SUMS OF RANDOM VARIABLES

Notations:

x and y are two random variables

$\bar{x} = \dfrac{\sum_{i=1}^{k} x_i}{k}$ is the expected value ($k \to \infty$)

$V(x) = \dfrac{\sum_{i=1}^{k} (x_i - \bar{x})^2}{k} = \overline{(x_i - \bar{x})^2}$ is the variance

Expectation of a sum:

$$\overline{(x+y)} = \frac{\sum_{i=1}^{k} x_i + y_i}{k} = \frac{\sum_{i=1}^{k} x_i}{k} + \frac{\sum_{i=1}^{k} y_i}{k} = \bar{x} + \bar{y}$$

Variance of a sum:

$$V(x+y) = \frac{\sum_{i=1}^{k} (x_i + y_i - \bar{x} - \bar{y})^2}{k} = \frac{\sum_{i=1}^{k} \left((x_i - \bar{x}) + (y_i - \bar{y})\right)^2}{k} =$$

$$\frac{\sum_{i=1}^{k} (x_i - \bar{x})^2 + 2(x_i - \bar{x})(y_i - \bar{y}) + (y_i - \bar{y})^2}{k} =$$

$$\frac{\sum_{i=1}^{k} (x_i - \bar{x})^2}{k} + \frac{2\sum_{i=1}^{k} (x_i - \bar{x})(y_i - \bar{y})}{k} + \frac{\sum_{i=1}^{k} (y_i - \bar{y})^2}{k} =$$

$$= V(x) + 2COV(x, y) + V(y)$$

If the two random variables *x* and *y* are independent:

$$COV(x, y) = 0$$

$$V(x+y) = V(x) + V(y)$$