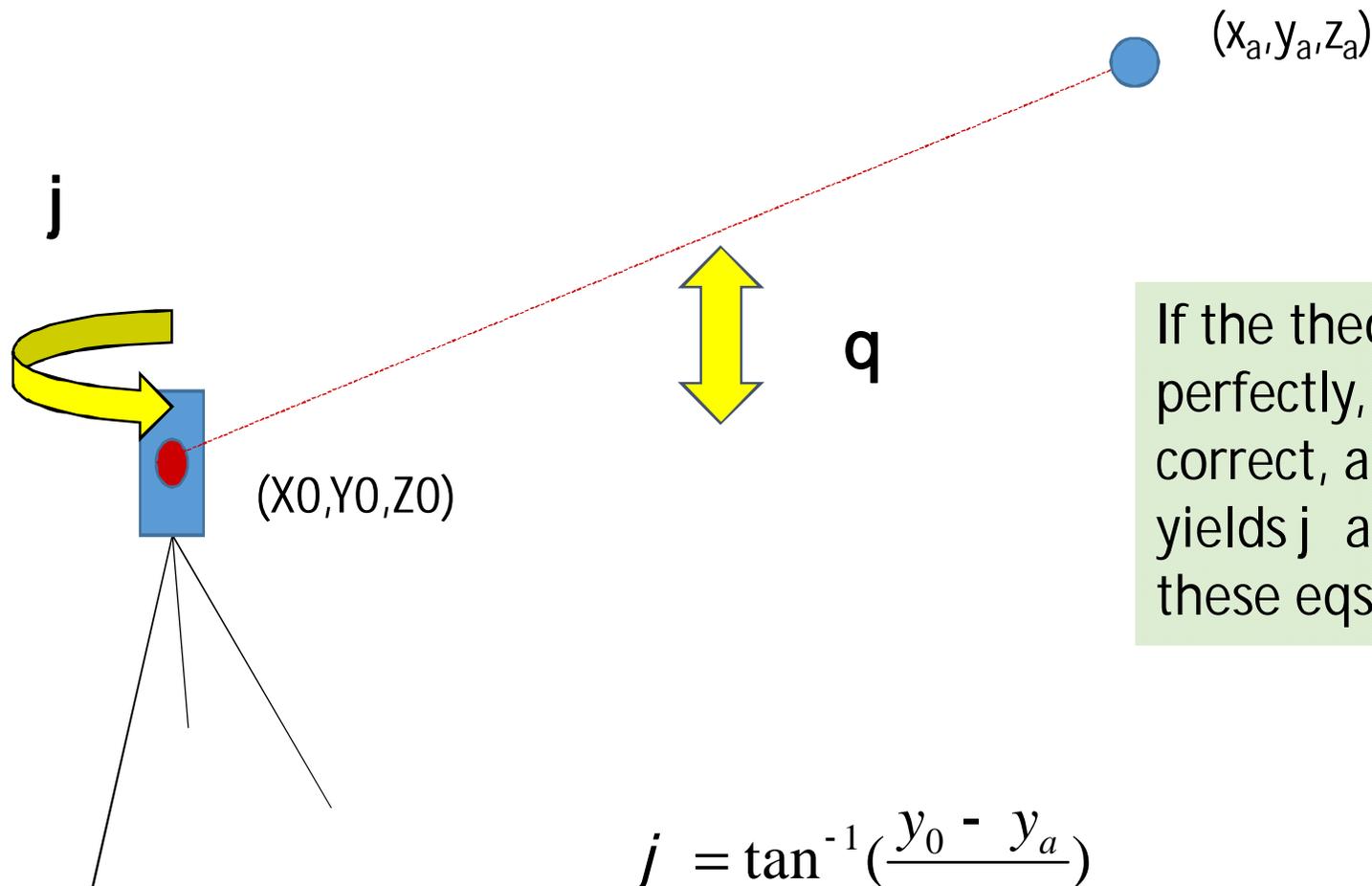
Tämän seurauksena molempien Teodoliittien XYZ akselit ovat yhdensuuntaiset.

Oletetaan että  $DX$  ja  $DZ$  voidaan mitata. Ensimmäinen on vaakaaetäisyys teodoliittien välillä, ja toinen voidaan esim. vaaita.



Kuva 5.2: Teodoliitin akselit ja kehät

# Matching actual observations with computed observations

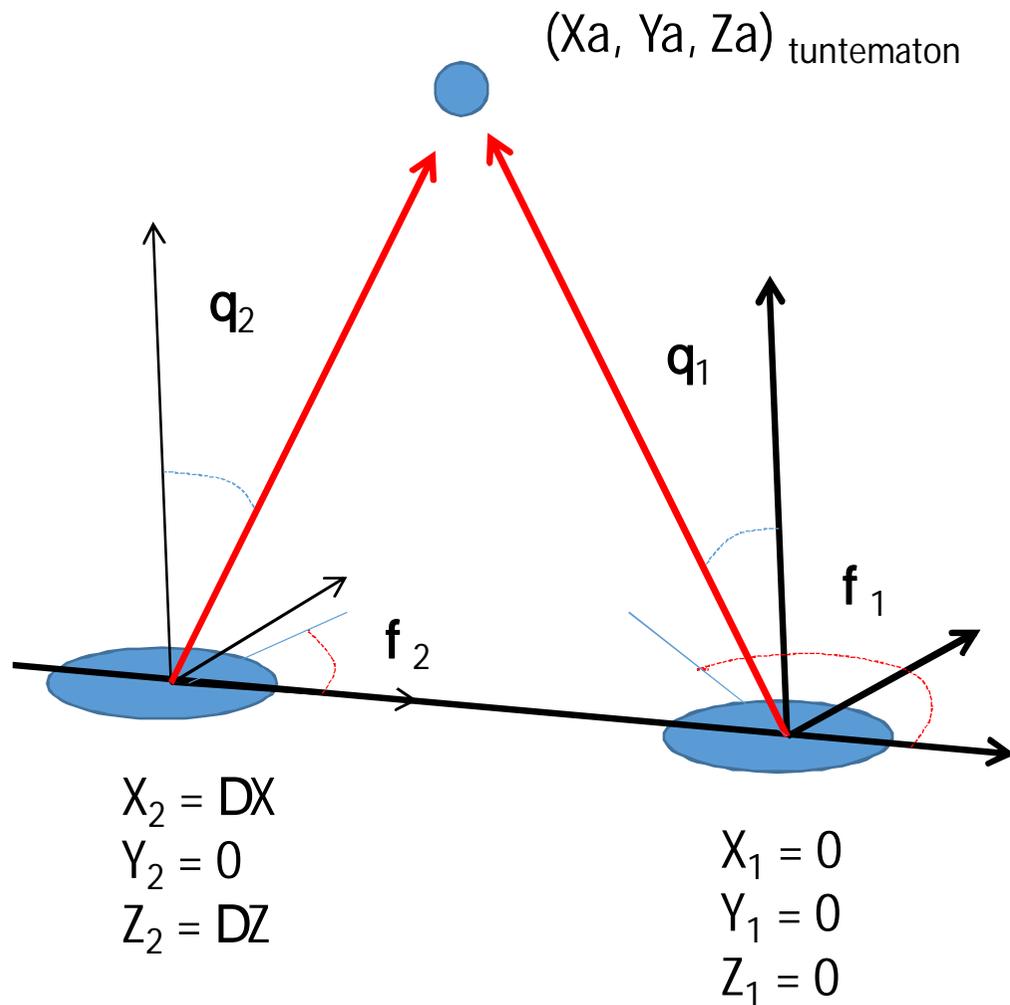


If the theodolite is aligned perfectly, and the position is correct, aiming at point  $P_a$  yields  $j$  and  $q$ , that match these eqs.

