Adaptive Dynamics Course
S.A.H. Geritz & É. Kisdi
Vienna 2007

The Evolution of Cannibalism

Consider a population of strategies $x_1,\ldots,x_k$ and corresponding population densities $n_1,\ldots,n_k$ living off a resource with density R. The strategy x_i is the proportion of time spent searching for the resource. The remaining proportion $1-x_i$ is spent attacking those individuals that are searching for the resource, i.e., attacking conspecific prey (cannibalism). The strategy space is thus $\boldsymbol{X}=[0,1]$. We assume that the resource dynamics are logistic, and that the attack on foraging individuals has a Holling-II functional response with a handling time that depends on the strategy. The latter reflects the assumption that the more time individuals spend attacking others, the better they learn the technique and hence they can handle the prey faster. The population dynamics are thus given by

$$\frac{dR}{dt}=rR\left(1-\frac{R}{K}\right)-\nu R\sum_{j=1}^{k}x_j n_j$$

$$\frac{dn_i}{dt}=\lambda\nu R x_i n_i-\beta x_i n_i\sum_{j=1}^{k}\left(\frac{(1-x_j)n_j}{1+\beta T(x_j)\sum_{l=1}^{k}x_l n_l}\right)+\frac{\gamma\beta(1-x_i)n_i\sum_{j=1}^{k}x_j n_j}{1+\beta T(x_i)\sum_{j=1}^{k}x_j n_j}-\delta n_i \quad (i=1,\ldots,k) \tag{1}$$

To simplify the system, and to avoid the problem of having to establish whether there is a stable equilibrium or not, we assume that the dynamics of R is fast compared to that of $n_1,\ldots,n_k$, so that we can substitute R in the equations for $n_1,\ldots,n_k$ by its quasi-equilibrium value

$$\hat{R}=K\left(1-\frac{\nu}{r}\sum_{j=1}^{k}x_j n_j\right) \tag{2}$$

The model can be cleaned up a little by letting $t'=\delta t$, $n_i'=\beta n_i$, $a=\lambda\nu K/\delta$, $b=\delta^{-1}$, $c=\gamma$ and $d=\nu/(r\beta)$. Dropping the primes, (1) with (2) becomes

$$\frac{dn_i}{dt} = a\left(1 - d\sum_{j=1}^{k} x_j n_j\right) x_i n_i +$$

$$-bx_i n_i \sum_{j=1}^{k}\left(\frac{(1-x_j)n_j}{1+T(x_j)\sum_{l=1}^{k} x_l n_l}\right) + \frac{bc(1-x_i)n_i \sum_{j=1}^{k} x_j n_j}{1+T(x_i)\sum_{j=1}^{k} x_j n_j} - n_i \quad (i = 1,\ldots,k) \tag{3}$$

For the handling time T we take

$$T(x) = x^p T_{\max} \quad (p \geq 0) \tag{4}$$

Different values of p correspond to different rates of learning how to attack and handle your population fellows.

The aim of the project is to study the evolution of the strategy x in relation to different parameter values. Good starting values are:

$$a = 10$$
$$b = 50$$
$$c = 0.1$$
$$d = 1$$
$$p = 10$$
$$T_{\max} = 1$$