Adaptive Dynamics Course
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Coevolution in a Predator-Prey System

In this project, we study the coevolution of predators with their prey. To model the population dynamics of the predator-prey system, we start with the standard model

$$\begin{aligned} \frac{dN}{dt} &= rN\left(1-\frac{N}{K}\right)-\beta PN \\ \frac{dP}{dt} &= \gamma\beta PN - dP \end{aligned} \tag{1}$$

where N and P are respectively the population densities of prey and predator, r and K are respectively the intrinsic growth rate and the carrying capacity of prey in absence of predation, β is the catch rate, γ is the conversion factor of consumed prey into predator offspring, and d is the predator death rate.

Predators of a given trait value can most efficiently catch and handle prey of a certain "matching" trait value (for example, predators of a certain size cannot catch prey that are too large for them and are also inefficient with too small prey). We scale the prey and predator trait values (x_1 and x_2, respectively) such that the best "matching" occurs for $x_1 = x_2$, and assume that the catch rate is a Gaussian function of the difference between the two traits,

$$\beta(x_1, x_2) = \exp\left(-\frac{(x_1 - x_2)^2}{2\sigma_\beta^2}\right) \tag{2}$$

Further, we assume that the prey carrying capacity depends on the trait value according to

$$K(x_1) = \exp\left(-\frac{x_1^2}{2\sigma_K^2}\right) \tag{3}$$

When several prey strategies are present, the competition coefficient between them may be constant but may also depend on the trait difference,

$$a(x_1^{(i)}, x_1^{(j)}) = \exp\left(-\frac{(x_1^{(i)} - x_1^{(j)})^2}{2\sigma_a^2}\right) \tag{4}$$

where $x_1^{(i)}$ is the trait value of the *i*th prey strategy. $1/\sigma_a = 0$ corresponds to a constant competition coefficient, $a(x_1^{(i)}, x_1^{(j)}) \equiv 1$.

Putting the above assumptions together, the population dynamics of *m* prey and *n* predator strategies are given by

$$\begin{aligned} \frac{dN_i}{dt} &= rN_i \left(1 - \frac{\sum_{k=1}^{m} a(x_1^{(i)}, x_1^{(k)}) N_k}{K(x_1^{(i)})} \right) - \sum_{k=1}^{n} \beta(x_1^{(i)}, x_2^{(k)}) P_k N_i \\ \frac{dP_j}{dt} &= \gamma \sum_{k=1}^{m} \beta(x_1^{(k)}, x_2^{(j)}) P_j N_k - dP_j \end{aligned} \tag{5}$$

for $i = 1, \ldots, m$ and $j = 1, \ldots, n$. Without predation ($P_j = 0$ for $j = 1, \ldots, n$), the model is the same as the Lotka-Volterra competition model studied in the lectures.

Assume first $1/\sigma_a = 0$ such that $a(x_1^{(i)}, x_1^{(j)}) \equiv 1$. Investigate the monomorphic evolution of prey and predator: Determine the singular point (x_1^*, x_2^*), its convergence and evolutionary stability. Recall that in two dimensions, convergence stability is not so straightforward as in one dimension; use the canonical equation if necessary and investigate whether a Hopf bifurcation is possible.

When (x_1^*, x_2^*) is convergence stable, is evolutionary branching possible? Can two prey strategies coexist? Two predator strategies? If you find within-species coexistence, obtain an isocline plot for the species while holding the trait value of the other species constant, and investigate possible singularities with two resident strategies in one species.

If time permits, investigate the effect of $1/\sigma_a > 0$ on evolutionary branching and coevolution.