$$j = \tan^{-1} \left( \frac{y_0 - y_a}{x_0 - x_a} \right)$$

$$q = \cos^{-1} \frac{(z_0 - z_a)}{\sqrt{(x_0 - x_a)^2 + (y_0 - y_a)^2 + (z_0 - z_a)^2}}$$

# Unknowns (XYZ) in a non-linear regression



Jos tuntematon piste on  $(X_a, Y_a, Z_a)$ , näkyy se atsimuutti ( $\mathbf{f}$ ) ja zeniitti ( $\mathbf{q}$ ) suunnissa, jotka ao.  $\tan^{-1}$  ja  $\cos^{-1}$  lausekkeet määräävät teodoliitin ja tuntemattoman pisteen koordinaateista.

$$\arctan\left(\frac{Y_i - Y_a}{X_i - X_a}\right) - f_i = 0$$

$$\arccos\left(\frac{Z_i - Z_a}{\sqrt{(X_i - X_a)^2 + (Y_i - Y_a)^2 + (Z_i - Z_a)^2}}\right) - q_i = 0$$

Havaitut suunnat ovat ( $\mathbf{f}_i$ ) ja ( $\mathbf{q}_i$ ). Jos lasketun ja havaitun ( $\arctan()$ ,  $\arccos()$ ) välillä on pieni ero, on sijainti havaintoon nähden uskottava.

Kolme tuntematonta

Yksi teodoliitti  $\mathcal{P}$  yhtälöpari

Kaksi teodoliittia  $\mathcal{P}$  neljä yhtälöä

# Epälineaarinen regressioanalyysi / PNS-tasointuslaskenta

Kaksi teodoliittia:

Tuntematon  $(X_a, Y_a, Z_a)$  tuottaa neljä poikkemaa:

$$y = [Df, Dq, Df, Dq]$$

Kuinka lasketaan parempi  $(X_a, Y_a, Z_a)$ , s.e. poikkemat pienenevät, eli "tuntematon istuu havaintoihin"?

PNS-tasointuslaskenta!

Tarvitaan havaintoyhtälöparista osittaisderivaatat:

$$dq/dX_a, dq/dY_a, dq/dZ_a$$

$$df/dX_a, df/dY_a, df/dZ_a$$

Muodostetaan  $4 \times 3$  havaintomatriisi (A) näistä, käyttämällä  $(X_a, Y_a, Z_a)$ :n sen hetkisiä arvioita/arvoja.

