

Evolution of Cooperation and Cheating

In this project, we investigate a simple model for the evolution of cooperative behaviour. The obvious problem with cooperation is that it is vulnerable to cheating, i.e., selfish individuals ("defectors") who take the benefit offered by others, but fail to return the benefits to others at their own cost, are at an advantage in a population of cooperators. Many models of game theory address the joint dynamics of cooperators and defectors and investigate which mechanisms can protect a population of cooperators from the invasion of defectors. In this project, however, we are concerned with the evolution of the very strategies "cooperator" and "defector". We thus assume that the level of cooperative investment is a continuous variable x , which is evolving, and which is scaled such that $x = 0$ invests nothing into the common good or to help others (but accepts what they give) and $x = 1$ gives maximal investment. Other model assumptions will be analogous to the so-called Snowdrift Game. For simplicity, we consider only pairwise interactions between individuals.

Assume that individuals encounter each other at random, and invest into cooperation according to their strategy x . Both parties share the same benefit, which is an increasing function of the sum of their investments, and both parties pay a cost, which is an increasing function of their own personal investment. In a population where strategies x_1, \dots, x_n occur at frequencies q_1, \dots, q_n , the expected payoff (=benefit minus cost) to an individual with strategy x_i is

$$P_i = \sum_{j=1}^n q_j B(x_i + x_j) - C(x_i) \quad (1)$$

where B and C are respectively the benefit and cost functions, which are both increasing and satisfy $B(0) = 0$ and $C(0) = 0$ (no investment implies no benefit and no cost).

The number of offspring of an individual is an increasing function of the payoff received, and also depends on population density (otherwise the population would grow exponentially). We assume that the number of offspring of an individual with strategy x_i can be written in the product form $W(P_i)F(N)$, where N is total population size, and that all parents die after reproduction such that the population in the next year is made up of only the offspring. The number of individuals with strategy x_i , denoted by N_i , changes according to

$$N_i(t+1) = W \left(\sum_{j=1}^n q_j(t) B(x_i + x_j) - C(x_i) \right) F(N(t)) N_i(t) \quad (2)$$

where $q_j(t) = N_j(t) / N(t)$ and $N(t) = \sum_{j=1}^n N_j(t)$. The relative frequency of individuals with strategy x_i changes according to

$$q_i(t+1) = \frac{W \left(\sum_{j=1}^n q_j(t) B(x_i + x_j) - C(x_i) \right)}{\sum_{k=1}^n q_k(t) W \left(\sum_{j=1}^n q_j(t) B(x_k + x_j) - C(x_k) \right)} q_i(t) \quad (3)$$

i.e., the dynamics of relative frequencies does not depend on function F . Under biologically realistic conditions (which do not permit a negative value of W) and using non-linear scaling of B and C , one can transform the function W into the piecewise linear function

$$W(P) = \begin{cases} 0 & \text{if } P < -a \\ a + P & \text{if } P \geq -a \end{cases} \quad (4)$$

where a may be positive or negative. The behaviour of the model thus depends on the shape of the (scaled) functions B and C .

Explore the adaptive dynamics of the cooperative investment x . Investigate under which conditions evolutionary branching or evolution to a single evolutionarily stable strategy occurs, and explore the evolution of dimorphic populations after evolutionary branching.