Monte Carlo simulations 2004. Exercise 1

To be handed in Tue 3.2, exercise session Thu 5.2. 10:15.

1. (4 p) Derive the probability

$$P_{hit} = \frac{2l}{\pi d}$$

for the Buffon's needle problem (see lecture notes).

If you want to have some fun, you can try the Buffon's needle java applet below http://stud2.tuwien.ac.at/~e9527412/.

In exercises 2-5, we consider the following random number generators, given in Numerical Recipes second edition (available in the web): a) The ANSI standard generator (if you use C), or the lowest-order ($ia = 106, ic = 1283, im = 6075$) quick-and-dirty generator (Fortran). b) The Park-Miller generator using the Schrage trick, but without the shuffle.

To make the results comparable, use 7 as the initial seed.

2. (6 p) Write a piece of code which calculates the repeat interval of the generators (by brute force i.e. going through the numbers before you get the original one back). Hint: you can use a "double" (C) or "double precision" (Fortran) variable to calculate the repeat interval; this data type almost always nowadays has a 15-digit mantissa, so it is enough to calculate the interval of any 32-bit integer (which only has up to 10 numbers).

3. (6 p) Generate a 2D distribution between (0-1 , 0-1) with the random-number generators, and plot them. Use 10000 points. Does the 2D distribution seem at least superficially OK?

4. (6 p) Generate again random points in 2D between 0 and 1, but print out only the ones lying below x=0.00001. Keep on doing this until you have obtained 10000 points, plot the result, and comment on it.

5. (6 p) Test of whether it is a good idea to attempt to improve on generators without much thought. Modify generator a) by changing the numbers a and c randomly to something which "feels" good to you. Then repeat the 2D test for the generator. Repeat this 5 times. Comment on the result.

Hint: If you do not have a favourite graphing and fitting system, the program `xgraph` *available in the science/tekno cluster in* `/usr/local/contrib/bin` *is enough to handle the graphical parts of this exercise.*