$$dq_1/dX_a, dq_1/dY_a, dq_1/dZ_a$$

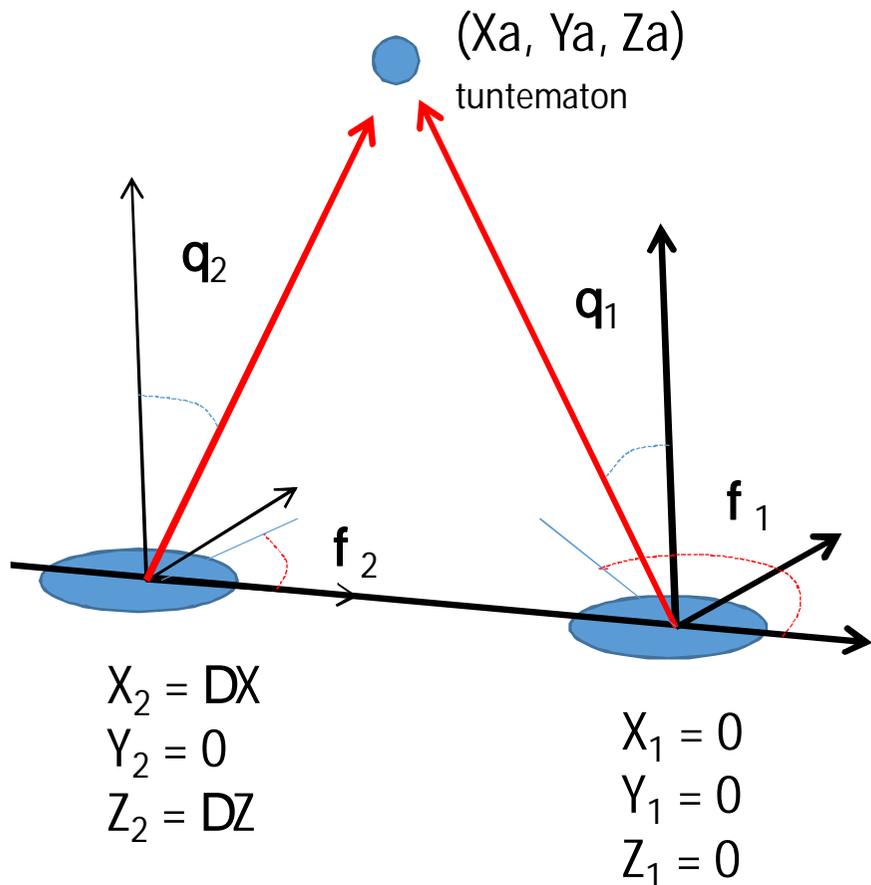
$$df_1/dX_a, df_1/dY_a, df_1/dZ_a$$

$$dq_2/dX_a, dq_2/dY_a, dq_2/dZ_a$$

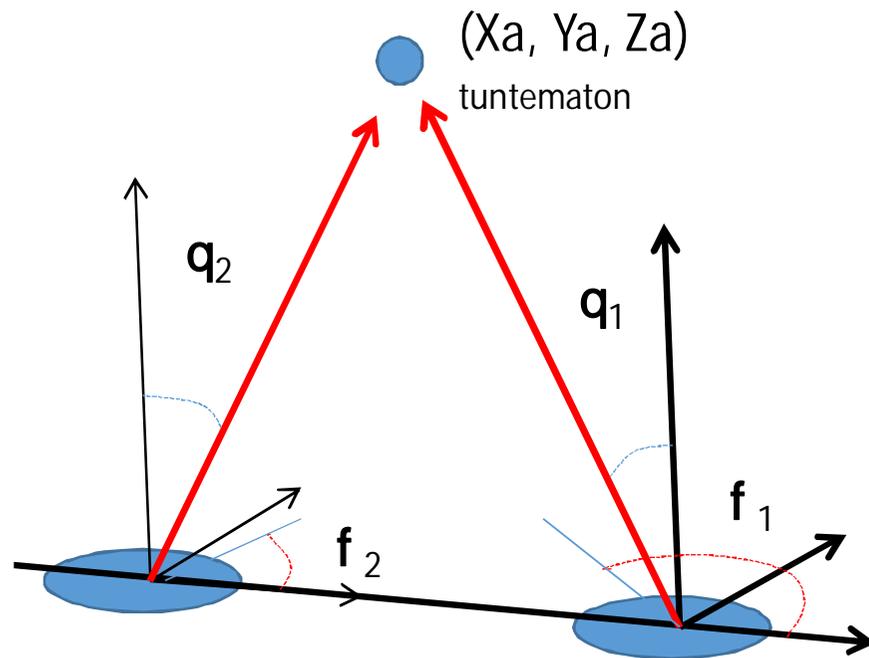
$$df_2/dX_a, df_2/dY_a, df_2/dZ_a$$

$$\text{Ratkaistaan } [DX, DY, DZ] = (A^T A)^{-1} A^T y$$

$$(X_a, Y_a, Z_a) + [DX, DY, DZ] = \text{parempi paikka.}$$

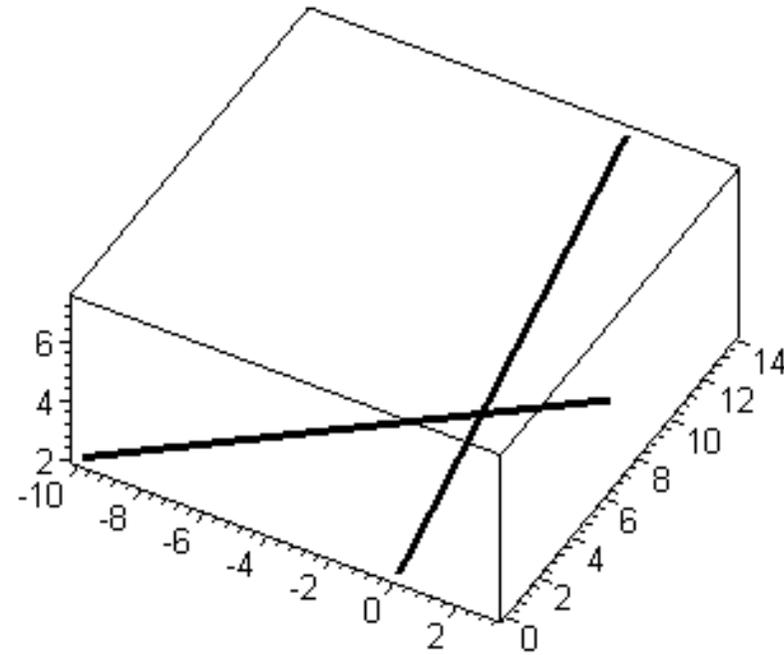


# Havaintoja enemmän kuin tuntemattomia, tarkkuusestimaatit



$X_2 = -10$   
 $Y_2 = 0$   
 $Z_2 = 0$

$X_1 = 0$   
 $Y_1 = 0$   
 $Z_1 = 0$



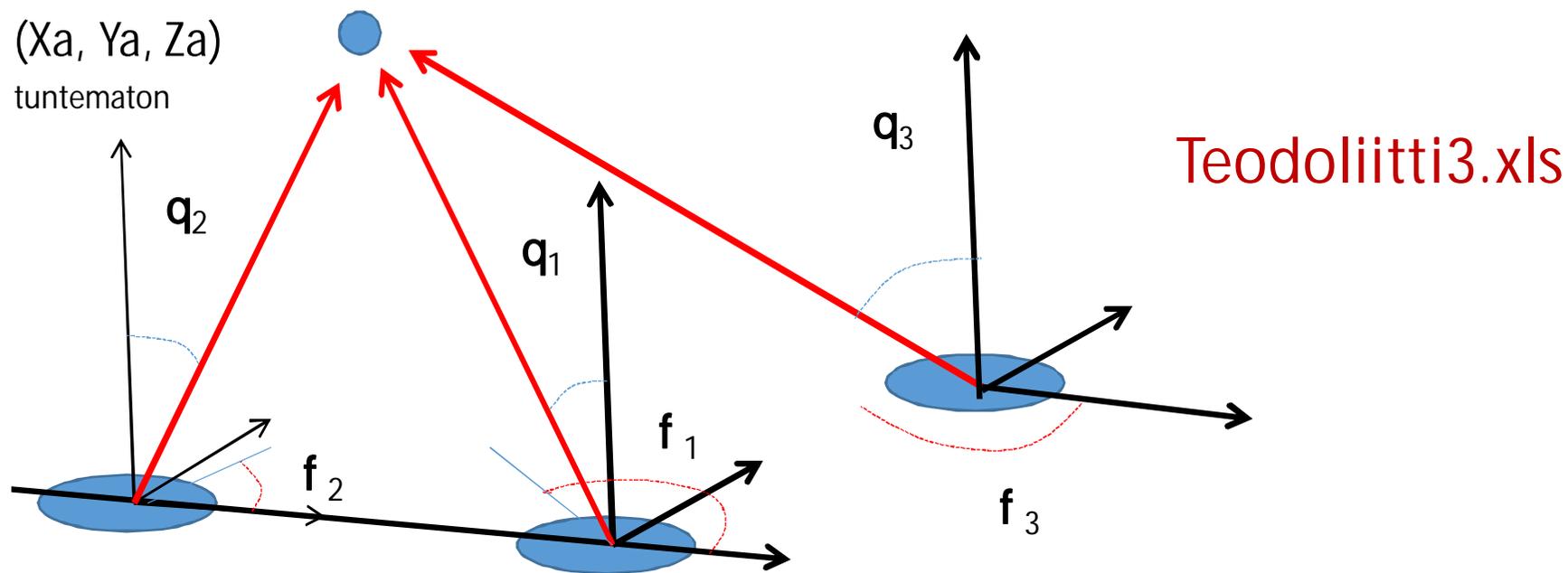
