Basics of Monte Carlo simulations 2006. Exercise 2

To be handed in Tue Feb 14, exercise session Thu Feb 16 10:15.

Return the exercises solution by email to the assistant eero.kesala@helsinki.fi. Return also the source codes used to solve the exercises; if the solutions involve more than about 5 files, pack them into a single .tar.gz or .zip package.

1. (12 p) Physical tests of random-number generators. A random walk is an important concept in both statistics and physics. The simplest possible random walk is the one-dimensional one, where a walker starts from 0 and takes exactly N steps of length 1 either to the left or right, with equal probability. It can be analytically shown that the average displacement of the walker after N steps $< x(N) >= 0$ and the average of the square of the displacement $< x^2(N) >= N$. These are exact results.

Write a program which simulates this 1D random walk to test random number generators. Perform walks to 100 steps, and repeat enough times to get a statistically good average $< x(N) >$ and $< x^2(N) >$. Consider the following random number generators: a) the Mersenne twister, b) the Park-Miller generator, c) the second $(ia = 211, ic = 1663, im = 7875)$ quick-and-dirty generator, and d) the higher-order $(im = 259200, ia = 7141, ic = 54773)$ quick-and-dirty generator.

Calculate $< x(N) >$ and $< x^2(N) >$ after 300 000 random walks of each generator. Repeat a few times with different seeds to get an idea of the statistical fluctuations (you do not need to calculate the uncertainty properly).

Report $< x(N) >$ and $< x^2(N) >$. Which generators fail the test, and which one is the worst? Return the code.

2. (6 p) In nuclear physics one of the most important distributions is given by the Breit-Wigner formula

$$\sigma(E) = a\frac{\Gamma^2}{(E - E_R)^2 + \Gamma^2/4}$$

which for instance gives the shape of single, isolated nuclear resonances at an energy E_R. Write a program which generates random numbers in this distribution using the analytical inversion method. Verify the correctness by plotting the distribution and an analytical prediction with the same parameter values on top of each other (use proper normalization). Hand in the code and plot.

3. (9 p) Write a program which generates random numbers distributed as

$$f(t) = \cos^2(t)e^{-t}$$

in the interval $[0, \infty]$ by the combined analytical-rejection method. Let the code also report the number of hits and misses. Verify the correctness by plotting the distribution and an analytical prediction with the same parameter values. Hand in code, plot, and report of number of hits and misses in a long run.

4. (6 p) In the recent Finnish presidential election, LL.M. Tarja Halonen was re-elected president by winning with 51.8 % of the popular vote over LL. M. Sauli Niinistö. In the Gallup polls preceding the election, typically about 1000 people were interviewed.

Write a Monte Carlo program which simulates taking a poll right before the election. Assume all interviewed people who give their opinion (those who haven't decided do not count) would have a probability to state they vote for Halonen of 51.8%, and that exactly 1000 people do give their opinion. a) Generate 20 poll results and look at the variation. After this generate enough poll results to be able to answer the following questions. b) What is the probability that a poll had predicted that Sauli Niinistö would win? c) What is the probability a poll would have given the correct result with a 1% accuracy, i.e. an answer between 51.3 and 52.3 %?

(n.b. this exercise can and may also be solved analytically).