

Evolutionary Diversification of a Predator

In this project, we study the evolution of a predator which has one important prey (which we model explicitly) but also consumes other resources (modelled implicitly). Moreover, individuals engage in direct conflicts (interference competition), which leads to increased death rate when predator density is higher. The evolving trait (x) is the fraction of time spent hunting for the prey rather than looking for other resources.

Denote the density of predators and prey with P and N , respectively. The ecological model is given by the following pair of differential equations:

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right) - \frac{cxNP}{1+chN} \tag{1}$$

$$\frac{dP}{dt} = \frac{ecxNP}{1+chN} + g(x)P - dP - kP^2$$

Without the predator, the prey grows logistically with intrinsic growth rate r and carrying capacity K . c is the catch rate of the predator and x (with values between 0 and 1) is the fraction of time the predator spends searching and handling the prey. The predator has Holling type II functional response with handling time h . Hence in one unit of time, each predator kills $cxN/(1+chN)$ prey individuals.

In the second equation describing the population dynamics of the predator, there are two birth and two death terms. The captured prey is turned into predator offspring with conversion factor e (first positive term). The predator is also obtaining other resources, and produces $g(x)$ offspring per unit of time from these (second positive term). We assume that $g(x)$ is a decreasing function, i.e., the more time the predator spends with the prey, the less other resources it can obtain. Density-independent death occurs at rate d (first negative term). Additionally, predators may kill one another in competitive fights: this corresponds to the second negative term of density-dependent death.

Because of the Holling type II functional response, this ecological model can produce cycling dynamics (with an unstable equilibrium within the limit cycle). The primary aim is to explore the adaptive dynamics of hunting time, x , when the population dynamical equilibrium is stable. This can be checked by solving (1) numerically, assuming an initial state near the equilibrium, or by evaluating the Jacobian of equations (1). Remember to check also that the equilibrium densities are positive (both prey and predator are viable).

To explore the evolution of hunting time, x , assume that a mutant predator with trait value y enters at a low density such that it does not disturb the population dynamical equilibrium of equations (1) noticeably. The dynamics of this mutant is given by the equation

$$\frac{dP_{mut}}{dt} = \frac{ecy\hat{N}P_{mut}}{1+ch\hat{N}} + g(y)P_{mut} - dP_{mut} - k\hat{P}P_{mut} \quad (2)$$

where \hat{N} and \hat{P} are the equilibrium densities determined by the resident population (from equations (1)) and P_{mut} is the (small) density of the mutant.

For the trade-off function $g(x)$, a possible choice is

$$g(x) = \alpha + \beta \exp(-\gamma x) \quad (3)$$

With positive β and γ , this function is convex decreasing, whereas with negative β and γ , the function is concave decreasing. (Other combinations result in increasing functions.) The linear trade-off $g(x) = A - Bx$ and its perturbations (adding or subtracting a small quadratic term $C(x - x^*)^2$) are also worth to check.

Explore the adaptive dynamics of hunting time via producing PIPs numerically. Try to find an example for evolutionary branching, and construct an isocline plot for this example. What types of trade-off functions lead to evolutionary branching vs to an ESS?

If time permits, produce a PIP for the alternative model

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right) - \frac{cxNP}{1+chN} \quad (4)$$

$$\frac{dP}{dt} = \frac{ecxNP}{1+chN} + g(x)P \left(1 - \frac{P}{L} \right)$$

In this model, the prey equation is the same as in (1). In the predator, however, we assume logistic growth using the alternative resources. For a fixed strategy x , this model is equivalent to the first model in (1): we can absorb d in (1) into $g(x)$ without loss of generality and we can define $g(x)/L=k$ to recover the quadratic last term of (1). The evolution of x is, however, different. See whether this latter model may be an optimisation model.