## observations

1.2	vertical angle
1.575	azimuth angle
1.4	vertical angle
0.48	azimuth angle

Teodoliitti2.xls

	Solution	SDEV	
X	-0.022	0.030	m
Y	5.174	0.072	m
Z	3.996	0.039	m

# Havaintoja enemmän kuin tuntemattomia, tarkkuusestimaatit



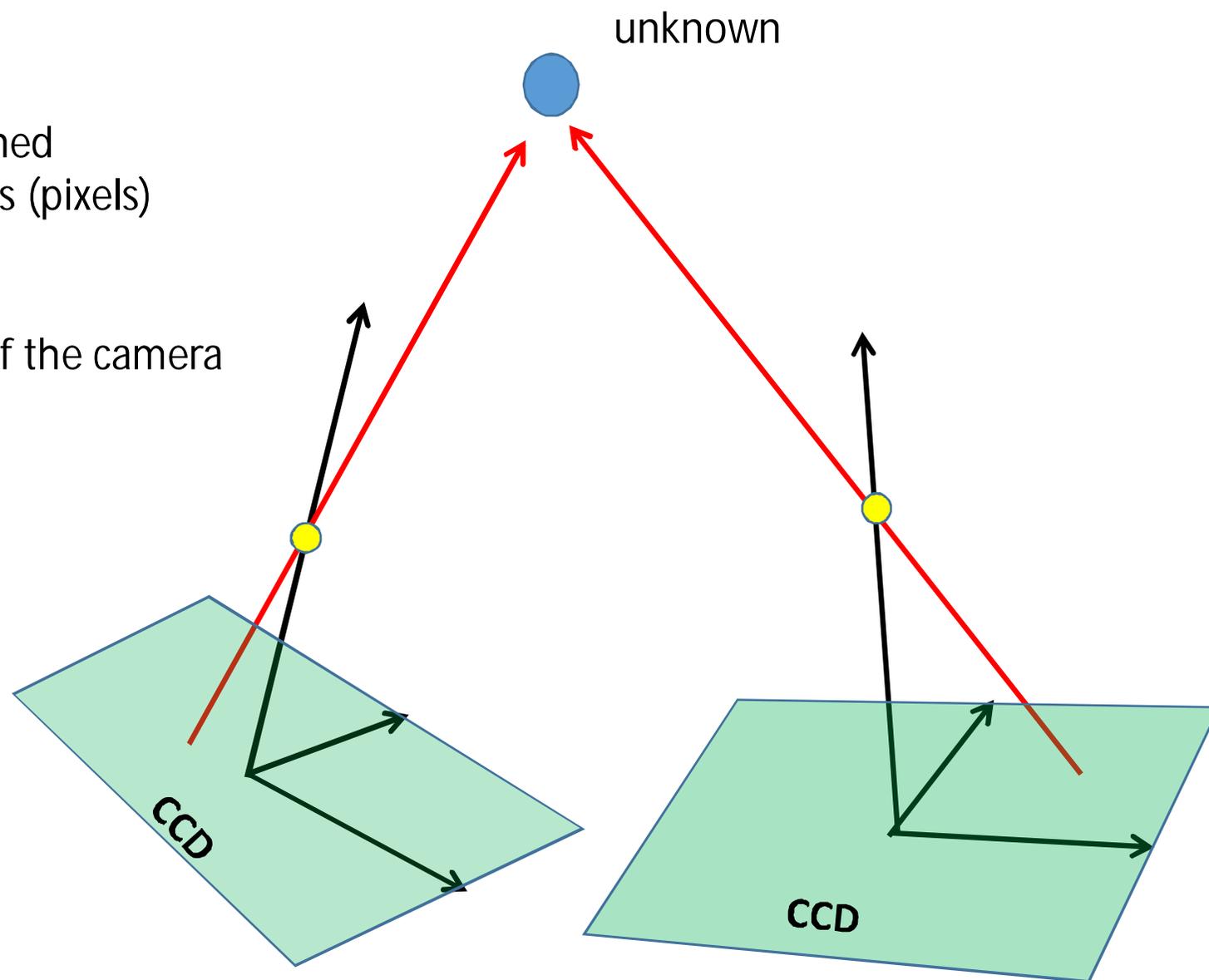
Painotettu pienimmän neliösumman tasoituslasku "kulmahavainnoista XYZ-koordinaateiksi". Kameran tapauksessa pikselikoordinaatit ovat "kulmahavainnoita kameran sisällä". Tasoituslaskennassa minimoidaan pikselivirheitä (xy), kun teodoliitissa minimoidaan kulmavirheitä.

