Computer practical 2

Download the Excel file AD_virulence.xls.

Important: enable macros (File tab → Options → Customize Ribbon → activate the Developer tab in the right panel; Developer tab → Macro Security → Enable all macros). This is necessary to use the pre-programmed functions

$$\beta(\alpha) = a\left(1 - e^{-c\alpha}\right) + b\alpha$$

for the transmission-virulence trade-off and

$$\rho(x) = A/\left(A + (1 - A)e^{-Bx}\right).$$

for the probability of successful superinfection (where *x* stands for the difference between the virulences). To use the function $\beta(\alpha)$, type beta([alpha],[a],[b],[c]), where [alpha] etc. is the coordinates of the cell that contains the value of α (e.g. B8 for [a]; remember to fix the coordinates when necessary). To use $\rho(x)$, type rho([first alpha]-[second alpha],[A],[B]). If you do not enable macros, then you have to type in the formulas for the functions each time they are used.

Exercise 1. The first worksheet ("functions") is fully programmed to show the functions $\beta(\alpha)$ and $\rho(x)$ (top two charts) as well as the equilibrium densities S and I of resident populations as functions of pathogen virulence (large chart). Briefly experiment with changing the parameters (in yellow cells) and investigate how the functions change. In particular, note that in function $\rho(x)$, parameter *A* equals to the intercept with the vertical axis ($\rho(0)$), whereas increasing *B* makes the function steeper. The latter corresponds to more asymmetric competition between strains (a small increase in virulence gives a large increase in the probability of successful superinfection).

Exercise 2. Take the parameter values $a = 30, b = 1, c = 3, b_0 = 1, \mu = 1$ as default. From the equilibrium density plot, find (approximately) the optimal virulence in absence of superinfection.

Exercise 3. The main goal of this computer practical is to obtain pairwise invasibility plots (PIPs) of virulence. In the worksheet "PIP", an empty table is set up for calculating the invasion fitness. The beginning of the table looks like this:

	0.2	0.4	0.6	0.8	1
0.2					
0.4					
0.6					
0.8					
1					

The numbers in the highlighted row (from cell I14 to AW14 in the worksheet) are resident virulence values, the numbers in the column (from H15 to H55) are mutant virulence values. Fill the table with the invasion fitness values.

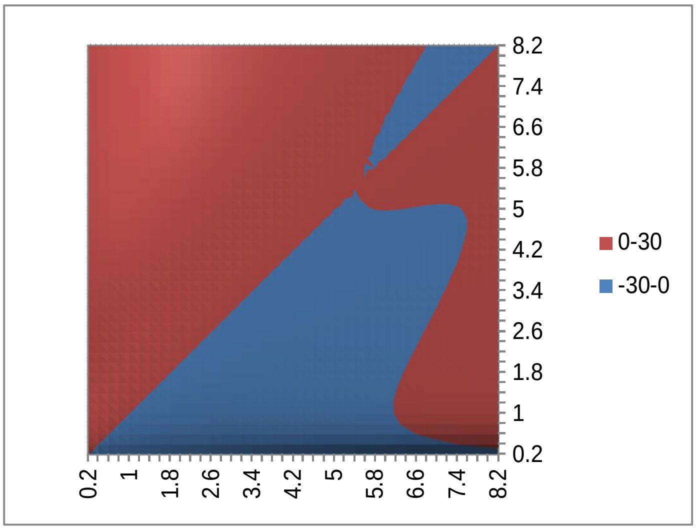
Hints: You will need the equilibrium densities of the resident population. To speed things up, these are already given in two rows above the resident virulences:

S(alpha)	0.0874	0.0655	0.0624
I(alpha)	0.7605	0.6675	0.586
	0.2	0.4	0.6
0.2			
0.4			

For example, in a resident population with $\alpha = 0.2$, the equilibrium density of susceptibles is 0.0874 and that of the infected is 0.7605. You can check the formulas in these cells to see that it corresponds to the equilibrium derived in the lecture. The same densities are shown in the "Equilibrium densities" chart of the "function" worksheet.

Remember to fix the coordinates for the parameters; for example, the value of a is always in cell B8, therefore this should be entered as B8. The resident virulence is always in row 14, hence it should be entered as I$14; this fixes the row (number) but lets the column (letter) vary when the entry is copied to other cells in the table. Analogously, the column must be fixed for the mutant, therefore it should be entered as $H15.

The chart on the left of the worksheet shows the PIP based on the data in the table. Technically, this is a contour chart of Excel (in the surface category of charts; it is a surface viewed directly from above), where the z-axis scale is manually set to minimum -30, maximum 30, major unit 30 so that the only contour line is at invasion fitness = 0. (With the scale left automatic, you would see also other contour lines; if you do see them, ignore them.) The horizontal axis stands for the resident virulence and the vertical axis stands for the mutant. With the default parameter values $a = 30, b = 1, c = 3, b_0 = 1, \mu = 1$ and $A = 0.1, B = 1$, you should see this PIP after filling the table with the invasion fitness values:



Exercise 4. Set A = 0 such that ρ is always zero. This means that superinfection is never successful, i.e., we recover the single-infection model. Verify that virulence evolves to the optimal value found in exercise 2.

Exercise 5. Set A = 0.1 and increase the value of B gradually from 0 to 1. Observe how the singularity shifts in the PIP, and explain why (recall that increasing B means increasingly asymmetric competition, see exercise 1).

Exercise 6. To better see the properties of the singularity, zoom in the neighbourhood of the singular point by changing the range of virulences on the horizontal and vertical axes (in the yellow cells of the table). Make sure that the two axes have always the same range. With the parameter values above ($a = 30, b = 1, c = 3, b_0 = 1, \mu = 1$ with A = 0.1), investigate at least B = 0.1 and B = 1 closely. Do you see a convergence stable ESS? an evolutionary branching point?

The remaining exercises are optional.

Exercise 7. Two strains of the pathogen (α_1, α_2) can mutually invade each other's resident population, and hence coexist, if $r_{\alpha 1}(\alpha_2) > 0$ (so that α_2 invades α_1) and also $r_{\alpha 2}(\alpha_1) > 0$ (so that α_1 invades α_2). Create a contour chart that shows which strains of the pathogen can coexist. *Hint:* with α_1 being the resident and α_2 the mutant, the existing table contains the values of $r_{\alpha 1}(\alpha_2)$. Create a second table for $r_{\alpha 2}(\alpha_1)$. A quick way to do this is to copy the existing table to the clipboard and paste it using Paste special → click Transpose. This converts rows of the original table to columns and *vice versa*; this is exactly what is needed. Next, prepare a third table where a cell is +1 if both previous tables have a positive element in the same position and -1 otherwise. The Excel syntax is IF(AND([first cell]>0,[second cell]>0),+1,-1). A contour chart drawn from the third table will show the pairs of α_1, α_2 that can coexist by mutual invasibility. Check that near an evolutionary branching point (found in exercise 6), there are strains that can coexist. Next, check the same near an ESS. If there are coexisting strains near an ESS, how will they coevolve? Based on the lecture, can you argue that these pairs are subject to stabilizing selection and therefore will evolve ever closer to the ESS (opposite of branching)?

Exercise 8. Investigate whether coexistence ever occurs with A=0 (no superinfection). Explain why it does or does not.

***** After the practical, disable macros in Excel. *****