Adaptive Dynamics Course
É. Kisdi, Oslo 2008

Evolutionary Arms Race and Disarmament

Consider a trait such as body size, which influences success in competitive contests with other members of the population. The evolution of such a trait is determined by two opposing forces: Being larger than the opponents is advantageous for winning the contests, thereby for obtaining resources and achieving high fecundity. On the other hand, large size entails some opponent-independent cost either in terms of resources used for maintenance or in reducing the chance for survival. In this project, we explore the evolution of body size (or weapon size or a similar trait) in a simple Lotka-Volterra type model.

Assume that the intrinsic growth rate, which measures population growth in absence of competition (i.e., at very low population densities), decreases with increasing size x due to the maintenance or survival cost of large size. The smallest possible value of x is then the "optimal" strategy, which produces the fastest growth and also the highest equilibrium density when it is the only strategy present. Whenever evolution leads to a larger trait value x^*, an *evolutionary arms race* occurs, i.e., the trait increases because being *larger* than others is advantageous even if being *large* is costly (just like in human arms races). In the examples below, we shall also encounter mutual *dis*arming when two strategies coevolve, along with other interesting coevolutionary phenomena.

Denote the trait value of the resident and mutant strategies respectively by x and y. The population dynamics of the mutant are given by

$$\frac{dN_y}{dt} = \left[r(y) - \alpha(y-x) N_x \right] N_y \qquad (1)$$

where N denotes population density. $r(x)$ is the intrinsic growth rate; because large size is costly in terms of reproduction or survival, we assume that $r(x)$ is a decreasing function. The competition coefficient, α, is a decreasing function of the *difference* in size. This is because large size by itself does not help winning a contest; being *larger* than the opponent does. For numerical work, assume the form

$$\alpha(y-x) = a + c\left[1 - \frac{1}{1 + v\exp\left(-k(y-x)\right)}\right] \qquad (2)$$

which has sigmoidal shape and saturates for $y - x \to \pm\infty$: if the difference in size is big, the larger contestant will almost certainly win any fights, and hence increasing the size difference further has hardly an effect on the outcome of competition. Note however that with $a > 0$, individuals feel some competition from smaller individuals even if the difference in size is big. This is because the resource may be exploited also without a fight, i.e., a small individual may find and consume a resource item without being noticed and challenged by anybody else.

To explore the adaptive dynamics of size (x), start by investigating the monomorphic evolutionary singularities. This can be done analytically, but illustrate the results also with PIPs. Construct isocline plots to explore the coevolution of two resident strategies. The following cases are worthy of investigation, but you can add others as well.

(i) Assume $r(x) = \beta - bx$ and $\alpha(y-x)$ as above, first with a=0. Choose parameters such that the population undergoes evolutionary branching. Construct the isocline plot for dimorphic evolution, and study the effect of increasing a on the pattern of coevolution of two strategies.

(ii) Assume $r(x) = b\left(\sqrt{x^2+d} - x\right) - \beta$, a convex decreasing function with parameters $b = 10$, $d = 3.5$, $\beta = 0.6$ and $\alpha(y-x)$ as above with $a = 0$, $c = 2$, $v = 0.7$ and $k = 0.24$. Show that there are several monomorphic evolutionary singularities, but eventually the population will settle on a monomorphic ESS even if it first becomes dimorphic via evolutionary branching.

(iii) Assume $r(x) = \beta - bx$ and $\alpha(y-x)$ as above with a=0. Construct an example such that an initially monomorphic population stays monomorphic, but an initially dimorphic population can evolve higher levels of polymorphism.