Painot = havaintotarkkuuksien käänteislukuja, suuri hajonta pieni paino.  
Esimerkeissä painot olivat samat pysty- ja vaaka-kulmille, kaikilla teodoliiteilla.  
Kuvahavainnoilla painot, esim. 5  $\mu$ m.

# General, image intersection

- Not levelled
- Position usually planned
- Lots of measurements (pixels)
- At shutter-release:
  - XYZ focal point
  - rX, rY, rZ rotations of the camera

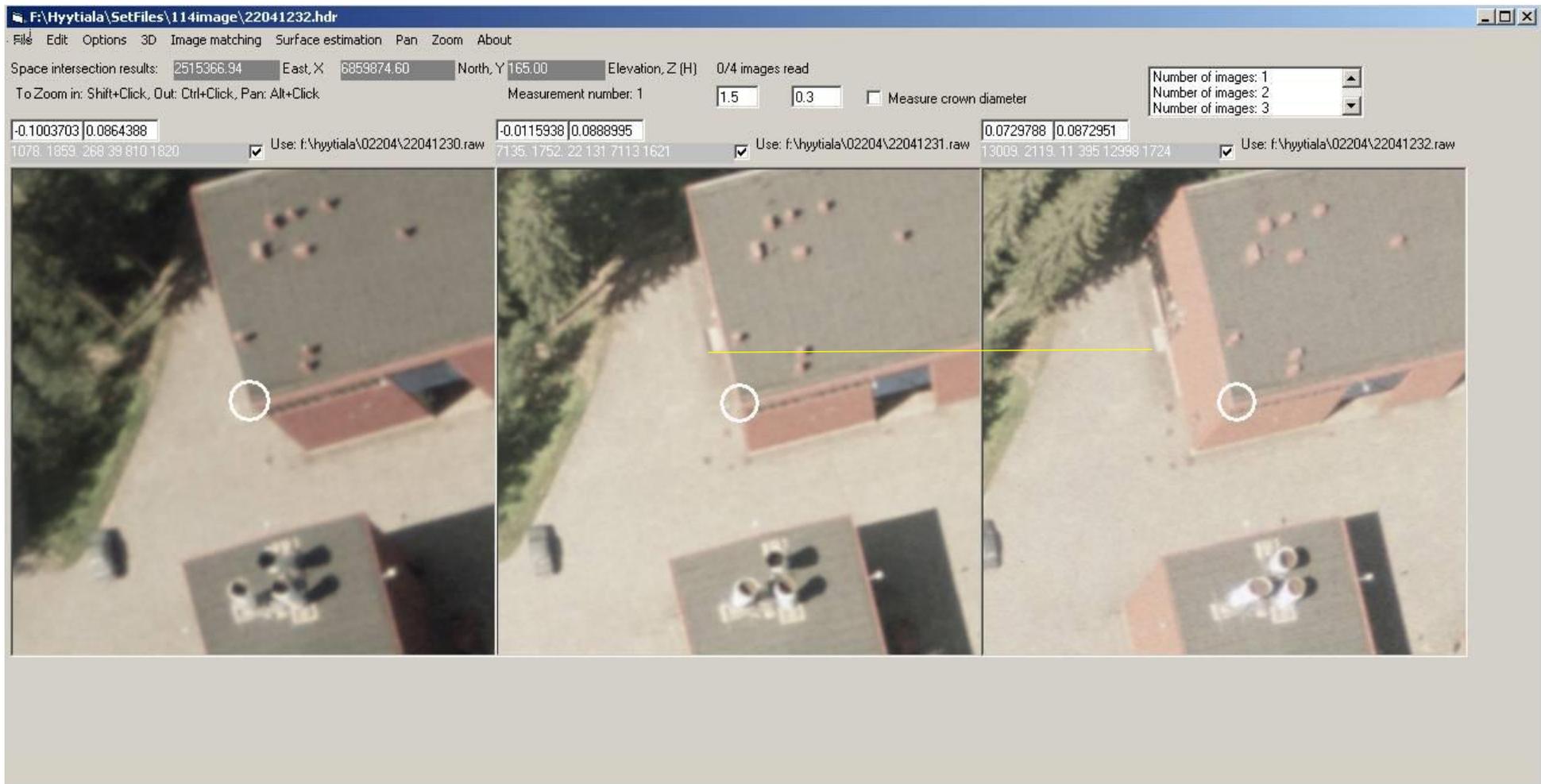
Correspondence  
Occlusion



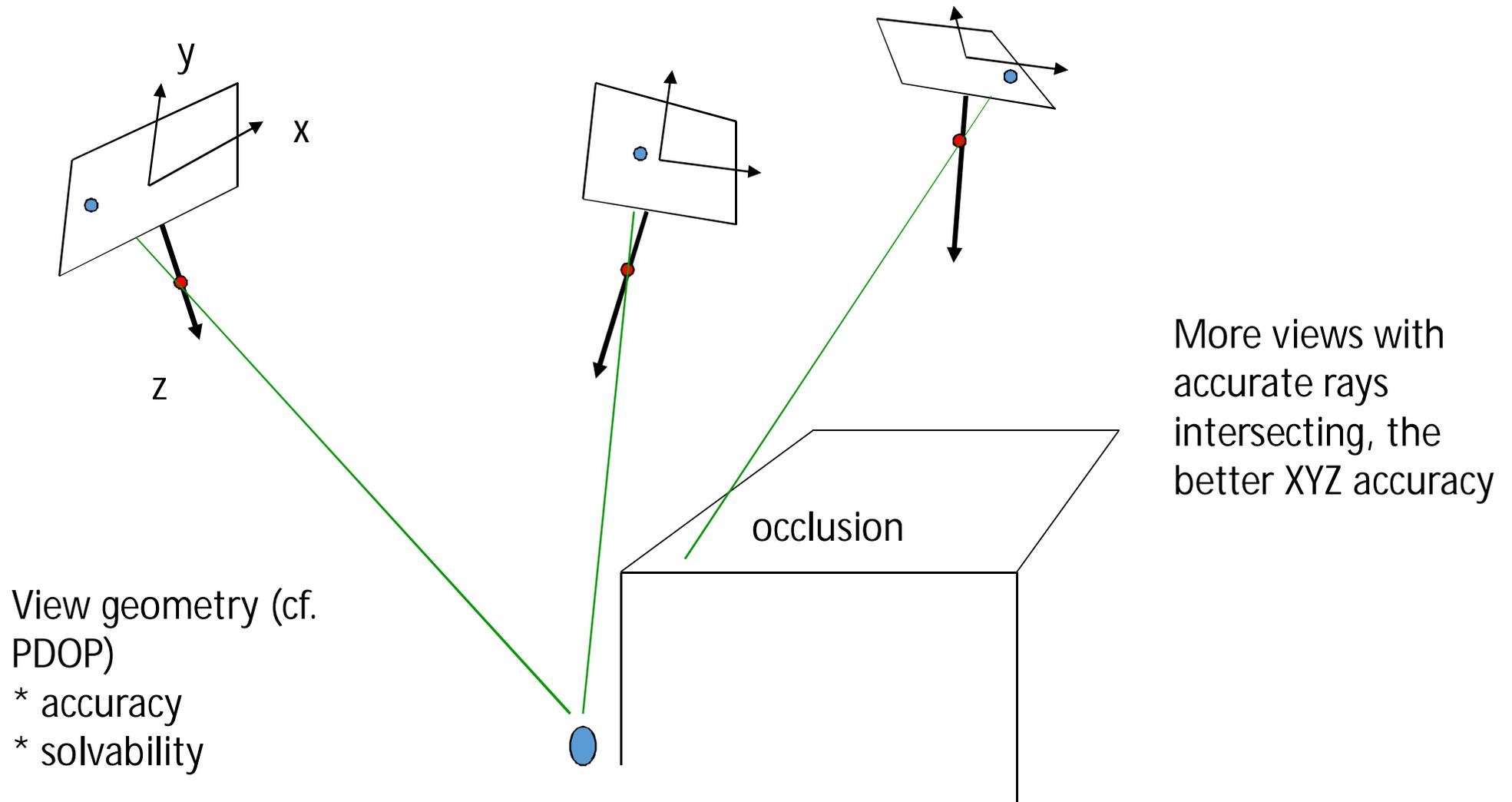
# Correspondence

## Occlusion

## Shading



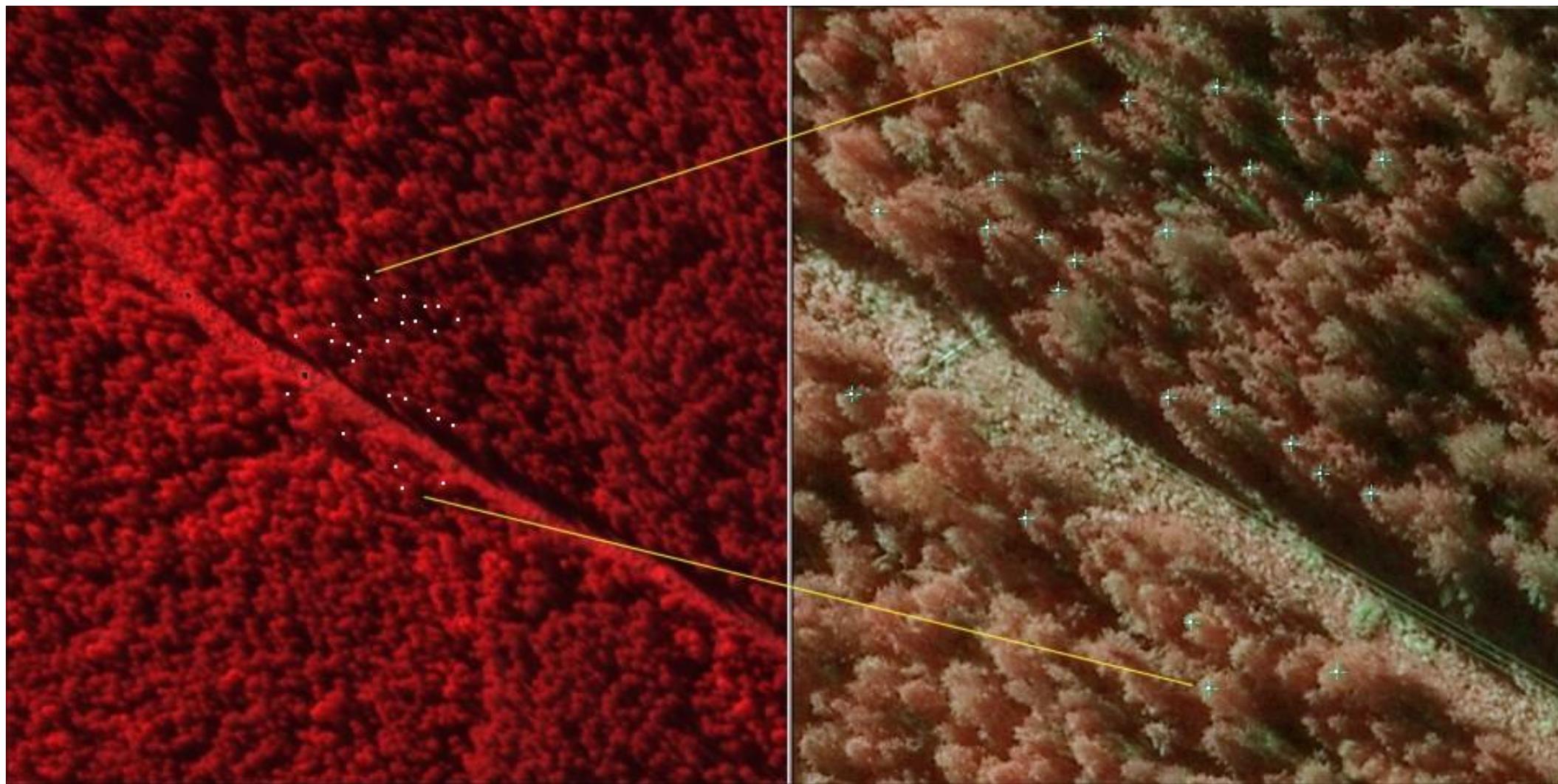
# 3D reconstruction in images



Correspondence problem  
(image matching)

