Predator-Prey Systems: Evolutionary Diversification of Prey

In this project, we compare the evolution of the prey species in two simple predator-prey systems. The two models are very similar, yet the results are very different.

In both cases, we consider a Lotka-Volterra type model of prey and predator, where the prey exhibits logistic population growth in absence of the predator. In Model 1, we assume the dynamics

$$\begin{aligned} \frac{dN}{dt} &= (r - hN)N - cPN \\ \frac{dP}{dt} &= ecNP - dP \end{aligned} \qquad (1)$$

where N and P are respectively prey and predator densities, r is the intrinsic growth rate of the prey, h measures how sensitive the prey is to crowding, and c is the catch rate of the predator. The predator transforms the consumed prey into predator offspring with efficiency e and the death rate of the predator is d.

In Model 2, we rewrite the prey equation into

$$\begin{aligned} \frac{dN}{dt} &= r(1 - N/K)N - cPN \\ \frac{dP}{dt} &= ecNP - dP \end{aligned} \qquad (2)$$

where $K = r/h$ is the prey carrying capacity. For any fixed set of parameters, the two models are obviously the same.

In both models, we assume that the prey can evolve safer strategies such that it can reduce the predator catch rate c, but this has a cost in terms of reproduction, i.e., it implies a smaller value of r. To give a concrete mechanism, c may be proportional to the fraction of time the prey is active, assuming that resting prey is well hidden from predators. With less time spent active (lower c) the prey can collect less food for itself and hence its intrinsic growth rate will be diminished (lower r). We thus assume that c and r are traded off such that $r = f(c)$ is an increasing function. Notice that in Model 1, the prey carrying capacity is r/h and hence is affected by c. In Model 2, however, K is a fixed model parameter.

Investigate the evolution of c under various trade-offs $r = f(c)$. A linear trade-off $f(c) = \alpha + \beta c$ (where β is positive such that $f(c)$ is increasing) is a good starting point; next you can perturb this trade-off by adding a small quadratic term $\gamma(x - x^*)^2$ and also explore other increasing functions. Remember to check which strategies are viable and result in stable positive equilibrium densities.

Focus first on Model 1. The singular strategy and its stability properties can be obtained analytically, but illustrate also with PIPs. Check under which conditions the singular strategy is biologically relevant ($c \geq 0$; cf. a negative predation rate does not make sense in eqs. (1)), and find out what happens if there is no biologically relevant singular strategy. Construct the isocline plot to explore dimorphic evolution in an example with evolutionary branching.

> If time permits, it is interesting to construct isocline plots also for an example with an ESS and for an example where the ESS is just to switch into a branching point (ESS-BP bifurcation), such that you can explore how the isoclines change when the singular strategy moves from an ESS to a branching point.

Next, construct PIPs for Model 2. Can you get evolutionary branching? Can you get any strategies coexisting? See if Model 2 is an optimisation model.