Basics of Monte Carlo simulations 2006. Exercise 3

To be handed in Tue Feb 28, exercise session Thu Mar 2 10:15.

Return the exercises solution by email to the assistant eero.kesala@helsinki.fi. Return also the source codes used to solve the exercises; if the solutions involve more than about 5 files, pack them into a single .tar.gz or .zip package.

1. (12 p) Write a program which reads in an arbitrary discrete distribution from a file with data in x_i, p_i format, with the x data evenly distributed, and generates random numbers in this distribution. The p_i data does not need to be normalized. The program should output the random distribution as well as the ratio between the random and original data.

Apply the program to generate 1000000 points in the distribution given in the file 'bimodal.dat' on the course web page, and return two plots: the original vs. the random data, and the fraction between the two data sets. Also return the source code.

2. (6 p) Write a program which evaluates the volume of an N-dimensional hypersphere using Monte Carlo simulation, with the hit-and-miss approach. N should be a parameter in the code. The code should also calculate the statistical uncertainty. Hand in the code and results.

3. (6 p) Write a program which evaluates the volume of an N-dimensional hyperspheres using Monte Carlo simulation, with the sampling approach. N should be a parameter in the code. You may use the analytical solution to get the base area in $N-1$-dimensions. It should also calculate the statistical uncertainty. Hand in the code and results.

4. (6 p) Consider the simple importance sampling routine given in lecture notes #5 for the integral

$$I = \int_0^1 e^{-x^2} dx$$

Rewrite the routine in your language of choice to use a good random number generator, then add on the possibility to use stratified sampling in addition to the importance sampling scheme, and see whether you gain accuracy by adding the stratified sampling. Always compare the two schemes with the same total number of sampled points N, and consider both very small and large values of N. Return the code, and a report of what (if any) gains you could achieve.