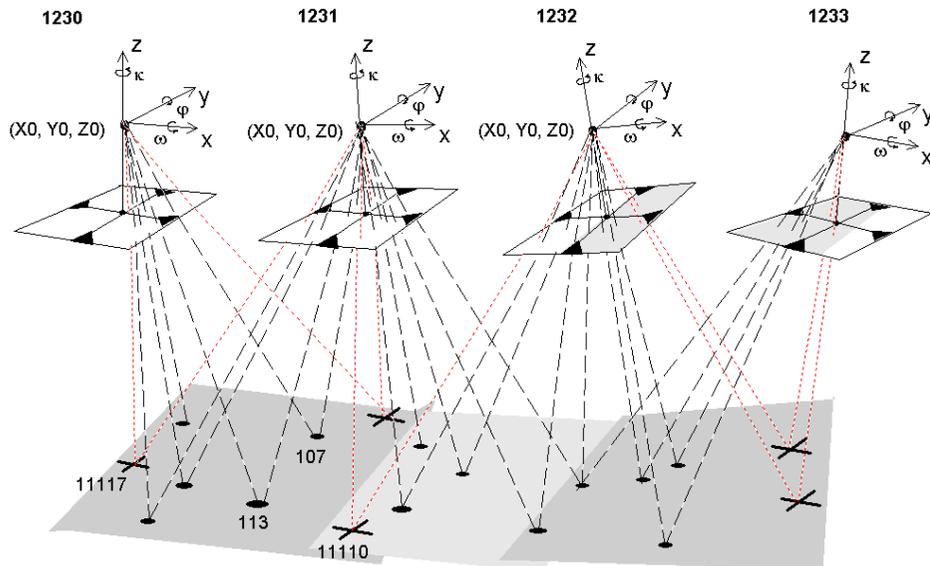
Not all points map to an image (not everything is seen) (cf. LiDAR!)  
Occlusion and shading

# Occlusion in closed-canopy forests Illumination conditions



# Photogrammetry – Aerial Triangulation

$$\begin{aligned} x - x_0 &= -c \frac{r_{11}(X - X_0) + r_{12}(Y - Y_0) + r_{13}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} \\ y - y_0 &= -c \frac{r_{21}(X - X_0) + r_{22}(Y - Y_0) + r_{23}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} \end{aligned}$$



Unknowns to be solved:

EO parameters (and IO + additional)

'Known':

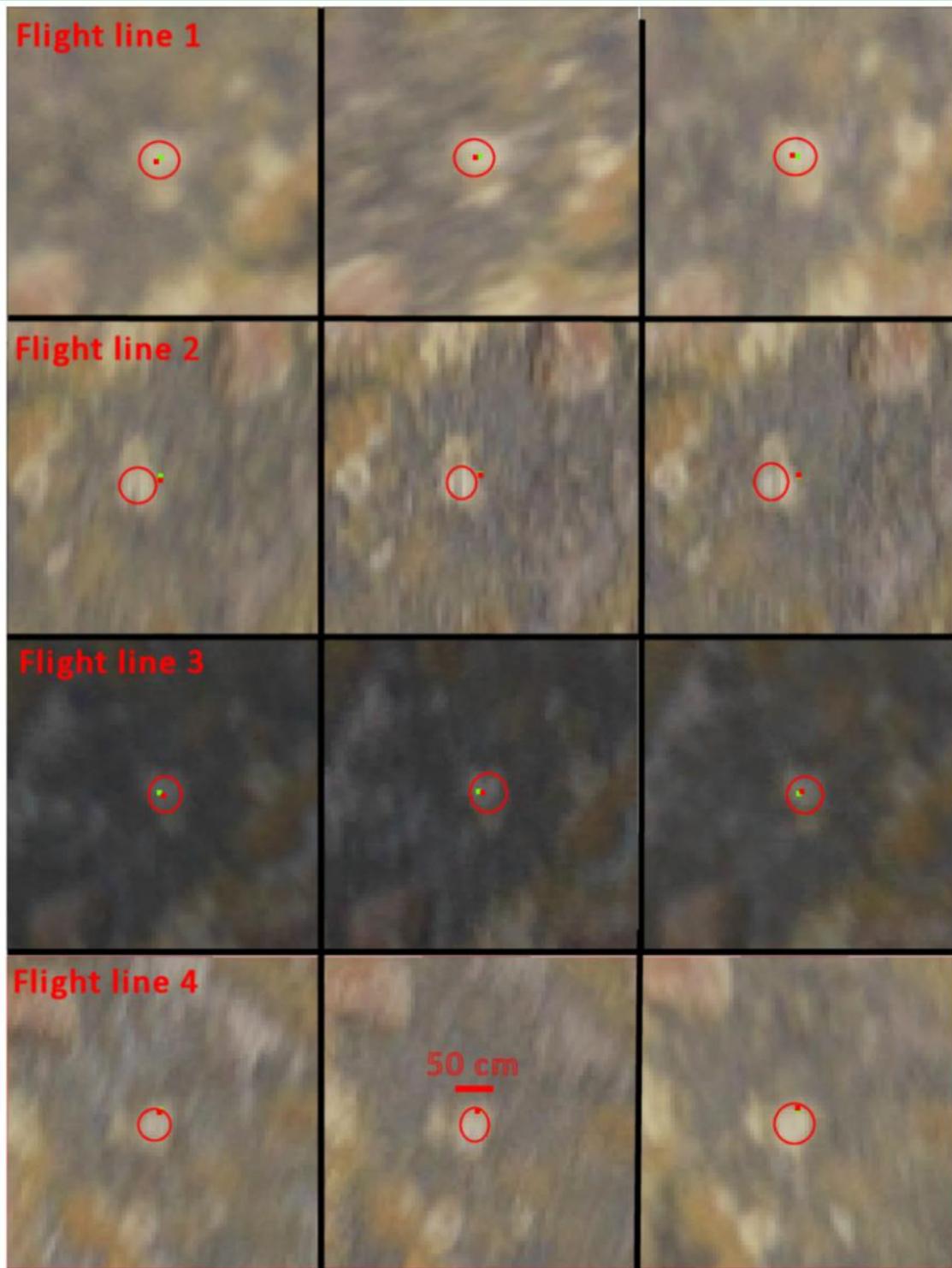
XYZ, XY, Z control points, distances, EO parameters (GNSS, imu), image points (of tie and control points).

Observation equations

(regression model components):

- 1) Image points
- 2) XYZ Distances
- 3) XY Distances
- 4) Z Distances
- 5) Observations of unknowns

