Basics of Monte Carlo simulations 2006. Exercise 4

To be handed in Fri Mar 3, exercise session Tue Mar 7 10:15.

Return the exercises solution by email to the assistant eero.kesala@helsinki.fi. Return also the source codes used to solve the exercises; if the solutions involve more than about 5 files, pack them into a single .tar.gz or .zip package.

1. (12p) The Maxwell-Boltzmann energy distribution for particles in a classical ideal gas is

$$n(E)dE = \frac{2}{\sqrt{\pi}} \frac{N}{V} \frac{\sqrt{E}}{(k_B T)^{3/2}} e^{-E/k_B T} dE \qquad (1)$$

Write an MCMC code which generates points in this distribution. Use the code to calculate the mean energy $< E >$ of the distribution. Compare with the analytical answer

$$\frac{< E >}{N} = \frac{3}{2} k_B T \qquad (2)$$

Roughly how many MC steps do you typically need to get within 5% of the analytical answer? Report what ΔE_{max} you used for this result.

Hint: remember that the MCMC method deals with proportionalities.

2. (12p) Consider the distribution `dsa_nonegative.dat` given on the course web page and Fig. 1. This is a γ-ray distribution from a Doppler-broadened nuclear lifetime state measured in the Accelerator laboratory [K. Arstila et al, Nucl. Instr. and Meth. in Phys. Res. B 101 (1995) 321-326] Assume you will repeat this measurement, but can generate only 10 or 100 counts of statistics in the repeated measurements. Using the procedure outlined in lecture notes section 6.1, generate enough synthetic data sets to determine the statistical uncertainty of the mean for such 10 or 100-count distributions. Note that the uncertainty may be unsymmetric. Also compare with the uncertainty of the mean calculated using Gaussian statistics for the individual distributions. Does the 10 and/or 100 count distribution error deviate significantly from the Gaussian error?

You may freely use your (or the assistants) solution to exercise 3.1 as a starting point.

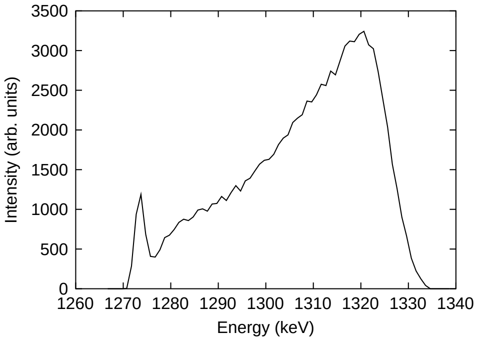

Figure 1: The DSA lineshape used in exc. 4.2.