Ⓒ Weighted non-linear regression



- \* Image blur
- \* Residual errors in aerial triangulation

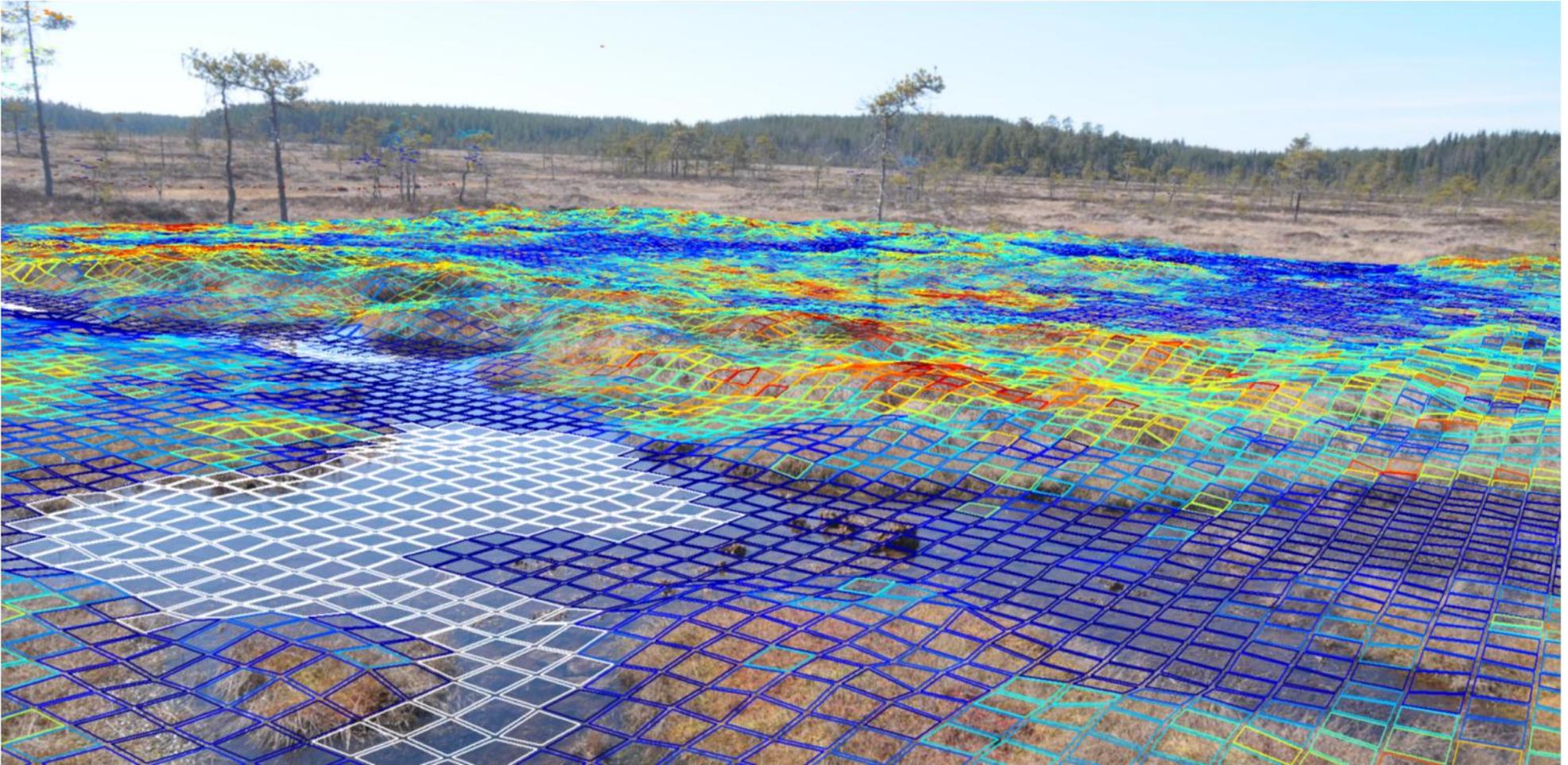
LiDAR WF amplitude values superimposed in an image pair



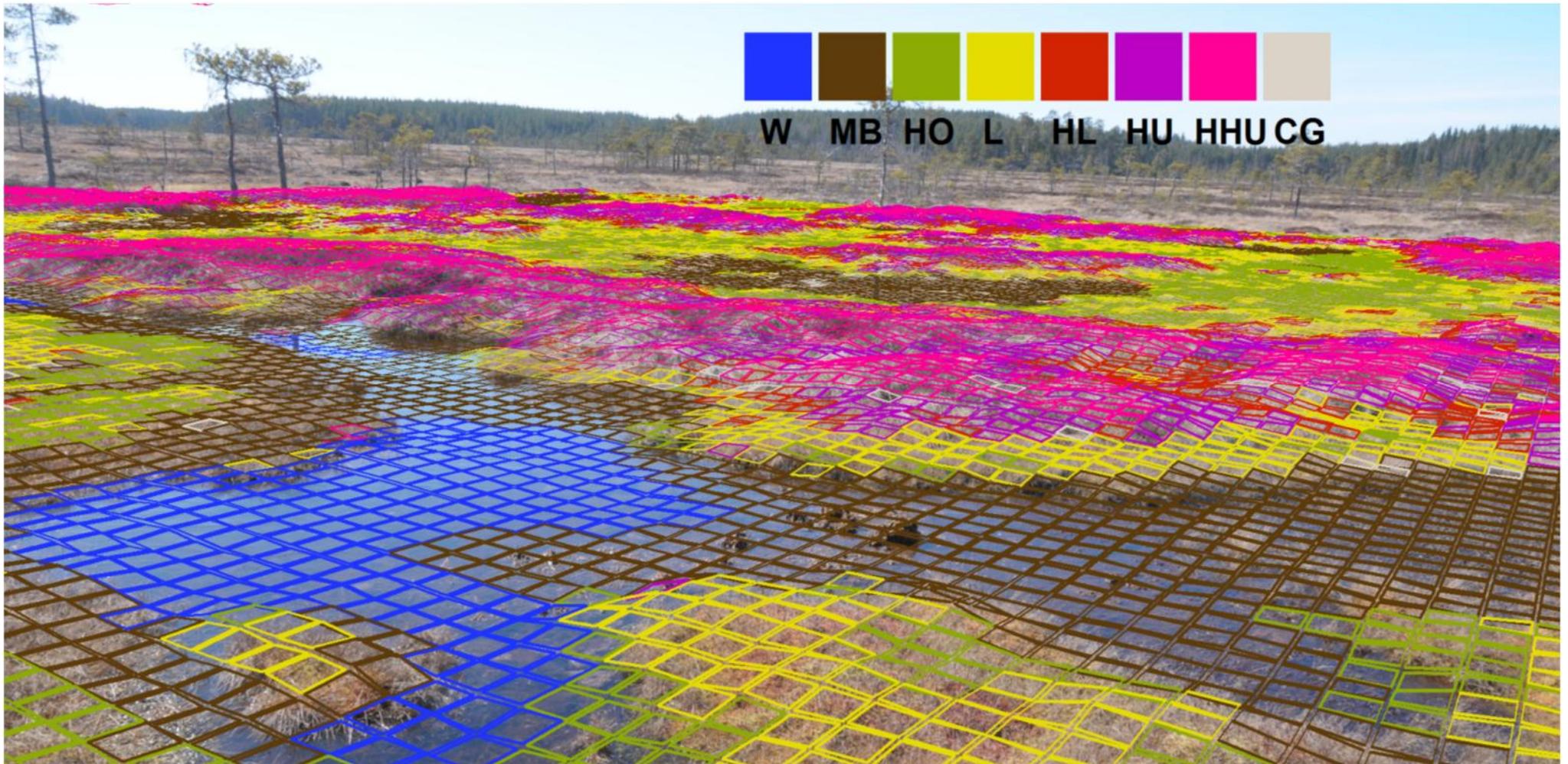
# LiDAR point data superimposed in a close-range view



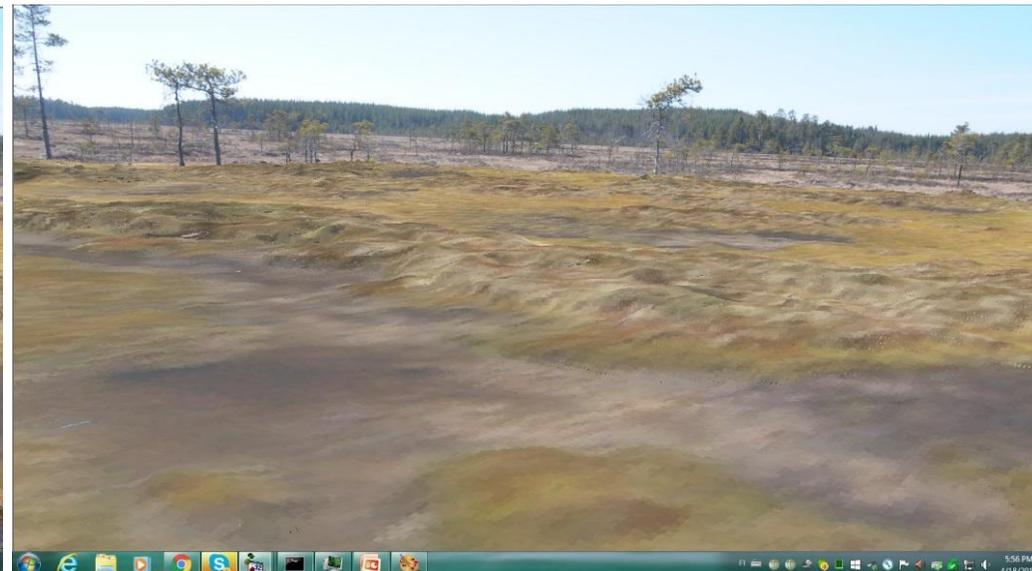
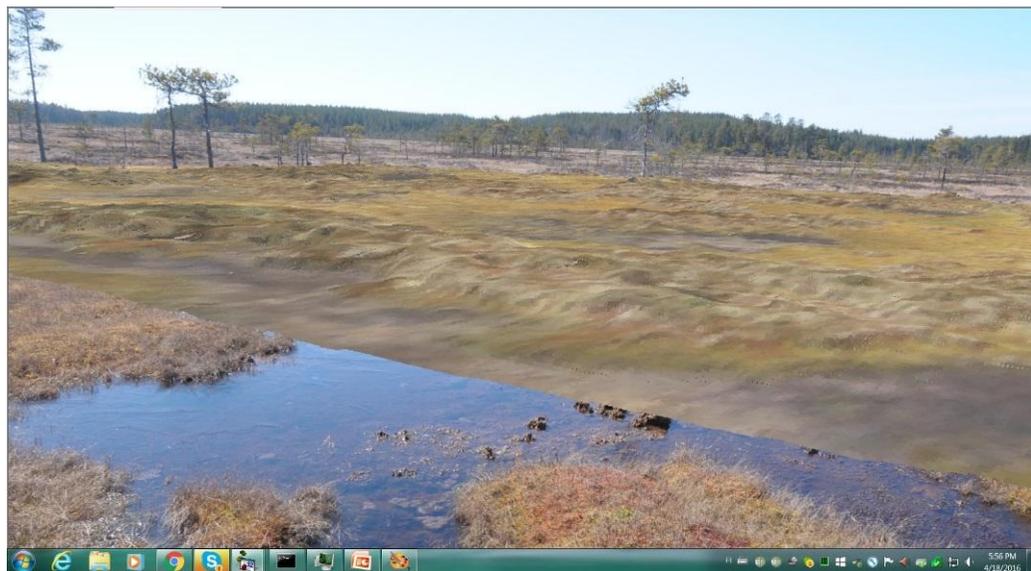
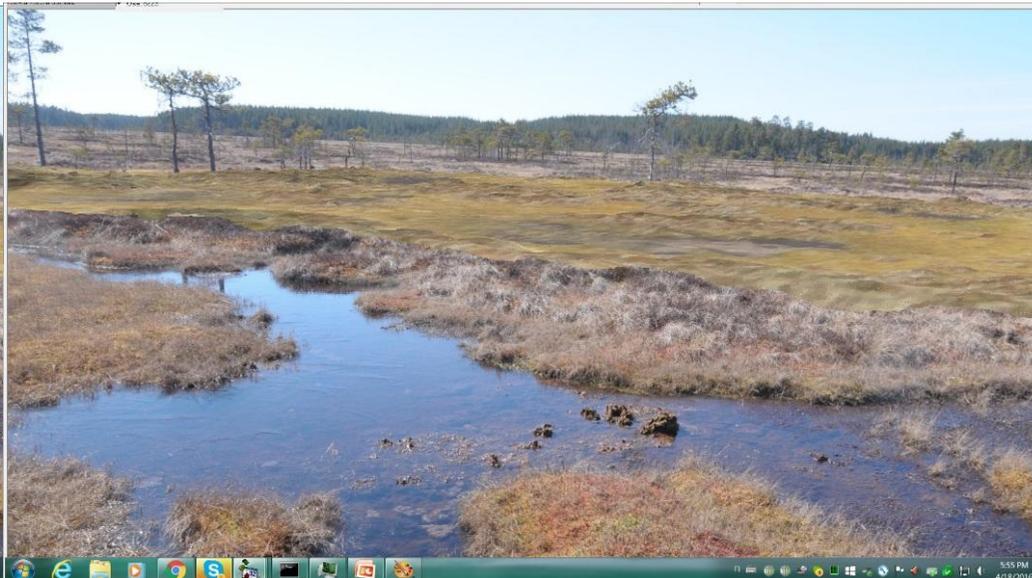
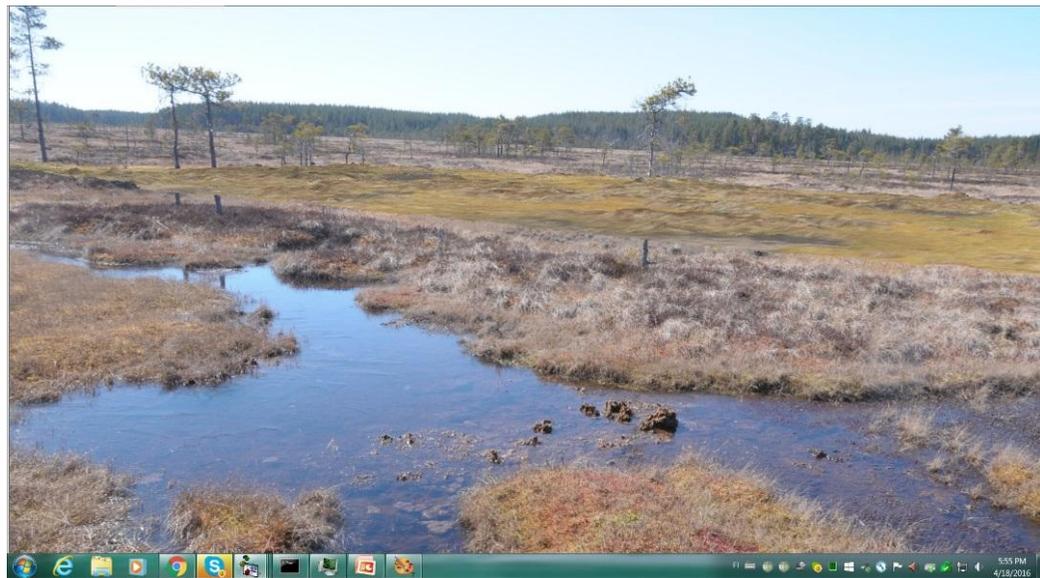
LiDAR intensity data interpolated in a 20x20-cm grid and data superimposed in a close-range view



Classification grid (20x20-cm grid) based on LiDAR and aerial image data - superimposed in a close-range view microforms on a patterned bog



Sequence showing warping of aerial image on a DEM, then projected to the close-range view



## Summary

Be it theodolite, camera, airborne or terrestrial LiDAR sensor – it is important to know the position of the instrument as well as the attitude, both require six parameters.

In image/theodolite the observations that establish the 'other end' of the 3D half-line are the position on the focal or the zenith and azimuth rotations of the theodolite's telescope. Camera has a rigid body, zoom may change focal length. Pixels view a large FOV, while it is cumbersome to turn the theodolite. LiDAR is turned (deflected) by a mirror, its attitude is very accurate.

Getting the XYZ coordinates requires the solution of the seventh parameter, which is the range (two-way travel time of the laser pulse) in LiDAR, or the intersection of the half-lines in the case of images.

LiDAR is without redundancy, no control over random errors. Two images already provide redundancy.

Occlusion influences less LiDAR, because...